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Session 9PD
Economic Scenario Generators

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Summary: A fundamental assumption in stochastic asset/liability modeling is the underlying economic scenario generator. In this session, panelists discuss:

- *Various scenario generation techniques*
- *Assumptions underlying bond and equity scenario generators*
- *Choosing suitable economic scenario generators for different application*

MR. MARC N. ALTSCHULL: I'm with Tillinghast–Towers Perrin, and I'll be moderating this session. Economic scenario generators are a fundamental assumption in stochastic asset/liability modeling (ALM).

Our session has three topic areas. First, we'll be talking about scenario generation techniques and theories, assumptions underlying bond and equity scenario generators, and selection of suitable economic scenario generators for given applications. We're hoping you'll have an increased understanding of the uses and limitations of economic scenario generators.

The charts referenced in the text appear at the end of manuscript

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Our presenters are Steve Sonlin from Swiss Re Investors, who will be speaking about how to choose a suitable economic scenario generator. Next will be Mark Tenney from Mathematical Finance Company, who will be talking about applying interest rate generators to uniform valuation system, fair-value liability, and dynamic financial analysis. We'll hear from Stephen Britt from Tillinghast–Towers Perrin, who will be talking about making scenario generators global.

Our first speaker is Stephen Sonlin. Steve is a senior vice president at Swiss Re Investors within the Falcon Asset Liability Management Unit. He's responsible for Swiss Re Investors' proprietary asset/liability and integrated risk-management modeling systems for insurance companies. Steve has been involved in the asset/liability management field for over 18 years. During his career, he has developed and used numerous economic and capital market simulation models to provide asset/liability management services to the life, property/casualty, and reinsurance industries as well as to pension, health and disability and endowment funds.

Steve is a graduate of Temple University with a B.A. in mathematics. He's a Chartered Financial Analyst (CFA) and a member of the Association of Investment Management and Research and the Baltimore Society of the Financial Analyst Federation. Steve has written numerous articles and papers on topics that include stochastic modeling, optimization, asset/liability management and financial risk management. He is a frequent lecturer at financial and actuarial conferences.

MR. STEPHEN SONLIN: I realize that this particular session is for advanced modeling professionals. I'm going to go through some basics, and I'm going to cover some terminology issues. I'm a practitioner. I've been doing asset/liability modeling and asset management for insurance companies. I regularly use asset models in my work.

One of the problems that I find is that the terminology is not very clear in this particular area. When people start talking about economic scenario generators, different people mean different things and they're applying them to different applications. I'm going to use this first session to straighten out some of the issues that we should be thinking about when we're using these models. This session is entitled Choosing A Suitable Economic Scenario Generator. I'll go through some different model applications, discuss some of the different scenario generators, and review some techniques used in building scenario generators. I'll end with some questions that you might ask yourself in trying to choose the right scenario generator for the right application.

I'll start by defining model types. There are numerous types of economic scenario generators. These different types are designed to address different applications. I've narrowed these into three types that I use pretty readily in my work in investment management and asset/liability management. There are many more classifications and there's a lot of overlap even in these classifications, so I'm not trying to put down a formal definition here. I'll just try to get across some concepts of what you can expect for different applications. The three types are: pricing, prediction, and risk analysis models. I'll now provide more detail on each of those types of models.

Pricing models are used to value interest-rate-derivative securities. They are intended to come up with a value as of today, or as of a particular point in time (time *t* equal to zero). These models are projected paths in coming up with that value, but their main purpose is to calculate the value of a derivative security as of today or as of a particular point in time. These models are calibrated to replicate a set of publicly traded bonds and options. That calibration is then used to price derivative securities that don't have an observable price.

Typically, and for mathematical ease, these models are designed to be arbitrage-free and risk-neutral. I'm sure these terms have come up a lot in your work. I'm going to come back to these terms a little bit later in my presentation. This is just a quick overview of what a pricing model is intended to do. I want to kind of differentiate pricing models from the other types of models that we use in our business.

The second type of models are prediction models. Prediction models are used to predict some economic or capital market value in the future. It's typically a point estimate type of an approach. In the investment management world, when we're managing funds for insurance companies, we often are asked to express our view on the economy and on the capital markets.

Some investment managers have neutral views. Other investment managers have views on where interest rates might be heading, how the stock market is valued, and they have systems that they put in place that they can use to help them develop their viewpoint on the marketplace. That allows them to make shifts in their investment portfolio. If they feel that interest rates are rising, they might want to shorten the duration of their fixed-income portfolio. If they feel that the stock market is overvalued, they might want to decrease their allocation to the equity markets.

