1997 VALUATION ACTUARY SYMPOSIUM PROCEEDINGS

SESSION 10

Asset Modeling Concepts

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ASSET MODELING CONCEPTS

MS. ELIZABETH A. WARD: I'd like to introduce Dave Canuel, my colleague. We're both from Charter Oak Capital Management, an investment firm that specializes in insurance company asset management. David and I are portfolio managers for Charter Oak. Dave graduated from Amherst College, has an MBA in finance from MIT Sloan School of Business, and an MS in computer science from Polytechnic Institute. Before joining Charter Oak, Dave directed quantitative research for Aeltus Investment Management's fixed-income portfolios and later co-managed Aetna's pass-through and collateralized mortgage obligation (CMO) portfolio, as well as Aetna's U.S. Treasury and future hedging positions. At Charter Oak, Dave specializes in mortgage-backed trading and quantitative analysis. I'm a graduate of the University of Rochester and a Fellow of the Society of Actuaries, and currently I specialize in corporate bond trading and asset/liability management. Both of us have assisted Charter Oak's clients with asset models for use in asset adequacy analysis.

Dave and I will address some of the highlights of what is required when developing an asset model for asset adequacy analysis for a life insurer. Given that smaller insurers are being pressured to perform asset adequacy analysis, our objective in this session is twofold: first, to point out assumptions that need to be addressed by the asset adequacy model and to suggest possible assumptions and sources of information, and second, to help you to understand what your investment manager or your department is doing to come up with projected cash flows and market values for your portfolio.

Although it's not obvious why it should matter, the purpose of the asset model is an important determinant in how to model the assets. There's a difference between an asset model that determines relative value as part of a buy/sell decision, and an asset model that projects cash flows and market values as part of cash-flow testing. This is the main reason why many asset modeling systems that are rated very highly by asset managers are also evaluated as incomplete by actuaries and vice versa.

In modeling assets for appointed actuaries per one state regulation, we must contemplate, at a minimum, portfolio descriptions including quality distributions and types of assets, investment and disinvestment assumptions, sources of asset data, asset evaluation basis, and assumptions made for default costs, call prepayments, market values upon disinvestment, purchase yields, and explanations for assumptions that differ materially from a prior opinion.

Similarly, Actuarial Standard of Practice No. 7 calls for consideration of any perimeter which is expected to affect the asset cash flows. Basically, we need to project cash flows and market values for each asset held at the starting valuation date and held at each projected valuation date. The disinvestment and reinvestment strategies define the rule as to which assets are added or deleted from the asset listing at future dates.

Dave and I will discuss default costs, bond call functions, prepayment functions in some detail, investment and disinvestment assumptions, and since it's relevant to many more actuaries this year than prior years, equity-indexed call options.

Throughout the modeling effort, it's important that the actuary, or the professional on which he or she relies, has valid studies for developing the various assumptions that go into cash-flow testing. These assumptions can't be arbitrary, but must be supported with a credible and/or audible study. Similarly, approximations may be appropriate when modeling a portfolio with many issues or when a smaller company is just getting started in valuation actuary analysis. For example, despite all of the discussion about how many scenarios should be run, if you don't have your assets modeled properly under the required seven or eight scenarios, the marginal benefit to your analysis is not improved significantly by adding 1,000 statistically generated interest rate scenarios. Get the basics right first, then look to enhance your asset model. Still, approximations must be sensible, not just convenient.

Since we can't calculate market values without cash flows, I'll open with comments on cash-flow projections and work our way into the basics of market value estimation. At any future point in time, you as the actuary, need to be able to project cash flows off of your asset portfolio. Not only do the cash flows need to vary by interest rate, but they also need to capture the risk of default.

Starting with C-1 risk, I've seen defaults incorporated into an asset model as a static deduction from investment income, or as a function of interest rates. The static rates usually vary by asset type and quality, like they do in the asset valuation reserve's (AVR) C-1 calculation. The deductions that are a function of interest rates are typically tied to short-term rates, since this is close to the rate that a firm would have to borrow to cover debt payments. Also defaults may be a function not only of current rates, but also of the change in rates over some pre-determined period of time. The deductions or parameters must be founded by historical studies.

Based on discussions with other actuaries and investment managers, oftentimes default assumptions are derived from a report created by Moody's Investors Service or Edward I. Altman. Moody's Investors Service annually produces historical default rates of corporate bond issuers. The most recent edition was published January of 1997 and covers issue years 1920 through 1966. Edward I. Altman of New York University's Leonard Stern's School of Business annually publishes defaults and returns on high-yield bonds. Their most recent report is dated January 1997 and covers 1971 through 1996 defaults.

Reasonable default statistics on other assets are more difficult to come by. In the case of real estate or equities, a default statistic like those developed for use in risk-based capital and the AVR may be more appropriate. In this case, an industry factor is adjusted by company experience or a specific company's experience subject to a limit. Stochastic modeling is another approach to accounting for defaults. The stochastic of distribution, however, is debatable. Some models I heard have parameters that vary by age, quality, sector, and public versus private market.

Another comment on default risk and per the regulations is to include an appropriate allocation of assets supporting the AVR. These AVR assets may not be applied for any other risks with respect to reserve adequacy, however. The amount of the assets used for the AVR must be disclosed in the

Actuarial Opinion and Memorandum. For assets that don't have options or contingencies, it's sufficient to project cash flows under one interest rate scenario, however, bonds with options or other contingencies are dependent on the interest path they travel and the potential paths that they may cover.

I'll start with a simple example of bonds with options. In reality, calls on bonds are a function of the financial situation and the management of a firm. An issuer will call its bonds if it makes economic sense, and if the issuer has time to recognize the opportunity. Economically, an issuer will call a bond if he can reissue it more cheaply than just paying the current debt service. In reality, an issuer must consider the interest rate at which you could reissue debt and the expenses that he must cover to reissue the debt. The interest rate of coupon rate is a function of the issuer's current bond rating and financial position within its industry, the yield curve, and the economic environment. As occurs so often in this form of modeling, we limit the number of variables. We try to capture the main trigger of the asset cash-flow variation. Commonly, calls are modeled by comparing the price at which the company is contractually permitted to call the bond, with the then-current market price for the bond.

