



SOCIETY OF ACTUARIES

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## AN ATTEMPT TO CONVERT AMERICAN ACTUARIES

by Hilary L. Seal

The history surrounding Gershenson's recipe for deaths among "existings" may be summed up in quotations spanning 72 years:

1898: "As the period of observation terminates at the close of a calendar year, the cases 'existing' are necessarily under observation for a portion only of the year of duration [age] then current, and some of the cases of death . . . during the last calendar year, would, if treated as 'existing', have in like manner completed only a portion of the year of duration [age] current at exit. In strictness, therefore, such cases should contribute to the number exposed to risk, not the full year of duration [age] current at exit, but only that portion of the year which actually fell within the period of observation."

Thomas G. Ackland, *J.I.A.* 33, 193

1943: ". . . it is evident that observed deaths before age  $x+1$  among the  $n_{x+k}$  entrants [at exact age  $x+k$ ] are to be 'exposed' for the period  $1-k$  and not for the full year."

Ralph E. Edwards,  
*T.A.S.A.* XLIV, 34

1943: ". . . it seems desirable to remark . . . that the entrants  $n_{x+k}$  obviously can be exposed for not more than the period  $1-k$  after entrance (whether they live or die) . . ."

Hugh H. Wolfenden,  
*T.A.S.A.* XLIV, 61

1945: ". . . the . . . exposed to risk . . . produces exactly the same number as would result from counting . . . a fraction of a unit for each . . . person corresponding to the fraction of the year of age during which he was both insured and under observation, *deaths in all cases being treated as if they had occurred at the end of the year of age.* [Italics in original]

Edward W. Marshall,  
*T.A.S.A.* XLVI, 38

1961: ". . . some students (and even some experts) . . . think of the exposure . . . as 'the distance from the point of entry to the end of the observation period, or to the end of the unit age interval, whichever end-point

occurs sooner'. That this line of thought is inconsistent with  $1-tq_{x+t} = (1-t)q . . .$  is easily seen by considering [an individual aged exactly 54.25 at entry who died at exact age 54-5/12 and would have been subject to observational cut-off at age 54.5]:

- The quoted line of thought would assign an exposure of one-fourth of a life-year . . .
- The two-step method would assign a *potential* of three-quarters of a life-year . . . . Furthermore, *no cancellation* would be required by the intervention of the end of the observation period, because this employee would not be one of the enders [i.e. existing]. The net exposure is therefore three-quarters of a life-year . . ."

Harry Gershenson, *Measurement of Mortality*, pp. 45-46

1970: ". . . consider a life D born on 1 July 1909, entering assurance on 1 May 1961 and dying on 1 November 1964. [The observation period ended on 31 December 1964]. . . . the amount of risk time during this age interval [55/56] . . . will be 1 year (notionally, because in calculating  $q$  [the observed rate of mortality] it is as if all the  $\theta_x$  [deaths at age  $x$  last birthday] were exposed for a full year."

B. Benjamin & H. W. Haycocks,  
*The Analysis of Mortality and Other Actuarial Statistics*, pp. 41-42

One's conclusion from all this is that in the 1940's American actuaries came to believe that Ackland had been wrong 45 years earlier, and that their adherence to this viewpoint converted the British later on. In fact, an "existing" entering at age  $x+a$  and scheduled to be lost to observation at age  $x+b$  ( $a$  and  $b$  both fractions) can only be "exposed" for  $b-a$  of a year at age  $x$  last birthday. Whether he lives or dies (the latter happening *after* exposure has taken place) can surely make no difference.

*Ed. Note: We are pleased to have this heretical contribution from our distinguished member now in Switzerland. Mr. Seal's solution is set forth mathematically in his 1977 paper, Multiple Decrements or Competing Risks, in Biometrika 64, 3, pp. 429-39; a reprint entrusted to this editor is available on request. □*

## BOOK REVIEW

*Report of The Universal Social Security Coverage Study Group.* 276 pp., Department of Health and Human Services, Washington, DC 20201. March 1980.

Reviewed by Sandor Goldstein

*Ed. Note: This is excerpted from a full review to appear in the Transactions.*

**The mission of the Universal Social Security Coverage Study Group was to examine the feasibility and desirability of mandatory Society Security for employees of the federal government, of state and local governments, and of private non-profit organizations, a proposal that, though frequently scuttled, has become of increasing concern to Congress.**

This report examines five major problems that result from lack of universal coverage, viz., (1) gaps in protection for workers who move between jobs covered and not covered by Social Security; (ii) gaps in benefit protection for non-covered workers; (iii) undeserved exemption of the non-covered from supporting a redistribution program with a social tilt favoring low-wage earners; (iv) windfall benefits to those who enter Social Security when already close to retirement—estimated to cost the system \$840 million a year; (v) losses suffered by those who contribute to Social Security for so short a time that they never become fully insured.

The leading choices available to policymakers are identified as requiring Social Security coverage, or alternatives that at least reduce existing gaps and windfalls.

The Study Group recognizes that most existing pension plans for government employees would have to be revised. New formulas could be designed to provide comparable benefits to "average" employees under any of three plan types (i) an "add-on" approach under which the new benefit would be proportional to salary but at a reduced level, unaffected by the amount of Social Security benefit; (ii) an "offset" approach under which the new plan's benefit would be reduced by a percentage of the Social Security benefit, or (iii) a "step-rate" under which a given percentage would be applied to earnings below a specified level, and a higher percentage to all earnings above it.

(Continued on page 5)