These viewpoints are typically based on an understanding of the historical relationships and conditions that exist in the economy and in the capital markets. Then various rules and judgments are applied to come up with a prediction of where rates and equity markets might be heading. These viewpoints can also be based on professional judgment and experience. These are not just purely data-driven models; they're used for tactical asset allocation applications, which is quite different than ALM.

My experience is in the area of asset/liability management. We're not trying to make predictions with ALM. We're not trying to say shorten your portfolio because I think that interest rates are going to be going up. Instead I want to say to go into a shorter duration portfolio because that makes sense within the confines of your liability structure and your operating objectives. So prediction models are more of a tactical application, and they do not have much of a role in asset/liability management, although, keep in mind that all assumptions do have some form of tactical aspect to them.

Finally, I'm going to define a model type that I'm labeling a risk-analysis model. These are the types of models that I use for asset/liability management. They're used to project economic and capital market values into the future, where the future can be multiple time periods. We're really looking to see what the distribution of interest rates is going to look like at the end of period one, two, three, or four in the future. Those periods could be monthly, quarterly, or annual, depending on the application and how detailed you want these projection models to be.

The parameters, instead of being calibrated like pricing models, are parameterized to reflect real world behavior. Real world behavior is how interest rates and how equity markets actually behave in the real world, and this is going to be contrasted with pricing models that are using risk-neutral and arbitrage-free valuation methodologies.

So the concept of real world behavior starts becoming important. In asset/liability management, we want to look at interest rates and the movements of interest rates. We want to look at the strategies that we're putting into place over various economic and capital market scenarios, where the economic and capital market scenarios have to reflect what we think will happen in the future. It's important to get the distributions right when you're looking at these types of risk-analysis models.

I'm going to concentrate now on risk-analysis models because those are the types of models that are used most frequently for ALM processes. I'm going to talk about three different types of structures that I have become aware of in my work over the years, and these structures take the form of regression models, lattice or tree structure models, and stochastic differential equations.

Regression models are the basis for some of the earliest risk-analysis models. Some people call them stochastic actuarial investment models. There's a very popular well-known model in the U.K., which is called the Wilke model (see Chart 1). It is a regression-based model. It's not totally a regression model, but it takes a lot of its structure from an analysis of historical data and uses a regression approach to structure the equations and then project them out into the future, similarly to how they behaved in the past. Regression models use standard regression techniques to define the structure of the underlying model.

Another structure used for risk analysis modeling is a lattice or a tree structure (see Chart 2). Lattices and tree structures come from some of the pricing model applications. Lattice or tree structure models are made up of nodes and branches. Chart 2 shows a recombining binomial type of lattice. There are many different lattice structures, but, in general, at each node, you branch out with an up and down scenario. You can also have tree structures with more than two branches.

An example of a risk-analysis model that uses this type of an approach is the Russell-Yasuda Kasai model.

The final type of structure for developing risk-analysis models is the use of stochastic differential equations, or mean-reverting equations (see Chart 3). In practice, these models are typically implemented as difference equations. Stochastic differential equations are continuous time forms of equations. Differential equations can be changed into difference equations so that we can do discrete period projections in the model.

At Swiss Re Investors, we have a proprietary investment model that we use for ALM. We call it the SRI Falcon Model. I worked with another one at Towers Perrin called CAP:Link. CAP:Link uses a similar type of an approach. I think Steve Britt will be talking about CAP:Link and extensions to CAP:Link to account for globalization of these issues.

If you're doing work with economic scenario generators, you're going to hear about the term arbitrage-free. Arbitrage-free, in its simplest form, is having an investment strategy where there is no initial investment, so there's zero cost. There's no possibility of a negative result, but there is a possibility of a positive result. So you have a positive expected return without any investment. You accomplish this by shorting, going long and going short for particular investments. So when you combine them, you get in as much money as you pay out. So there's no initial cost to you. There is some positive expectation of a future benefit and no possibility of a loss.

When this situation occurs, it is referred to as arbitrage, and there are people out there who salivate when they see arbitrage opportunities. They take advantage of the arbitrage opportunity, and they quickly remove those opportunities from the marketplace. There are arbitragers out there. There are arbitrage situations that occur in the real world but they're very infrequent.