If the difference is greater than some stated margin which incorporates the expense of reissuing the bond, then they would call the bond. You can have a call function in which partial calls are also contemplated. Some of the vendor systems for cash-flow testing don't allow for partial calls, but it's important to capture the characteristics of the bond. A gimmick might be to model the potential partial calls as separate callable bonds. Also a simple modification to this methodology could cover voluntary sinking funds. In this situation, the issuer has the option to increase the amount of the annual sinking fund requirement; therefore, it's similar to a partial call.

Many bonds have call schedules such that the issuer may call the bonds at various dates with corresponding premium prices. These prices often grade down to par on or before the maturity date. The call function must address these premiums if this type of bond is held in the portfolio. Put schedules are usually modeled as basically the flip of calls; that is, instead of the call option being exercised, if the current market price is greater than the call price plus the margin, the put is exercised from the current market prices less than the put price plus the margin. The margin for the put exercise should be smaller than that of a call, since there is less of an expense incurred by the holder of a put bond than by the issuer of a callable bond.

The world would be a simple place if callable and putable bonds were the only options we had to model; however, since there is a yield premium on more complex options, there is a demand for more complex bonds. In this month [September 1997], the largest amount ($25 billion) of publicly issued asset-backed securities sold in one month is expected to be sold in the market. According to *Lehman Brothers Weekly Relative Value Report*, more asset-backed securities were issued in 1996 than investment grade corporate bonds. It's more likely that valuation actuaries will have to model asset-backed securities this year. Unfortunately for us modelers though, the structure of the asset-backed securities ranges from relatively simple to much more complex. Relatively simple might be when we look at credit card receivable collateral backed bonds; more complicated would be something like the new collateralized bond obligations or tax-lien-backed securities. After all, if there are cash flows, Wall Street can turn whatever it is into an asset-backed security; and that's if they haven't done it already.

In the early days of asset-backed securities, when credit cards and auto loan receivables were about all there was, projecting the cash flows was relatively simple. As these structures have become more complex, the need to pay attention to the securities details has grown.

What is the collateral? The collateral type determines the payment and/or type of prepayment schedule, as well as the default and/or delinquency risk. The structure of the bond is very important if your model is going to be accurate. What determines how much and when the bondholder gets paid? What are the early amortization events? These, combined with the tranche of the deal owned, affect the loss functions, the recovery levels (if there are any), delinquencies, and prepayments. Basically, an asset-backed security is a claim on a portion of a group of similar cash flows. For example, credit card receivable asset-backed securities are backed by a pool of current credit card bills, as well as the future bills generated by predetermined credit card holders. The cash flows associated with the credit card bills are then grouped and assigned to classes of bond holders according to the structure of the deal or their tranche. The pass-through structure of asset-backed securities paid cash flows on a pro rata basis. The pay-through structure of these securities arranged the cash flows by tranche or senior subordinated levels. Asset-backed securities are structures to insulate investors against currently foreseeable events. The rating agencies require certain protections that vary with the type of security for a tranche to get a particular rating. The rating requirements are very important to the issuers and therefore, to the structure of the bond. For example, 8% credit support might be sufficient for a prime pool to achieve a AAA rating. A wide variety of credit enhancement is used, such as excess spread protection, subordination, reserve funds, overcollateralization, letters of credit, and surety bonds. The credit enhancement typically varies by asset-backed type.

Credit card receivables and some other forms are further protected by payout events. Payout events are events that occur during the life of the security that would weaken the credit quality of the security. When one of these events occurs, the security begins amortizing immediately. The investor receives pro rata principal payments over the amortization period. This, too, protects the investor against principal loss or delay.

To evaluate credit risk on asset-backed securities, you need to consider the collateral characteristics and underwriting, credit support, the method prescribed for allocating defaults and salvage value and other features. Another element that is important to the credit risk of some asset-backed types is the servicer and trustee. In many cases the servicer manages the assets as the originator and the underwriter. Since the servicer must remit cash to the trustee, there is a risk associated with the time it takes for the servicer to pass on this cash flow. The difference in rating between the servicer and the receivables transaction affects the credit quality. An addition for default risk may be required depending on the comfort the modeler has; but the added collateral required by the rating agencies is sufficient to cover the credit risk. Also, as part of the servicing fee, a substitute servicer is willing to step in should the original servicer no longer be available or able. The risk associated with the trustee is that the trustee doesn't forward the payments to the bond holders or certificate holders. Some of these factors must be incorporated into a model and others may be simplified. It really is a function of the asset-backed security that you're trying to model, whether or not some of these varying elements are necessary to be considered in your model or not. Some of the newer and riskier securities do require looking at the servicer.

To protect cash flows, consider prepayment rates for the collateral, underlying collateral, interest payment conventions, terms of payment to other tranches and other features of the receivables. Clearly the prepayment rate is important in the case of securities such as auto loans, where the cash flows are paid to the investor as they are received from the loan holder. However, the expected prepayments are more or less of a concern depending on the collateral. The option to refinance is less valuable in a consumer loan -- an asset-backed type of loan we're seeing issued quite often this month -- than in a mortgage.

In the case of other receivables, the interest payment convention is also important. Auto receivable contracts are often amortized according to the Rule of 78, as opposed to what in the business they call the actuarial or compound interest method used for mortgage-backed pass-throughs. Under the Rule of 78, the interest portion of each loan payment is calculated by multiplying the total scheduled interest by a factor that's based on the number of payments. In the case of the second payment in a one-year loan, the factor would be the number of payments minus the current payment plus one, divided by the sum from one to the number of payments. In this case, it would be the second payment in a 12-payment one-year loan. Twelve minus two plus one, divided by the sum one plus two, plus all the way up to twelve, the denominator adds up to 78. Now you remember why it's called the Rule of 78.

Clearly the terms of payment to other tranches are also important. As in the case of tranches of a CMO, funds don't always flow sequentially from one class to another. Examples of other items of varying degrees of interest to an asset-backed modeler are the presence of a clean up call, the servicing fees, and the amortization of principal payment. The only good way to project cash flows is to review the prospectus of the deal, except in the case of the most liquid sectors of the asset-backed market where the deal structures follow convention and relevant statistics are very similar among most issuers. You need to read the prospectus to understand exactly how the dynamics of the cash flows work. It never hurts to verify that the deal is not an exception. Each year an assortment of new structural features is introduced to the market, also. So you probably want to consult your investment manager to identify the assumptions that need to be made and what the triggers are to the variations in cash flows.

