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**Session 30TS**  
**Complex Liability Modeling Issues**

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*Summary: The instructors discuss the development of realistic liability models for use in asset adequacy analysis, asset/liability management, pricing/repricing, financial reporting, and embedded value calculations or other corporate purposes. The primary focus is on modeling issues for products with embedded options, such as minimum living or death benefit guarantees on both variable life and annuities; secondary guarantees on both fixed and variable universal life; annuity death benefit enhancements; and issues for variable immediate annuities. Topics include analysis of the special risks posed by these features and products, selecting realistic dynamic assumptions, techniques used in modeling these features and products, stochastic analysis and presenting results to best communicate risks.*

**MS. PATRICIA L. RENZI:** I'm the product manager for the MG-ALFA Actuarial Projection System distributed by Milliman U.S.A. I've been involved with the development and use of commercial modeling systems and actuarial modeling systems for a little over 15 years now. While many of the issues associated with the development of models haven't really changed much over that time, we've introduced a new set of risks over the past several years with some of the product innovations that require a careful evaluation and have raised some additional modeling challenges, which is what we plan to discuss in this session.

I'm going to begin with a more general discussion of modeling issues. Craig Morrow from Hartford Life will discuss the specifics of guaranteed minimum death benefit (GMDB) and guaranteed minimum income benefit (GMIB) modeling. Craig has 24 years of experience in the insurance industry, spending the last 18 years at Hartford Life. Craig presently works in the corporate area and has primary responsibility for cash-flow testing and investment strategies. He's an FSA, an EA, and a chartered financial analyst (CFA.)

Our comments are really designed to try to raise some issues and provide some general background for discussion. We have left a fairly significant amount of time for some questions. What we're hoping for is to provide an opportunity to discuss some of your issues on modeling complex liability structures, some of the challenges that you're facing, and facilitate a sharing of ideas and experiences.

I want to keep in mind some of the general challenges that we have with modeling and why we build models. To set the stage for our discussion, I would like to define why we model. The most obvious purpose of modeling is practicality. Models generally provide a faster, more flexible and more manageable structure to use and maintain.

Data management is cleaner, more auditable, and often more readily manipulated for sensitivity testing. With the introduction of more complex risks in the products we sell, the need for sensitivity testing escalates. This creates a challenge in balancing speed and comfort that your model is robust enough to fully capture the risks.

What are the objectives that we have in building models? We're building models to make sure that we understand and can quantify the risks inherent in the products that we have. The purpose of the models that you're building is going to be a big driver of the model granularity and in how you set your different modeling points. It will also affect the number of scenarios and sensitivities you will run.

For example, if you are trying to evaluate the impact of different reinsurance arrangements on a variable annuity block of business, the model you build will be different than it would if you

were pricing a variable annuity and trying to initially determine the cost of a death benefit option. Additionally, if you're building your model to price a product versus looking at short-term forecasting of earnings vs. solvency testing, the size of the model and the definition of that model is going to be very different. You need to keep in mind what the purpose of the model is as you're defining it and building it.

How do we define the different model points and what is appropriate to group together. How do we go about doing that? Certainly, the standard model point definitions that you have in any actuarial model are things like plan gender, risk class, issue-age duration, and different policy structures. If you're looking at a block of business, you're going to need to make sure that you're accurately reflecting things like insurance period, premium period, distribution channel, and so on. Certainly, these basics are very familiar to you and are considered in any actuarial model definition.

Additional considerations that we have with some of the new products that have emerged in the last ten years or so are funding levels on universal life contracts and fund selection and transfer behavior on variable life and annuities. You're going to need to make sure that you're considering your level of funding in the model building. Typically, it's enough to say that you are to have low-funded, medium-funded and high-funded policies for UL business, but you cannot lump everything together and ignore the impact of funding level on cash flow and behavior. The funding levels are key in driving many of the benefits and behaviors, so it's critical to make sure that you're capturing the differences that you have across the business.

For features like GMDBs, you're going to want to make sure that you're capturing the differentiation across the entire block of business based on how "in the money" the particular option is. You need to thoroughly evaluate the block of business and determine groupings based off of the status or value of the option on the projection date. This can increase model sizes, but is critical in capturing the future value.