I talked about pricing models being arbitrage-free. I also talked about them being risk-neutral. Risk-neutral, in its simplest form, says that every asset class has the same expected return. You're indifferent to risk. If you're indifferent to risk, you do not have to be compensated for taking on risk and, thus, everything will have the same expected return. This has a lot of nice mathematical properties, especially in the pricing world, because, if everything has the same expected return, we know what rate to use for discounting cash flows. We can discount them at the risk-free rate because everything earns the risk-free rate. There's no additional premium for taking on risk. The mathematical properties of pricing models take advantage of this condition, and it makes for a convenient pricing model application.

In a risk-analysis world, it wouldn't be too much fun if we were risk-neutral. If we were risk-neutral and no one cared about risk, or if we were indifferent to risk, I would certainly be out of a job. Doing ALM analysis and risk analysis would not be a very important profession. In the real world, we have to consider models that are not risk-neutral. We have to consider risk, and we have to consider the premiums that are paid to the people who are taking on that risk.

That brings us to a new kind of classification where we want a realistic representation of things that might happen in the future. In this world, different asset classes have different expected returns, which is, in fact, the type of world we live in. We live in what is referred to as a risk-averse world. People expect compensation for taking on risk, and that's exactly what risk-analysis models are trying to evaluate. Equity markets and equity returns are traditionally higher than fixed-income returns, and long bond returns are typically higher than cash returns. This is the type of world we need to assess. We need to assess the risk in this world and the return in this world.

That brings us to the fundamental theorem of asset pricing. What does it mean when you're dealing with risk-neutral models, real world probabilities, and arbitrage-free models? Essentially, the fundamental theorem of asset pricing says that any arbitrage-free model can be transformed into a risk-neutral model by changing the probabilities of the various scenarios. Therefore, if that transformation can occur, we can say that the model is arbitrage-free.

So all risk-neutral models are, by definition, arbitrage-free. It's not true that all arbitrage-free models are risk-neutral, so you have to separate out the concepts of risk-neutral and arbitrage-free. Most pricing models utilize both of those concepts. They are risk-neutral, and they are arbitrage-free, but they're different concepts. Many people get confused between the terminology here, so they use the term arbitrage-free and risk-neutral synonymously. Actually, they're two very different concepts with different characteristics.

So is arbitrage-free required for risk analysis modeling? It's certainly required for pricing models. All pricing models are put together such that they are arbitrage-free and they are risk-neutral. When using risk-analysis models, the question becomes, do they have to be arbitrage-free? I think you can get a lot of opinions on this particular question. It's certainly reasonable. You don't necessarily want models where you can make profit without investing any money. We would like that situation to pop up more in our personal investment opportunities, but do we really want our models to have it?

So it's certainly a reasonable property, and it's a required property for pricing. But producing arbitrage-free multifactor interest rate models that are realistic and can be used for risk-analysis modeling is an extremely difficult thing to do. So the pricing model methodologies that are put into place for risk-analysis modeling are actually very constraining on how these models are developed. These constraints often result in us not being able to have other characteristics that we want in a real world, risk-analysis model. My experience shows that the standard of being arbitrage-free or guaranteeing arbitrage-free for risk-analysis models has an extremely high cost, and that cost is that we lose a lot of our ability to make a reasonable or realistic risk-analysis model.

I think Mark Tenney might be talking about some models that are going to do this in a more realistic or easier way. There's work being done in the area to take risk-analysis models and pricing models and somehow bring them together to gain the strength from both sides of the modeling process. In my experience, the standard of arbitrage-free for risk analysis is much too costly right now. However, you don't want models that have large amounts of arbitrage in them.

When we look at risk-analysis models within Swiss Re, what we have is a series of tests that we put together on our modeling, and we want all those tests to check out within the model. Arbitrage-free is certainly one thing we look at. At this point, we haven't found a technique where we can guarantee that the model will be arbitrage-free and still give us the flexibility that we need to reflect the types of real world conditions that we want to reflect.

My final topic is choosing the right model for the right job. Remember, any model is an approximation. It relies on assumptions, so you have to consider the application before you actually choose a model. Pricing models, prediction models, and risk analysis models all have different modeling objectives. They all have different modeling requirements, so you have to know what it is you're going to do before you can pick the appropriate model. Just because one model is effective for one particular application, does not mean that model is going to be good for everything. There is no such thing, at least that I have found, as one model that fits all applications.

What do you want to do when you're trying to choose the right model? Ask yourself a lot of questions. Here's a series of questions you want to be asking yourself about the application. You want to understand what the model and what the economic scenario generator is intended to do. Is it a model for pricing? Is it a model for risk analysis? Is it a model for prediction? What is the structure of the model? You have to apply your objectives and your standards, and your criteria to that of the models that you're looking at.