In the case of some types of asset-backed securities and in most cases of mortgage-backed securities, prepayments are the items that must be modeled correctly if you're going to project the cash flows and come close to calculating the market value correctly. Dave will describe a prepayment model and how it might be modified to capture the prepayment risk embedded in some of these securities.

MR. DAVID E. CANUEL: I'm going to be talking about the steps that you would undertake in implementing a model to forecast prepayments. I'll be presenting the functional form that we use. We model prepayments as a sum at its most basic level of two things, what we call demographic and then rate-related refinancing factors. When no refinance incentive is present, we refer to those as demographic prepayments. These occur when people decide that they want to move to a bigger or smaller house. They change jobs, or they move to a different city. It's the type of thing where there's no rate coming into play explicitly. In regard to prepayments on pools where there is a refinance incentive, we say that those pools are typically trading at a premium in the market because rates are below the coupon rate. Those are "refi's." Generally what people are doing there is taking out a mortgage at a lower rate and incurring whatever expenses are necessary for them to do that.

The way we model these is we take the discount or the demographic prepayment and we model those separately using a subset of the data; then we combine the two data sets together when we model the full prepayment function, which includes the premium or the rate-related prepayment. Because, as you can imagine, even for the rate-related prepayments, there are demographic prepayments that are occurring simultaneously. So both of those are occurring simultaneously in the data set where there are rate-related refinances.

We use a nonlinear, least-squares regression package. I use one called EVIEWS which is nice because typically, in something like SAS, you have to enter the partial derivatives of the function that you're estimating. I can't remember the last time I got all of those right. So EVIEWS does that for you and tests numerically for the direction that the gradient is supposed to go. I find that a lot easier.

We do separate regressions for 30-year and 15-year collateral. We separate out conventionals and balloons. The prepayment functions are obviously different for all of them. We use a data set that looks at production year by coupon, by month going back to the early 1970s.

First, when looking at the demographic refinances, we could see several components to it. Essentially, the first one, and this is a little bit counterintuitive, is to see that there's a rate-related component. I had just said rate related refinances are when you have some sort of economic incentive to go out and refinance your mortgage. If you think about people changing jobs and so forth, even when they have a coupon on their mortgage that is below the current market rate, then, to extent to which the coupon is below the current market rate, it provides people with a relative disincentive to move to a bigger house or a smaller one. They're willing to stay in a nonoptimal situation when they have an economic incentive to do so. What we find is that rates do play a role in this sort of prepayment. It takes the form of a double exponential, where this is measured as the difference between the coupon on the mortgage and the current refinancing coupon in which we take as a point and a half above the current seven-year rate.

The second thing is, although there are some refinances that are due to the people in the homes passing away, there's also a relatively important seasonal component. Seasonal prepayments are the biggest variation month to month in discount or demographic prepayments. That varies by up to

20%. The prepayments in June and July are 20% above the average for the rest of the year; January, February, and March are about 20% below the rest of the year. That's a pretty big harmonic there.

There are some other factors that get a bit more obscure as we go on; but seasoning refers to the tendency of relatively new mortgages to pay pretty slowly. That's based on the frequency with which people want to undertake changes in their lives. They just moved into a new house, and they don't necessarily change right away. That thing is really going to take about two to three years or maybe longer to work through. The prepayments during the first couple years of a mortgage's life are very low, say 20% of the eventual demographic level in the first year, then it's 40%, 60%, and it moves up to where the mortgage is fully seasoned somewhere after three years and four years. If you think about what happens after three or four years, in a typical case, you can get probably 10% more appreciation or you have some paydown into the mortgage or a combination of those two things.

Finally, we have a couple more obscure things: partial prepayments and then dispersion. Partial prepayments essentially proxy to the fact that mortgages, which prepaid very rapidly when there was a refinance incentive, tend to do so even when that refinance incentive goes away. There are possibly a lot of reasons for that. Maybe people are in the habit of prepaying, but a lot of it has to do with the fact that, as you may know, when you prepay your mortgage partially, that's a curtailment. It takes time off the end of the mortgage; but you still owe next month's payment when next month rolls around, refinance incentive or not. What actually happens is when people partially prepay and then they return the schedule, subsequent scheduled payments actually are prepayments. This is capturing some of that. It's measuring previous curtailments if you will.

Finally, dispersion reflects the fact that in any pool of mortgages, there's a distribution of mortgage ages and mortgage coupons. As the mortgage is paid down and gradually approaches the end of their lives, what you tend to see is that the factor starts to matter a lot, and people start to clean up their mortgage balances and that type of thing. What we're calling the factor there is the zero prepayment factor on the mortgage; it's what the outstanding balance (in a percentage) would be if the mortgage hadn't paid down. As that starts to approach zero, we have a little positive coefficient out in front, which becomes a more important consideration.

What I'm going to do now is show you what some of these terms would look like. Chart 1 is the seasoning, and that double exponential on the rates in Chart 2 is actually a flexible form. You see that a lot in prepayment functions, and you'll see it a lot in the functions that we produce. Depending upon the values of the coefficient, it can be made to look a lot of different ways. In Chart 1 is the seasoning, and it starts out low. After a period of three years, it ramps up to a coefficient or a factor of one. The strength of the coefficients on these matter, and that's the whole point of doing the econometric estimation.

Chart 3 shows what the whole thing looks like combined. This is for a pool with a moderate disincentive. I've shown a couple of them there, a 25-basis-point refinance penalty, and then a 75-basis-point refinance penalty, and it shows the seasoning effect. The rate effect is shown by the level of the graph, and then the seasonal is in there, and you can see that that's the most pronounced factor. It gives you more of a sawtooth pattern throughout the year. This is only for the first nine years of the mortgage life, but there's a modest upward slope to that harmonic, and that continues through the life of the mortgage simply due to those last factors: the dispersion factor and then the partial prepayments that occur through the life of the mortgage, if people double up on their payments for one reason or another.