Fund allocation and fund performance are another big issue that we have in modeling variable contracts. Certainly, you're not going to model every fund offered or track the allocation across

funds by policy. How we group funds together and how we decide what the average allocation is for the block of business that we're looking at is going to be critical in evaluating variable products.

I'm going to talk a little bit about fund modeling and how that can be approached. Obviously, when you have a large number of funds that require simplification, what you're really going to do is clarify the risk and the risk/return dynamics across all of the different funds that you have within your portfolio and then group those together.

One of the options that you have is to look at some different index funds that are out there. There are funds that you can track in the market, and for which you have historical information on so that you can generate expected returns on those into the future. You need to be able to identify or look at some existing funds that are out there that you can really get some good statistics on so that you can use that to be able to feed your stochastic projections of what the fund performance is going to be moving forward.

As an example, assume our objective is to collapse the funds into six indices. By doing a regression analysis for each of the actual funds, you can assign each fund to be represented by one of these six indices, reserving the risk and return characteristics of the actual funds. Next, determine the allocation of funds for each policy across the indices. Using this distribution, you can group policies by level of volatility to create a model that accurately reflects the risk profile of the investment options and the investors.

To demonstrate the importance of retaining volatility level as a model characteristic, let's look at a variable annuity with a simple GMDB benefit. The average GMDB net cash flow as a percentage of the initial account value is 0.67% when we ignore volatility level in the modeling. However, the average for high volatility policies is 0.1% while the low volatility is, 1.34%, demonstrating the importance of maintaining this characteristic.

When you're evaluating variable products, the risk that the particular policyholders are taking should be an additional model point.

Step back from the specifics and talk again about the general challenges in creating models of products with embedded options. To reiterate, a large number of scenarios are needed to be able to get credible results. Obviously, that means that you're going to need an efficient calculation engine and some very good modeling techniques. You're not going to be able to run extremely large models, so you need to really understand the drivers of the products that you have, and understand the behaviors of the policyholders within the block of business that you're looking at. Group things appropriately so that you're creating models in which you're not hiding or missing some of the big drivers of cash flows.

Behavior models are key but, at this point, data for the determination of credible behavior models is limited. It's certainly difficult to get our hands around it and to feel comfortable that we can accurately predict when policyholders are going to exercise an option. How are market returns and new product innovations going to drive policyholder behavior? How will the value of a GMDB option impact lapse behavior? There are more questions than answers, but we need to define behavior models for deposits, loans, withdrawals, lapses, fund transfers and any other elective options offered to the policyholder.

To summarize, the key questions you need to be asking when constructing your models include: Are there certain things that I can eliminate? For this particular product, can I get by with a minimal number of ages? Is the risk classification or gender going to be a big driver, or can I just forget about these more standard modeling issues and focus on the fund allocation, additional behavior models, etc.? Once again, you need to understand what the drivers are of the options in the product and make sure that you're focusing on the right things when you're building your models. Finally, spend the effort to gather as much information and insight into the behavior models as possible.

The products that we have, and the different benefits that have been added increase the need for good behavioral models to capture the options. Again, there's not a lot of empirical evidence out there as to what those models should be, so sensitivity testing is necessary. Also, don't forget to include the modeling of corporate decision analysis. How will you define setting credited rates on general account funds, what are the investment strategies, reinsurance strategies, and so on?

Are you going to hedge the risks internally? Are you going to incorporate different reinsurance structures on top of the risks? All of those things are additional considerations and key issues in building models to fully evaluate the risks and the value and the cash flows of products with these complex structures.

As I mentioned, Craig is going to get into some more specifics of modeling GMDB and GMIB structures. Then we will have some time for questions and discussion.

**MR. CRAIG D. MORROW:** Even though this session is on complex liability modeling issues, I want to start with a brief discussion on what I call noncomplex liabilities. We don't want to forget these in our modeling. I'll then move on to some complex liability risks that are associated with GMDBs and variable annuity guaranteed life benefits (VAGLBs). I'll then get into some modeling techniques that we've used for GMDB and VAGLBs. Then I want to spend some time on realistic dynamic assumptions. I have some recent experience I want to share with you on policyholder behavior that I think you'll find surprising. I'll finish with my thoughts on stochastic analysis and presenting results.