I'm giving you a practitioner's point of view with respect to models and concepts. For a more in-depth analysis, I have some references at the back of my paper. There are some general references that I've found useful when I was first looking into building these types of models. Finally, there are references for each of the specific models that I talk about in my presentation.

MR. ALTSCHULL: Our next speaker is Mark Tenney. He's the president of Mathematical Finance Company, a financial software firm that provides software tools for simulation of interest rates, multiple equity indices, inflation, unemployment, and other variables for the U.S. and other economies. He's currently working with Columbia University on the application of its finder software for low discrepancy sequences to the insurance industry for use in scenario generations. He is also working with the AAA Committee on a Uniform Valuation System. The committee is using his scenario generator for its modeling work.

He worked on a two-year dynamic financial analysis project for a global direct insurer and reinsurer in both property and casualty and life insurance that covered alternative risk-transfer reinsurance and finite risk reinsurance. He co-authored papers with David Beaglehole that resulted in solutions for the state price of the Cox, Ingersoll and Ross line of one and multifactor models. They also developed closed-form solutions for the multivaried quadratic interest rate model that he and David invented. He is the developer of the double-mean-reverting process.

MR. MARK S. TENNEY: As Marc pointed out, I'll be talking about applying interest rate generators to a uniform valuation system, to the fair value of liabilities, and to dynamic financial analysis, all of which are fairly well related to each other. I'll try to elucidate the links a little clearly in my talk as well as shed light on some of these issues that Steve has talked about as to the desirability of an arbitrage-free model and the transformation from the real probability measure to the risk-neutral probability measure.

Let's discuss dynamic financial analysis. This is what property and casualty (P&C) people tend to talk about. We tend to add the words dynamic financial condition analysis, although I think the condition should be dropped. I find it is an extra complexity. It's whole company modeling of assets and liabilities. You have purchases and sales during these scenarios, and new liabilities could be issued. You have to have some functions for your crediting rate. In general, it's the stochastic analysis of business risk drivers.

The variables simulated are all those relevant to the business. There is interest rate risk, which is what we're probably talking about, and there are also equity markets (particularly if we're looking at variable annuities), equity-indexed annuities, inflation, mortality, default, expenses, premium, and claims in general, if we're thinking in a P&C context.

The uniform valuation system essentially takes dynamic financial analysis as its tool and applies that to answer the question of how much we need in assets in order to pay off our liabilities some percentage of the time. It can be used for capital-based solvency. It applies conceptually to the company as a whole. We're determining the capital needed by the entire company as opposed to the capital needed for each little block of business. However, you could apply it to a line of business. Suppose you apply it to lines of business and get a risk-based capital for each line of business based on a percentile type definition, say 95%. Add up those risk-based capitals from the different lines. That number will be greater than the risk-based capital based on the same 95th percentile rule applied to a simulation of the entire company. We're all probably familiar with that, but it's a critical point to keep in mind.

The factors that go into it are, of course, the investment strategy. When you get one of these simulation results, and you see strange tails that are very large and fat, that slowly go off into infinity, it often means that the investment strategy was not tailored to the liabilities. What if you just throw an investment strategy into your dynamic financial analysis model without matching your liabilities very well. It will then require you to have greater capital. Now that's actually a desirable feature because, if you're not matching your investments to your liabilities, or you're not thinking in terms of hedging plus or minus some allowed variation to pick up some investment return, then you are, in fact, taking much more risk and should have more capital.

Your crediting policy and other future business decisions of pricing become a big issue. The overall issue of how you price an insurance liability is before us now with the fair value of the liability issue. As you're issuing new policies in a scenario, if you simply invented a premium formula that doesn't relate to any of the random factors, then your issue prices of your liabilities are, in essence, mis-specified.

So as we move into the problem of using these systems to set company capital and possibly the fair value of liabilities, we need to become aware that it's a recursive problem. But if you're issuing liabilities during the scenario, the premium that you're charging really needs to relate in some rational economic way to the random factors that are forcing the model. So those would be interest rates, equity rates, or whatever else might be random. So what is the need for the uniform valuation system (UVS)? Current reserving capital methods are formula driven and don't necessarily relate to the actual risks in distribution of future events. Now a great deal of work is done to try to make these formulas be very good, and they're not just dreamed up out of the air and then plopped down. Some very careful simulation work is usually done, and people try to tailor formulas that will work as if you had done a simulation. The problem is it's not an automatically self-adjusting process. It really can't work perfectly. There are some flaws to it. As new products come along, there tend to be gaps until those products get analyzed. Once they are, it's still not a perfect solution.