The more interesting part of this and the part that comes into play when you're doing cash-flow testing is the modeling of prepayments that are related to rate. You can note that the double e exponential function is involved again. The coefficient out in front is typically large and positive. It's 85 or 90, which really reflects the absolute maximum percentage prepayment rate you could get in a year if the situation was ideal and there was an enormous incentive to refinance. Then both data 2 and data 3 in that function are less than zero, and those are the kinds of things that determine the shape and position of what we call the refinancing S curve.

CHART 1
Seasoning -- Discounts

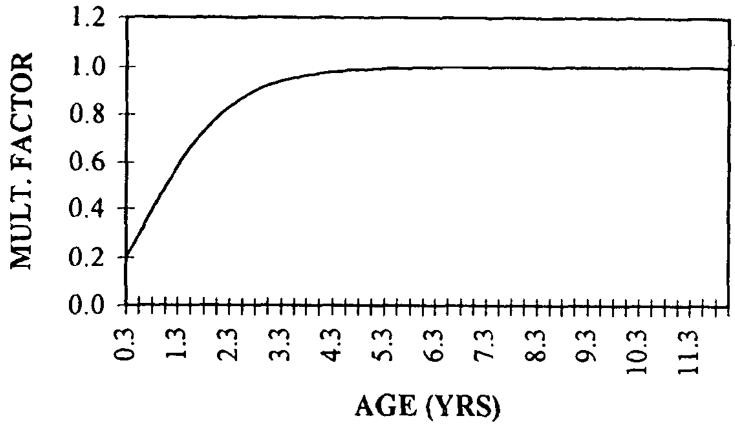


CHART 2
Rate -- Discounts

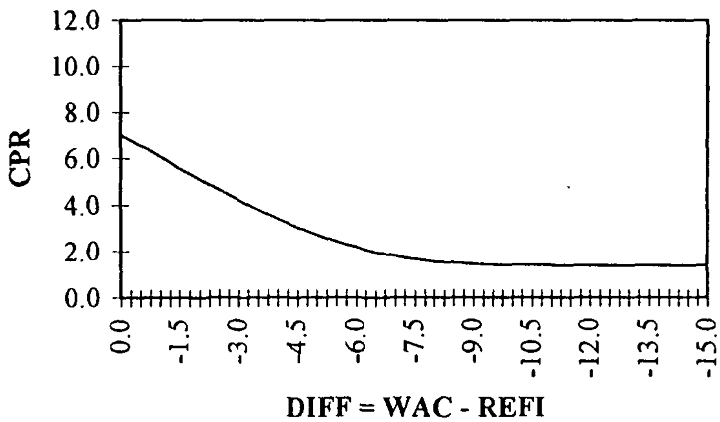
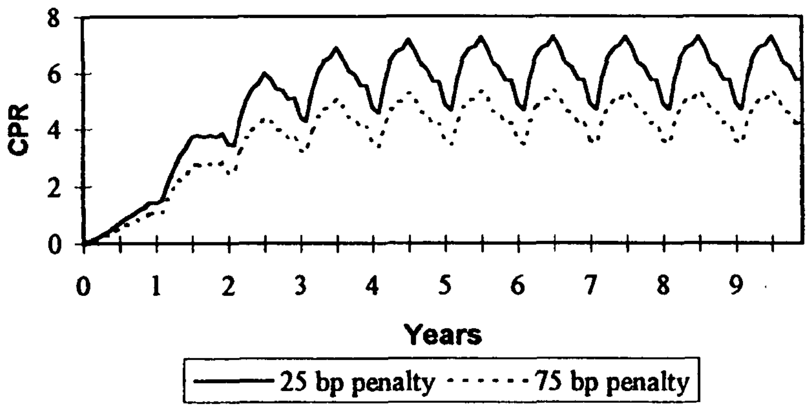


CHART 3
Demographic Prepays



In another double exponential financial, the variable is now measuring a positive number between the coupon you're paying on the mortgage and the current refinancing rate, and then you have negative numbers on beta 2 and beta 3. It fairly accurately measures the way refinances occur when there's a positive rate incentive. There's a gradual "fairing in" at first, and then as you get what we call "in the money" by 100 to 150 basis points, the refinances pop, and they increase very rapidly until you get to 300 or 350 basis points in the money. Then they are at a very high level, and they stay that way.

There are a few other pieces in that function which matter. It's a fairly complicated function. It is not just the absolute difference, but we also try to capture the net present value incentive for people to refinance. So we take that difference between the current mortgage rate and the coupon people are paying on the mortgage, and take that as a percentage of the current mortgage rate. You're looking at almost a perpetuity there. We find that gives us a better fit. Then there are a couple other pieces that reflect the recent change in rates. Those are D up and D down. What we find is that if rates have increased recently and there's a positive incentive to refinance, everybody hurries up and says let's lock it in and take the money and run, so they do a refinancing. What we also find is that, if there's a refinance, but rates are headed down, people say let's wait a little while and see if we can save a little bit more. So they don't refinance. What you basically see here are greed and fear operating right next to each other in the prepayment function, and they both end up with significant coefficients.

Finally, steepness plays a role, where people in 30-year mortgages refinance into balloons or ARMs, which both have shorter terms, so what one would expect to see is that, when the term structure is steep and there's some possibility of saying I'll go from my 30-year into a 15-year balloon and keep my payment the same, then that's what you find people doing.

Finally, the final variable is what we call LOW. It just says that if in the past this mortgage pool had some refinance incentives, generally some of that goes away through subsequent refinancing waves, through a thing we call "burnout," and what it really takes is for rates to hit a new record low for the life of that pool to achieve the same kind of spike in refinances that you had seen before. This captures the extent to which the current rates are below the previous record rates experienced by that pool. It's a media effect. I know you've probably seen over the past few years how mortgage rates hit a new low. You've seen in *USA Today* the stories about refinances. This more or less proxies for that type of media attention, and that type of behavior on the part of homeowners is in response to it.