This is a session on complex liabilities, but let's not forget the non-complex ones. Quite often in life, it's the little things that come back to bite us. If you look at history, we had loan options in life insurance policies back in the 1960s and 1970s. How many folks thought when they were pricing those products that these were significant options, and that there was much risk there? Then the 1980s hit us and money market rates reached 20%. Policyholders could borrow at a 5-7% loan rate on their life insurance policy and turn around and put the money in a money market fund earning 20%. It was arbitrage; folks exercised it. Insurance companies got hit hard. They incurred large substantial capital losses.

As for interest rate floors, I remember pricing annuities back in the 1970s and 1980s thinking that the 3% and 4% floors weren't much of an issue. I think quite a number of companies have even 5% and 6% floors in their life insurance policies and their annuity contracts. How many folks out there now are crediting those rates? Given where new money rates are, I suspect that if rates stay at that level, we're not going to be getting the target spreads that we hoped for in pricing.

I do have one more item that I want to chat about that I would call a noncomplex liability. That's the return-of-premium guaranteed minimum death benefit. When I was pricing that back in the 1970s and 1980s, we had a cost in there for it, but we never really modeled it. We didn't think it was worth that much. You don't worry today about those contracts that you sold back in the 1970s and 1980s. It's the contracts that you sold in the past two to three years that are in the money. If you want to get reinsurance today on a return on premium (ROP) benefit, you'll find that it is not cheap. As you model the complex liabilities, let's not forget the noncomplex ones.

As for GMDBs, we have our familiar return on premiums, the ratchets, the maximum account value (MAV), the roll-ups, and the earnings protection benefits (EPBs). The risk, of course, is a function of market rates, mortality rates, and lapse rates. I think the insurance industry has a pretty good history of managing many risks. We're pretty good at managing mortality risks and interest rate risks, but we're talking about equity market risks, and that's relatively new for us. I think that increases the risk to us.

I do want to add another risk in there that I want to call volatility or regulatory risk. Let me give you an example. You've got the MAV death benefit and most of us would call the net amount at risk to be equal to the MAV benefit minus the account value. Let's assume the market drops 10%. The account value was sitting at \$100. The MAV was \$110, the account value drops to \$90, and, all of a sudden, our MAV has doubled. It has gone up by 100%.

I think most of us today aren't so concerned about the level of claims that we're paying under these guaranteed minimum death benefits, we're probably more concerned about the volatility of the reserves. The AG-34 is quite volatile, and that probably has a much bigger impact on your statutory income statement than the death benefit claims themselves.

As for VAGLBs, we have our guaranteed minimum accumulation benefit (GMAB), our GMIBs, our guaranteed minimum withdrawal benefits (GMWBs), and guaranteed payout annuity floors (GPAFs). Again, the risk is a function of market returns and, to a greater extent, policyholder behavior. You have lapses and you also have other risks such as interest rate risks. Look at your GMIBs today. You have low interest rates and a low equity market; it's a double whammy. You

also have liquidity risks and concentration risks. Look at the GMABs. Concentration risk is defined as having a large block of business that's in the money. It could be all exercised at once. You can diversify this risk by having GMDBs with diverse maturity dates. Election rates or policyholder behavior is very difficult to model, and we have very little experience to date. I want to get into behavior later and give you some experience that we've seen on the GMDB side.

I'd like to discuss modeling techniques and guaranteed minimum death benefits. I think it's safe to say that if you're modeling just the death benefit portion, and you're looking at a return of premium benefit, you can probably aggregate your data to a larger extent than if you're looking at a MAV death benefit. We have a large block of GMDB business, and we are modeling a good portion of it. We were trying to get a better handle on the risk, and we had 400 input sells. We ran the model, and we couldn't validate it; the AG-34 reserve was off. The AG-34 reserve was about 75% of the actual. Given the magnitude of the AG-34 reserve, that 25% differential was substantial. We had to figure out what was going on. We spent about two days looking at the model. We thought it was the logic in our AG-34 calculation, but it wasn't; it was the input data.

When we aggregated the data, the one thing we looked at was the ratio of the account value to the MAV. I think some folks call that "in-the-moniness." As for the buckets, we had a much broader range than what you see there. For example, in category A, the bucket was from 91% to 100%. Bucket B was probably 101% to 110%. We went back and doubled the number of buckets. Lo and behold, our AG-34 reserve came within 5% of the actual. Be careful when you aggregate data.