The actuary's judgment is not involved. In the P&C world, the actuary has to certify the reserves. In the life world, we have formulas. Involving the actuary's judgment to say that the capital is sufficient means that there is some human paying attention to the system who's responsible for saying that we do, in fact, have sufficient capital given what our business plan is. The standard may be maintaining the company as an ongoing concern, not merely to scrape by with the minimum of solvency.

Let's discuss the issues in UVS interest rate simulation. We're doing a long-term simulation of the entire yield curve, so maturity is a bond from 0 to 30 years over a 30-year horizon, and we will be, of course, reissuing bonds during that horizon. There are additional economic variables and the relationship to the yield curve. So we might have inflation. If the level of rates is higher or when curves are inverted, inflation tends to be higher. We also have and modeled unemployment and multiple equity indices which have a certain correlation.

As we apply this, you need to have a model that you can think with. It should be one that can be used by the person at the firm who produces scenarios and sends them to everyone else. In addition, the firm should have a conceptual understanding of the model that it's using and what its implications are (whether they are for pricing business, risk, and so forth). It needs to be a model that everyone in the firm can understand. They need to understand what's driving the model.

You have the real world probability version of the model that Steve talked about. You also have the risk-neutral version where you have essentially changed the parameters of your model to go from the real probability to the risk-neutral. You alter certain parameters. If it's mean-reverting, what is the ultimate target rate of interest rates? That number will be higher for risk-neutral than for real. Real might be 5% or 5.5%. Risk-neutral might be at 7% or 7.5%. That reflects a risk premium against people who hold long-term bonds. They are taking interest rate risks and they're being compensated by pricing those bonds as though the risk-neutral measure rates were headed to 7.5%. That gives us the upward slope of the yield curve.

Similarly, if it's a multi-factor model, there's a risk premium for yield-curve-shaped risk, so there is the risk that short rates could exceed long rates. Those risk-premium functions don't get much discussion. They're discussed a bit in some of the more theoretical papers and in the academic literature, but in the applied world they tend to be forgotten. We talked about the risk-neutral version of the model or even a separate risk-neutral model in the real world version as though they were two separate models but had no real relationship to each other. However, they really are very much related, and they're related by the issue of what additional expected return you get for taking an additional unit of risk that receives a risk premium.

Take the case of a one-factor model whose one factor is the short-term rate. For the volatility of that short-term rate over each instance in time, you get a certain risk premium. If you take one unit of short-term volatility risk, you get a certain coefficient. If you take two units, you get twice as much. The value is going to drive the results about your business in a very large way. If your business plan is to take short-term interest rate risk or, in general, to have your assets longer in duration than your liabilities, say by a year or two, then you're basically saying that you're getting a risk premium for taking that risk. So the parameter in your model that is the value of that risk premium is crucial to all the results of the simulation. If you don't even know what that number is, then you're running the simulation without even understanding the key number.

You can set up a complicated portfolio of assets and liabilities but, if the stochastic driver is an interest rate variable that receives a certain risk premium, that all boils down to how much of that risk you are taking and how much of a reward you get for it. It really comes down to that one parameter and not to the complexity of all these different bonds you might hold. It really comes down to the exposure to each stochastic risk factor and what reward you get for holding that risk.

Let's discuss the UVS requirements on an interest rate model. The first things we might think about are these realistic probability things we've thought about in the past. Dave Becker and Steve Craighead have greatly influenced my thinking. There is the ability to replicate historical yield curves; there are things like frequencies of yield curve inversions as well as general qualitative relationships. There's frequency distribution of short and long rates. If we decide to look over some period of time, that's a subjective decision. Let's say it's 1955 to 1999 or 2000. How often were rates between 5% and 6% for three months, three years, or even ten years? So we need to do that for different points on the curve.

We want to capture episodes where we have high rates for a long period of time or low rates for a long period of time. We need to capture those episodes if we're going to capture things like the risk of guarantees or the risks of high interest rates or inversions.

The fifth point that needs to be formulated, in terms of the small set of meaningful economic variables, then comes back to relating to those episodes. What periods do we calibrate? How important is one period to low rates of the 1950s versus the high rates of the early 1980s? Is our current experience now fundamentally different than the past because the federal reserve finally learned how to master the economy and can never slip away from that? That has been the theory and common opinion, basically from the time the Federal Reserve Act was first enacted.