Finally, there's a season factor which is like that in the demographic prepayments. It's something that fairs in gradually. It's a little bit faster for rate-related refinancing in that it's not related to moving or other life changes; it's just a matter of going out and refinancing your mortgage. Then there's what we call burnout, which is that tendency for what people used to call the hot money, to all refinance, leaving people who will never refinance still in the pot. I don't think that is necessarily the case. I think what you are seeing there is people with big incentives, they have high balance mortgages to refinance, and then other people are left in the pool, those with smaller balances, so that the present value of refinancing is just not as big, because there's a dispersion of balances in the pool as well, and also people who've experienced some kind of problem. They actually might have burned down a corner of their house or something and the value of the house isn't as high and so they couldn't refinance. Perhaps they've lost their job, so they don't meet the income requirements. There's all this going on in these pools. In particular, in a place like New England during the early 1990s, you saw everybody with a house under water, but strangely enough they weren't refinancing. It wasn't because they were stupid; it was because they were smart.

Finally, aging is also a factor in rate-related refinances. What you tend to see is that as the variable *V* factor goes down; again it's the zero prepayment factor for people who have not prepaid at all. What you tend to see is, as that factor goes down, the pool becomes less sensitive to rate in terms of the refinance. So since that *V* factor variable is less than one, something that compounds that would be a coefficient greater than one.

We've discussed the rate-related piece. There's the seasoning piece, and then the burnout piece gradually decreasing through time. To illustrate this, I have our model's forecast compared with actuals (Chart 4). The boxes on the right show that variable DIFF, which measures the distance between the current refinance rate and the coupon on the mortgage. The dotted line is showing actual prepaid, and then the solid line underneath is showing the forecast. You can see that we do a pretty good job in our model of capturing that initial ramp up in prepayments. We're keeping the forecast low even though that variable DIFF is rising, but then, once you hit the real peak in refinances in 1993, we fall apart in that and so do a lot of prepay models. This is a standard thing to see; you underestimate that peak in 1993, and you do a lot better than that in the most recent decline in rates which started in 1995 and has carried through since that time, with the exception of a period in 1996.

CHART 4
Sample Performance of Model

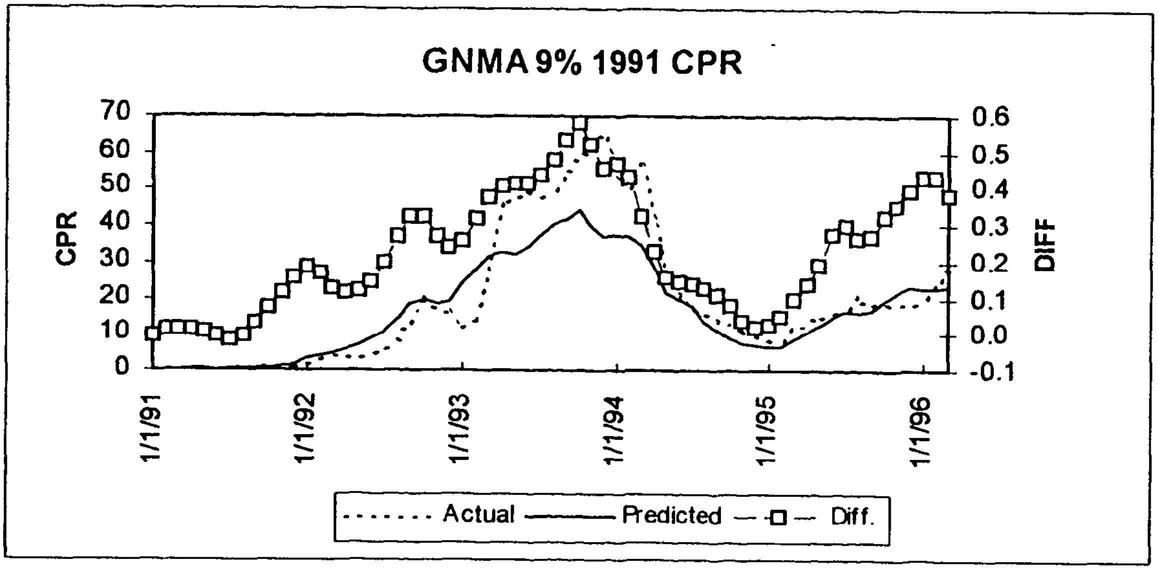


Chart 4 is showing some of the strong points as well as the weak points of prepayment models in general and looking at investing in mortgages. You don't know what the weak points in your analytical methods are. In 1991-92, it wasn't forecastable that models were going to do such a poor job of capturing actual prepayments. But how do you avoid problems like that? My main job as a portfolio manager is to avoid situations where model weaknesses of this type can hurt you. So I try to avoid things like current coupon mortgages and the type of thing where there's a lot of leverage to a particular prepayment. It points to what I think the real risk in the mortgage market is, not that a certain rate scenario will produce certain prepayment outcomes via the forecast, because that's something you can hedge against. If a bond is a problem for you, what do you do? Just sell it, no problem at all. The real problem is that you misestimate what your hedges are going to be because you're using a model that has some weaknesses, and I think every model does. That's the risk that is there, but isn't captured via any cash-flow testing or really any other forward-looking analysis that I know. Let's just say, through time, we're doing a much better job of forecasting prepayments than we did in the past, but I think anybody who tells you that all the questions are answered or all the potential surprises are going is really just pulling the wool over your eyes.

I have a little bit more detail on modeling prepayments via the *S* curve. I'd like to say first you have those three main components in the heart of the prepayment function: beta 1 governs its total height, beta 2 governs more or less it's position, and then beta 3 governs its steepness. Those things are rough characterizations of what those coefficients accomplish, but that's generally what they do. As it turns out, measures of value, like option-adjusted spread and effective duration are highly sensitive to those.

I'll describe what changing those coefficients does. These are relatively moderate changes. One of the problems that you encounter when you do this type of modeling is that pretty modest changes to one part of your prepay function, like the discount piece, can produce changes of some magnitude in your S curve. It's not as though you're modeling a surface that looks like a valley where those coefficients lie, so that every time you do the estimation you home in on those points. It's much more of a flat, very uneven surface, with a lot of local minima. You arrive at those, but you could arrive at some other ones that are close, but not the same, and they can result in fairly material differences in your prepayments.

Changing beta 2 can move the lower piece of that curve in and out and essentially affect your refinance sensitivity at relatively low levels of rate incentives, whereas changing beta 3 also changes the low level refinance incentive. It does, however, make the curve a lot steeper and can affect your prepayments at higher levels of refinance incentives.