Reinsurance contracts seem to be unique. You may have full coverage or partial coverage. It could be multiple insurers and that can apply at the seriatim level. The reinsurance premiums are often a function of the net amount at risk at the seriatim level. There are often minimums and maximums that are based on the account value, and those are at the group level.

VAGLBs. Let's look at Guaranteed Minimum Withdrawal Benefits (GMWBs) and MMMM. I think all of us now realize that the industry is moving towards stochastic reserves and stochastic risk-based capital (RBC). We were looking at a GMWB benefit, and we knew we could model



the reserves on a stochastic basis. However, the capital was an issue. At interim periods, we had stochastic on stochastic problems. What we did is we said, “Why don’t we define RBC as a function of the account value?” We set it as a percentage of the account value. That percentage varied depending upon the contract duration, how long the contract was in force, and the percentage that was in the money. That helped us solve the so-called stochastic on stochastic problem. Stochastic reserves and capital are coming, so I would suggest that you begin modeling on a stochastic basis.

I do want to back up just for a minute and mention one more thing on modeling techniques. I think it’s important that you model GMDBs and VAGLBs by themselves. I think you can learn a lot about the risks. But then you need to step back and model the entire contract. You’re going to find offsets in there. For example, you might have some contracts where you have a GMDB benefit and what we call an earnings protection benefit (EPB). The GMDB benefit is in the money when the market is down, but the EPB benefit is in the money when the market is up.

On a similar note, you might have death benefits and living benefits in some of your contracts. Look at the mortality risks. If you have a GMDB benefit, the high mortality rates can hurt you; whereas, if you have a GMIB benefit in that contract, the low rates can hurt you.

Realistic dynamic assumptions and market risks. You need to have fat-tail scenarios. GMDBs and VAGLBs have significant tail risks. These products have cliff-type risk profiles where, for most of the scenarios, you’re going to get lucrative returns. In those rare fat tails, you’re going to get awful returns. I have some experience to share with you folks. We have a large portion of individual variable annuity business. How many of you folks would think that lapse rates would increase as the market increases? How many think that lapse rates would increase as the market decreases? We saw both cases. In the 1990s, as the market was going up, some of the brokers and agents went back to their policyholders and said, “Hey, you know what? I gave you good advice on buying this annuity. It’s done quite well, but guess what? We can move it to Company ABC and get you a 5% bonus, so let’s do so.” So, we saw a lot of 1035 exchange lapses increased.

With the recent downturn in the market, we are also seeing lapses increase, but they're not from 1035 exchanges. I don't think the agents and brokers are talking to their policyholders that much. I suspect a lot of you that have stockbrokers aren't getting a lot of calls from them either. What I think has happened is that folks are just upset with the market. They don't know what to do, and they're pulling their money out and going to, perhaps, the bank. I don't know why. It doesn't make sense to me.

If you think about it, back in the 1980s, when money market rates hit 20%, a lot of folks exercised the loan option because of the arbitrage there. Why would folks in variable annuity contracts with the market down move out? If you're going to go to a bank, what are you going to get for a return? Banks aren't crediting high rates, and CD rates are low. You could move it to our general account and get a 3-4% guaranteed rate. That's a pretty good deal. Recent behavior has not been tied to the value of the GMDB benefit. The one thing we know about the behavior of policyholders is that we don't know what it's going to be.

Stochastic analysis. No single scenario has any realistic chance of occurring. That shouldn't surprise any of us in this room. But you do get a distribution of possible results. I would suggest that you really stress test your assumptions, especially with respect to policyholder behavior. What I like to do is run stochastic results and look at the bad scenarios. I like to pull those out and do further analysis on them. You can learn some neat things. You might learn that there's a limit as to how much of a certain line of business you want to write. Better yet, you might determine that if you tweak the product provisions a little bit, you might not mind the risks quite so much and you might want to write more of it.