With moderate changes in those coefficients, I have used our prepayment model in what we would call the base case or the way we estimate it and made some fairly modest changes to it. We dramatically changed not only the levels of effective durations, the levels of option-adjusted spreads, and the forecast prepay rates, but these were also changed in different ways for different coupons, depending upon how close they are to the prepayment costs. Those things definitely matter. Aside from those technical issues, there are a lot of reasons to believe that the coefficients don't stay in the same place through time, they change a lot. All you need to do is think about, over the past several years, how the mortgage banking industry has gotten so much more efficient than it used to be, and how they go calling you and how your banker calls you to refinance. You can look at the servicing fees on mortgages. They've gone from 75 basis points down to 40 basis points now. The mortgage banking industry is tremendously competitive.

There is a much wider dissemination of refinance opportunities in the 1990s than there was in the late 1980s. The advent of shorter duration mortgage alternatives increased the steepness of that S curve and made that little steepness variable I told you about before more significant. It used to be you didn't even need to include that, but now you do. Including these things in the model is important, and even for things like the seasoning variable and the burnout variable, we find that mortgages season a lot faster than they used to and they burnout a lot more slowly. For all those coefficients that I told you about, what we have in there are actually dummy variables that proxy for the time period of origination of the mortgage. We're modeling separately the more recent issuance so that we can get a much better sense for the new mortgages versus the old ones and the different sensitivities applied to the two of them.

I think this gives you an idea of some of the pitfalls that you face, and this is for the stuff on which you have data available. There are a bunch of things that are bought all the time by portfolio managers, including me, where the prepayments are a lot more nebulous. Common ones are jumbo loans. There are a lot of places in the country where you can't buy a house for conforming mortgage prices. There are a lot of expensive homes in Atlanta. Larger mortgages are pretty commonplace, and California really is the place where most of those are concentrated. Those mortgages prepay a little differently. Other nonagency issuance, like "low doc" and "no doc" things don't conform to agency underwriting standards. There are investor-owned properties, that is, people who go in and say I'd like the mortgage, but I don't want to tell you how much money I'm making. That happens all the time. People who have a large cash income for one reason or another can get mortgages but they pay a little bit higher rate. There's a continuum from there into home equity.

Home equity used to be sort of second liens on homes, but now it's really first liens for people who are referred to politely as being credit impaired. There are programs designed to write mortgages for what they are calling B or C borrowers. That's really what the home equity market has metamorphosed into. Those people prepay very differently.

Then finally there are manufactured housing, commercial mortgage-backed securities. For commercials definitely, unless there's a lockout in the loan, there's potentially a rate incentive there. The strategy for modeling the prepayments is going to vary by the type of mortgage that you're looking at, but one problem is that data availability is pretty much an issue for these things. A lot of the brokerage house prepayment models for jumbos and nonagency mortgages are derived through proprietary relationships that they have with the mortgage issuers. Prudential with Pru Home is an example. They're not going to give you all Pru Home's prepayment data. In some cases, you can adjust the prepayment parameters on your existing agency model and you can tweak it. I laid out what they were for you, but based on your intuition, you can figure out how things ought to change, and you can make adjustments on your own. As you move from out of 30-year and 15-year mortgages, that gets to be a lot more problematic.

In the case of jumbos, that's actually a fairly easy one to do. The rate sensitivity is going to depend a lot on the average balance of the loan, but if there are loans that are $200,000, $250,000 and up, they'll prepay like classic Citicorp, Pru Home, or that type of jumbo deal. Those will tend to exhibit a couple of characteristics. One is that beta 3 term is going to be more negative. There will be a lot more sensitivity to refinance at high levels, and that popping up as the mortgage goes in the money is going to be a lot more pronounced. At the same time, the refinance incentive will disappear almost completely when that option goes out of the money, and the base level of prepayments will be low. When there's no refinance incentive present, people with jumbo mortgages tend not to have a real high demographic prepayment. They're already in the home of their dreams, why move? In general you can say these mortgages are going to season more rapidly, so you can adjust that parameter in order to dampen that out. They'll tend to burn out much more slowly, so you can even get rid of that parameter completely and completely eliminate that from the prepayment function.

How do you actually tell whether the changes that you've made in your prepayment function are correct? One way to validate it is to go back in and say I'm going to model a few deals and see if the option adjusted spreads and effective durations that I come out with are roughly comparable to what I would get in the agency market plus say 15 basis points, considering the fact that I'm in something a lot less liquid. You can model a few deals that way and say I have something that makes sense to me intuitively, and then also make sense from a quantitative perspective on the way out looking at valuation measures.

When you get to nonstandard or nonconforming loans, typically the prepayment rate and sensitivity is going to depend a little bit on the purpose of the loan and type and credit quality of the borrower. People of lower credit quality will tend to exhibit lower rate sensitivity, but for a lot of those subcategories, it's hard to tell exactly what the credit quality of the borrower is. In general, they tend to exhibit lower rate sensitivity. They tend to exhibit higher demographic prepayments, almost twice what you get in an agency. They tend to exhibit more rapid seasonings, 12 to 18 months rather than 30 to 36. The loan size is very important. Lower loan size decreases that data 2 and data 3 and reduces the rate sensitivity. High loan-to-value ratio tends also to dampen prepayments. That changes through time. If you have home appreciation, the loan to value ratio is going to go down anyway.

Finally, I thought I'd talk about commercial mortgage-backed securities briefly. They're a class unto themselves. In my view, the only appropriate way to model those is on a loan-by-loan basis. You ought to reflect the lock-out and make whole provisions in the bonds. As far as I'm concerned, the best approach to take there is a stochastic distribution on the values of the properties, so you can tell either they're going to send you your check with your mortgage payments, or they're going to send you an envelope with the keys in it. That modeling of the distribution of property values will reflect that tendency for the default option to be exercised a little bit efficiently on those properties, too.

As the owner of the higher-rated tranche in a commercial mortgage-backed security deal, you do not get the keys in the mail, but those defaults and foreclosures and then recoveries are seen by you as prepayments. There is an economic impact, and it's really a debatable issue as to what the correlation of that phenomenon is going to be with the level of interest rate, but there is one. Then the call options would take place when there's a positive refinance incentive. Typically, I think the most reasonable assumption is something pretty close to efficient exercise.

MS. WARD: A question that we were asked to answer is how might an actuary model reinvestments or disinvestments. The issue of reinvestments is very important; because when we're talking about projections that go beyond one year, which these all do, reinvestments become a significant part of determining whether or not you really do have the cash flow on hand. Whatever assumptions you make, they are almost overwhelming versus some of these other assumptions.

As stated in Actuarial Standard of Practice No. 7, a reinvestment strategy should reflect current investment strategy. In addition, I believe it should be relatively simple. For example, unless the structure is an integral part of your investment strategy, I often omit CMOs from my reinvestment strategies because I don't want to have to guess what CMO structure will be available in a future market, nor exactly how the cash-flow dynamics are going to work. Unfortunately, the reinvestment strategy does have a significant impact on asset adequacy analysis, and we really do have to review the implications of any assumptions or simplifications.

Some common reinvestment strategies that I've seen call for reinvesting in a particular mix of securities, particular durations, a strategy that maximizes yields or durations or a combination such as a mix of securities that has a predetermined duration or that causes the total portfolio to remain at a particular duration. In any case, you also need to make sure that your strategy doesn't cause you to purchase too much of one particular asset type. Although it isn't necessarily something that you would think of first when looking at your asset adequacy analysis, if you have even too much of corporate bonds in some states (if you own 100% in noncallable corporate bonds), you've violated the investment regulation and therefore, 40% of your portfolio would be called a nonadmitted asset. That also needs to be considered as part of your reinvestment strategy.

In all these situations, the model must contain expected future market prices on the securities; basically, the value of the security should equal the sum of the present value of the zero-coupon components. It's probably best to reduce the security to its cash flows or component cash flows, and then take the present value of each of those cash flows.

To calculate the market value of these instruments, we need the cash-flow projections from the then valuation date in the future or forward spot yield curve, plus the yield spread applicable for the securities to discount the cash flows. The cash flows are projected as we described before. That is, if the security has options or contingencies, we need cash flows under varying interest rate scenarios. For each of these varying interest rate scenarios, we must calculate the spot yield curve. The spot yield curve is essentially the yield on a zero coupon that has the same maturity as the security's cash flows. We will have a discount rate for each cash flow rather than just one rate.

The spot rates can be derived from the projected yield curve, just using basic arbitrage arguments. First we're given the yield curve, and this is usually stated in bond equivalent terms; remember, that is compounded semi-annually. Be sure that you know in what form your system expects the rates to be. Since Treasury bills, which are Treasuries of maturity less than or equal to one year, don't have coupons, their yield rates merely need to be converted to annualized form.

As an example, I'll describe how to calculate the one-and-a-half-year spot rate. The one-and-a-half-year Treasury in this example has a 5 7/8 semi-annual coupon. The projected cash flows are simple. If we discount each cash flow by a zero-coupon rate by rules of arbitrage, the price must equal the present value of the flows. Therefore, we have one equation and can solve for the one-and-a-half-year spot rate, which is the foundation of our discount rate. We must add the appropriate yield spread to capture the risk inherent in the bond over that of the U.S. Treasury.

In the case of noncallable bonds, yield spread statistics can be developed from databases of Wall Street indexes or studies. The problem with using these studies and calculating an average spread matrix for each asset class maturity and quality ratin is that the current environment may vary significantly from the exposure period. That is, if you blindly use historical yield spread data, without weighing the current environment more heavily, you may end up with lower market values than currently exist. That would be the case if you were to use a current study, since current yield spreads are lower than we saw in the late 1980s and early 1990s. Using these studies without adjustment would not only cause a sudden blip in the asset portfolio's market values, but also a sudden decrease in the cost and an increase in the yield of the reinvestments. That's not desirable, and is not what was intended when some of these yield spreads were picked.

Some vendor systems try to aid the user by allowing interest rate spreads to be a function of current interest rates plus a constant. The issue then becomes not only defining the static spread, but also what is the dynamic solution between interest rates and yield spreads. Recently, a number of relationships that were thought to be true due to their high correlation between spreads and interest rate levels have been in essence, disproved, because now the correlation is so low. Although some of these relationships are still believed to be true or rather they're still included in models, I really think that you need to validate your assumptions regularly to knock out some of these assumptions. There are basic assumptions that when interest rates start heading up, we're going to see yield spreads exceed GAAP. Interest rates have gone up a bit, and we haven't seen the yield spread move an inch.

As discussed before, cash flows and securities with options are dependent on their interest rate path. A common simplification is to use option-adjusted yield spread. The option-adjusted spread represents the single yield spread applied to all spot rates under all the scenarios, such that the probability weighted, discounted cash flows equal the current market price. In other words, the option-adjusted spread is the spread added to the Treasury spot rate, and then used to discount the securities cash flows. The option adjusted spread levels also may be derived from historical studies.

For more information on option adjusted spreads, I recommend that you read a book like *Introduction to Option Adjusted Spread Analysis*. It's available through Bloomberg at no cost. The *Handbook of Fixed Income Securities* gives a good introduction to option-adjusted spreads, also.

Gliding is not appropriate. Assuming that an option price starts at the current market price and is just going to make a termination value, and it follows some curve to get between those two points, is really not accurate. You need a model that produces option values at all points in time.

A disinvestment strategy is important if funds are required to cover negative cash flows at any point in the projection period. Determining which securities are liquidated first can be tough if your investment strategy does not mention this topic. Some common methods are borrowing, purchasing negative assets, and selling assets according to some rule. Sometimes the disinvestment rule is obvious as a result of the financial statement of management. The order of priority ranking of disinvestments must be considered, since a security's liquidity may make the sale a slower, longer process, thereby increasing the transaction cost of the sale. Liquidity drives down the price that you can get for your own assets. Also liquidity levels change with market conditions, asset structure, and the relative size of the transaction.

In 1993, some CMOs were hard to sell at what had been fair levels just the year before. Generally, the more complicated the asset structure, the lower the level of liquidity, since it takes more sophistication and time for the buyer to understand the value of the bonds. The relative size is a factor for a couple reasons. By size, I mean the size of the issue as well as the size the you actually own. First, the sale of a $200 million piece of a particular security might change market prices, since there are a limited number of investors available to purchase that bond in that quantity; even if you break it down, it may take time to sell the bond and the broker/dealer wants to cover the market risk while he owns it.