Another thing I get from stochastic analysis are what I call relationships or rules of thumb. About two years ago, I had a student working for me, and he built a slick little model to look at death benefit risks. It would run an ROP for you, a MAV, a ratchet, and we had 200 stochastic scenarios. But we also had the mortality rates and the lapse rates moving on a stochastic basis. For the lapse rates, we put in some jumps so that they would jump up or down depending upon where the market was. One of the things that we learned right away from it that kind of surprised me was that high lapses, early on, normally decrease your risk for MAV risks. However, you

want just the reverse for your return on premium (ROP) benefits. Again, that's just a rule of thumb that we got from the stochastic analysis. Another rule is please be sure to validate your model.

Presenting results: The Man from Mars View. There was an actuary about 20 years ago that I knew that moved from the so-called traditional actuarial role into investor relations. After about six months to a year, he gave a presentation to all the actuarial students. He said that one important thing he learned was that he had to take his knowledge about the products, the risks, and the results and put those into terms that a man from Mars could understand. He had to put the information into simple English. I think we need to do that, too. I like graphs. I think a picture is worth 1,000 words.

I think we need to be careful when looking at present values and a lot of the metrics that we use in pricing. Take internal rates of return (IRR) for example. That's a return over the life of the contract. How many folks outside the actuarial field really understand that? We often look for present value of book profits, and present value of distributable cash flows. Those are impacted by the rate we discount them at. The higher the rate, the lower weight we give to later year results. I like showing year-by-year results. I think most folks understand that pretty quickly, especially for the first five years. I think if you can show the volatility and the impact to the income statement, that says an awful lot. One of my bosses liked to show the results on an earnings per share basis, and I thought that was pretty clever. That put it in English pretty quickly.

**FROM THE FLOOR:** I built a pricing projection risk management model for the variable annuities, specifically for GMDB, GMIB, and EPB. I used six computers running simultaneously for the hedging strategy for the company and for risk management purposes.

When I was developing the model, the major issue that I had that I haven't solved so far is the fund transfer from one period to another, from one fund to another. Any experiences on this that can help us?

**MR. MORROW:** I would vary the heck out of it. When I look at our 401(k) business and our business in the 457 market, the transfer behavior differs quite a bit. In the 401(k) market, we see very little transfers when the market goes way up or way down. In fact, for the contributions coming in, the allocations haven't really changed historically. But when I look at the 457 market, the transfers really move as the market moves and so do the allocations.

When we run models with transfers, we end up really stress testing it, and, quite often, we will reverse our assumptions to assume that it's the opposite behavior of what we normally think. I think that all of us are going to be surprised by future policyholder behavior.

**FROM THE FLOOR:** One of the things that I have struggled with, and I don't think it was addressed in the session, has to do with just projecting the GMDB benefit. In situations where that is based on the greater of a number of components, it seems to me that you have a different situation, and that is if a product has a ratchet and a rollout. Which of those is governing how in the money it is right now? You'll have a big influence on how it projects into the future. Do you have any comments or thoughts on that subtlety?

**MR. MORROW:** Could you repeat that please?

**FROM THE FLOOR:** Say you have a product that has a death benefit that is based on the greater of a ratchet or a rollout. When you project it into the future, it matters quite a bit whether its current "in the moniness" is based on the ratchet or the rollout. If it's based on the ratchet benefit, and you project a flat scenario, then that death benefit isn't going to go up at all. Instead, if its degree of being in the money is governed by the rollout and you project a flat scenario, it's still going to have an increase in death benefits.

**MR. MORROW:** Right.

**FROM THE FLOOR:** It seems to me that you have to distinguish which characteristics are governing the degree of it being in the money. That gets further complicated in my second question. It has to do with it being based on a rollout benefit. How do you capture the prior history in your model as far as premiums and partial withdrawals?

**MR. MORROW:** We don't have that problem. I haven't modeled those. I know modeling just the MAV death benefit combined with return-of-premium was difficult for us for a while. The logic is tough to program. I wish you a lot of luck there.

**FROM THE FLOOR:** Modeling GMDBs and GMIBs is very difficult when you use an Excel spreadsheet. That's why we use C++. What we usually do is create a large array of a number of scenarios, time periods, benefit designs, and so on. You have four or five scenarios for a variable with five or six dimensions containing a number of scenarios, the number of benefits assigned, the number of time periods, and so on. When you move forward in funds, you have your historical values from times zero up to the current time. That's how we did it in Excel.