Also the liquidity affects the market price if the dealer amount being sold is very small. Some investors will not invest in issues that are below a certain size. Similarly, there is less liquidity if you're selling less than $1 million of a bond than if you sell more than $1 million. There's a similar liquidity gradation at the $5 million mark.

Regardless of the rules chosen, liquidity affects the market values. Even if you don't need to sell assets to cover projected liability cash flows, many regulators do want to see the market value of assets at future valuation dates. That also comes into the issue of what we are talking about with the market value on a bid or on an offered level. I expect that the market value should be that of what the market's willing to pay for it, as opposed to what you actually think it's worth.

With this in mind and given that we can't cover all asset types, we'd like to go through one more asset modeling example that may be of interest to a growing number of you. How do you model some of the portfolios required to support an equity-indexed product?

MR. CANUEL: In my opinion, this problem is one that is of broader applicability than equity-indexed annuities alone. It's the problem of modeling cash flows from a perspective of modeling for doing cash-flow analysis. We're looking at modeling securities because cash flows are contingent upon some risk variables other than the level of interest rates. In equity-indexed annuities, we're looking at an asset payoff that is going to depend upon the performance of an underlying stock index. Maybe it's not linear, maybe it's just saying if Standard & Poor's (S&P) outperforms a certain strike price, you get a coupon of eight, if it underperforms it, you get a coupon of zero and your principal gets deferred. That's not a linear payoff with the S&P, but it's definitely a contingent claim on the S&P. It's going to arise in a variety of circumstances.

I've been in situations where currency linked notes -- where the payoff was related to dollar Yen -- were used. I've seen European yield curve notes where the payoff was related to a basket of European yield curves, where we looked at the lira, etc., and looked at how the steepness of those varied. You got a larger coupon if they got steeper and a lower coupon if they flattened out, which they did. The problem there is not in establishing the relationshipos between the value of the risk variable and dollar interest rates. All you need is a data series, and you can come up with any correlations there. You can plug that into the interest rate scenario you're using, and you can get a deterministic relationship out the same way you can in mortgage prepayments. The real risk is not in that deterministic relationship in getting it right. There's an error term there, and you're really not sure how large that is and how it works. That's the thing that really is the problem here. If it has a correlation of 0.2 with interest rates or whatever variable, it has no correlation. If it's low enough, you might as well just say it's zero.

Once you establish that relationship, if that's what you want to do, you can very easily incorporate a terminal asset value in your cash-flow projections and model the payoff of that security. The real solution to this problem is looking at a distribution of outcomes and supplementing those eight scenarios that are based on interest rates with stochastic analysis. You would specifiy what your volatility terms are and the correlations between your two distributions. You get out a set of distributions for your asseets, either cash flows or probably I would use a total return over year horizon and say compare that to my liabilities and see as a supplement to the cash-flow analysis, what I get out in terms of a distribution on surplus.

I've done this for somebody that we used as a prospect, where I looked at their asset portfolio and then their liability portfolio. I was able to generate a distribution for their surplus, and it was what I would call the economic value of surplus and how that would change over the course of the year based on some changes that we were recommending in their portfolio. The disaster here is actually the low risk results. It's not what they had in the first place.

Especially when you're looking at things where you have a second or a third or a fourth set of risk parameters, like you would have in commercial real estate at play, that's going to produce distribution of outcomes for you that are not going to be captured in any really deterministic way. This is the direction that I would move in modeling the risk associated with something like the stock-related instruments that are backing up an equity-indexed portfolio. In the case of a life insurance company general account or a property/casualty general account, the things that you have in there include commercial mortgage-backed securities, including currency notes, European bull notes, and anything of that type.

MS. WARD: Throughout all of the modeling it's important to keep adequate documentation so that, as you're working to try to define your asset model or an appropriate asset model for some of these newer security types that are included in your asset model, you have a good foundation for why you've done it the way you have. With a number of these asset types that are newer, including asset types that are backing particular liabilities, there is no one correct way. Unfortunately for us, the asset modelers, we can't just open up a handbook and say this is the way I have to handle it. We do have to be creative, and turn this more into an art than a science.

It is absolutely important to keep adequate documentation. Sources in addition to the Valuation Actuary Symposium that will help you and your invvestment department in documenting your asset model to the satisfaction of regulators can be found in the National Association of Insurance Commissioners (NAIC) Actuarial Opinion and Memorandum, Actuarial Standards of Practice Nos. 7 and 14, the Practice Notes, the *Dynamic Financial Condition Analysis Handbook* and even from upcoming Society of Actuary seminars like the Dynamic Financial Condition Analysis.

I also remind you that such documentation sufficient to determine the procedures followed and results generated must be retained for seven years. Now, I'd like to open the discussion up for any quesitons.

MR. DOUGLAS C. DOLL: You made a good point when you were talking about the mortgage prepayment models. One of the risks that we can't quantify is the risk that the model is long in its underlying assumptions. The conditions may be different in the future, and we saw that in 1993. Under our cash-flow testing that we do for reserve adequacy purposes, we're supposed to be testing for moderately adverse conditions. Has somebody ever come to you and said, I'd like to model moderately adverse conditions for my mortgage prepayments? I had a situation like that at a company for which we were using an outside company to do the CMO projections. The results were very sensitive to the prepayment function. I asked them to give me moderately adverse prepayments, but they weren't used to addressing that kind of quesiton. What would you use if somebody asked you to do that?

MR. CANUEL: As an asset manager, actually we had thought about exactly that issue in terms of doing relative value. What we typically would do is ramp up the volatility in the model, and that makes the option worth more and makes it exercised against you more often. What you're asking is something a little bit different, because you're looking at a specific scenario. What I would do in that case is essentially just agree on a benchmark coupon. In the current environment, I would say that would be 7.5 because that's current coupon. I would identify what a ramp up in speed would be for that and adjust my data 2, data 3 combination to give me that. It's not a very good answer. It's going to affect all the other coupons a little bit differently, but you know what direction you want to go in terms of adjustments of your parameters.