**MR. MORROW:** That's a lot of tracking.

**FROM THE FLOOR:** Yeah.

**MS. RENZI:** Are you saying that in your model you actually had all of the history for each of the policies as part of the model?

**FROM THE FLOOR:** Yes.

**MR. MORROW:** How do you find modeling reinsurance? I mean I found that very, very difficult in our models. Some of the software out there just for valuing statutory reserves do a great job on the AG-34 reserves, but as soon as you throw in reinsurance, they just don't work.

**FROM THE FLOOR:** We were successful in doing what you call GMDB stochastic modeling, which we actually use when we do risk management and what you call economic hedging. When

I first asked the Chief Executive Officer if I can get six or seven of the fastest computers with the biggest memory, he was actually smiling at me and asked, "Are you joking?" I said that I know what to do with these kind of products. So I was successful in getting seven of the fastest computers in the industry.

So when we were doing stochastic within stochastic modeling, you have to know what you call the beginning account value and the beginning GMDB value at each point in time as you move forward. You have to actually keep track of this because when you do stochastic projecting at each point in the future, you have what the starting point is, and then you model it downwards.

There are two kinds of projections. One is horizontal projection. As you go forward, you stop at each time period. Then you do a vertical projection at its time period to calculate the statutory reserves for its policy. So it's very complicated. It's basically used if you are integrating the benefits. If you have the combination of EPB, GMIB or GMDB in one policy, you have to do it simultaneously.

**MR. MORROW:** Problem.

**FROM THE FLOOR:** Yes, problem.

**MR. MORROW:** As I said, we have a slick little model that we built in Excel just to get us relationships. We also have a model in Excel that we run blocks of GMDB business through, and it's slow. You used six computers. We've used 12 over a weekend on it. It's a real monster. We also have some software that isn't C++ that we can run overnight on one PC. I think you're right. The platform that you use can make a big difference in speed.

**FROM THE FLOOR:** I'd like to say that we're still modeling some of the business that Victor left behind. Ours is simpler than what you're doing, but it's still difficult.

I'd just like for you to go into a little more depth about stochastic modeling of RBC. Everything I do has the surplus calculated in it. It just does the function of a factor times whatever relevant metric the RBC calculation has. We've obtained several different calculations depending on whether we're looking at the statutory RBC or an S&P formula or a homegrown analysis of risk that maybe isn't captured in those RBC formulas. I'm just wondering what comment you have on that.

**MR. MORROW:** I was inches away from escaping clean. Truthfully, I didn't work that much in the VAGLBs; it was another unit so I don't have much knowledge there.

**MS. RENZI:** I just want to make sure I understand your question. Right now, you're incorporating RBC into the model by just projecting your policy benefits and cash flows forward. Then the RBC is just a percentage of whatever.

**FROM THE FLOOR:** Right.

**MS. RENZI:** There is account value, reserve, or whatever it's going to be, or a combination of those things as opposed to having a stochastic calculation for risk-based capital. The only thing I can comment on is what companies are doing in the market. From my experience, I'm not aware of anyone who is doing anything other than that at this point. Victor was just mentioning about the reserve calculations that it is not really a stochastic on stochastic at this point due to AG-34. You do have the vertical piece within that, and then you are adding a stochastic part on top of that.

The only thing I can equate it to is the requirements in Canada, in which your reserves require a stochastic projection at each point in time. If you are doing stochastic projections, you're having to then do stochastic within stochastic. While that is certainly where they are headed, the implementation and practical issues in doing that are really sort of stumping people. It's slowing things down.

You can calculate your reserves today but to be able to calculate reserves on a stochastic basis within a stochastic projection is difficult. We're already dealing with six computers running simultaneously (or twelve running over the weekend) to be able to do what we're doing. There are some practical issues in the implementation. You're going to have to end up, within your projection, coming up with what you think those factors are. Maybe you'll need a grid where you'll have to do some stochastic analysis. Then you'll be able to say, "If this is what this particular relationship is at this point on each of my stochastic paths, then here's what the factors should be." That's the only instance that I can think of that that would make it practical to implement something like that. At this point, from my experience, I haven't really seen anyone doing anything other than what you mentioned you're doing.

**MR. MORROW:** I can tell you that when we developed our chart, the reserve was based on a 60% Conditional Tail Expectation (CTE) formula and the RBC was a 90% CTE.