Each year everyone has said the Federal Reserve can finally manage the system in order to avoid inflation and depression. That has uniformly been the concept, and it has been wrong up until the last five years. So you might want a model you can think about, just in case Greenspan is replaced and the next person is not quite so God-like. In that case, you might want to think about these episodes in the past.

As you think about them, you're also essentially making some decisions when you parameterize how high or low rates can get or what the probabilities are. You're also influencing these risk-premium functions, and it's important to think about that as you do this work because, if the risk premium is high for bearing a risk, then you don't need as much in assets to cover the liabilities to get to a certain percentile. If you're looking at a 95th percentile or a 98th percentile for solvency, then, if you get a bigger pick-up for bearing risk, it's going to push you a bit in the direction of needing less initial assets. We need to be very much aware that this is the assumption that's driving the model results and not get lost in the complexity of all of our bonds and all our liabilities. So the model needs to accurately quantify the length and duration of low rates and the volatility of rates.

So the parameters of the model are going to control its behavior. We can think of those in familiar terms as the speed of mean version of the level of rates from low or high to some normal level. If the yield curve shape is inverted or extremely steep, how fast does it get back to a normal shape? As you think about volatility and short-term and long-term rates, don't think just about these as probabilities we calibrate to the past. We're just reproducing statistics. These decisions are also saying what the probability of your expected returns are. So you're making decisions on your risk-premium function as you set these probabilities.

Let's say you don't use an arbitrage-free model for pricing assets and, in general, for the simulation. Let's say you're issuing assets in your DFA model during this scenario. The prices of those assets don't relate to your probabilities about interest rate movements and some risk-premium functions because your model is not arbitrage-free and, therefore, those risk premium functions don't exist. Then, you're basically making up your issue prices. At a minimum, you'd have to think a lot more about whether those were sensible prices.

The fundamental problem is if you don't have an arbitrage-free model, then because there's the possibility of making a return with zero investment and zero risk, it means, in essence, the risk-premium function has a portion or has an infinity in it. So the risk-premium function doesn't exist as a well-defined function. You say that for bearing a certain level of short-term rate risk or long-term rate risk, you get a certain return. There is no such trade-off that's properly defined in a mathematical sense if the model contains arbitrage, which means that, fundamentally, there's no meaning to the results.

So you need to use an arbitrage-free model. You need to have these risk-premium functions well-defined. So there's, in essence, three things we can think of and two sets of parameters. We have the parameters for the real probabilities: there is the speed of mean version, the level of rates, and the slope of the curve. The same thing applies to risk-neutral. Then what goes between the two is the risk-premium functions. The risk-premium functions we can think of relate historically to expected returns or realized returns on different bond strategies. The risk-neutral functions determine how well we can price cross sections of yield curves or other securities. The real probabilities are how often rates are high or low, or whether yield curves are inverted.

We have three different things to look at and two sets of parameters. If we look at only two and calibrate them separately without thinking about it, one of them has to give away and be poor. You really have a very delicate balancing act to come up with a proper model. If you don't do that well, then the whole result of the exercise is not very meaningful.

The interest-rate model we're using is the double-mean-reverting process. As Steve pointed out, it is an arbitrage-free model in the academic sense that you can't make risk-less profits with no possibility of loss. It's used for the interest rate model generation, not for asset pricing in the scenario. That's one limitation. However, I don't believe we're issuing anything in the horizon other than Treasury bonds; therefore, it should not be a problem.

So this is the double-mean reverting process. It's on the logarithm of the short-term rate, which is mean-reverting to a target. The target mean reverts to an ultimate target.

Some additional economic variable are multiple stock indices, dividend yield, unemployment, and bond and stock portfolios.

Let's discuss the convergence issues. How many scenarios are enough? We've tried to use 50 to 100 and 250. For many purposes, that's adequate, but, as we get to the point now of setting capital for a company, it shouldn't depend on things like the choice of a random numeracy. It should be robust. If we mean it's capital set at the 98th percentile or the 95th percentile, it should be meaningful that we've actually calculated that and not something that could have been several percentiles off or have a large error. If we want the 98th percentile with 1,000 scenarios, that means its 20 scenarios beyond the 98th percentile point. If we're not using something like low discrepancy sequences, then 1,000 Monte Carlo scenarios are probably not enough.

There's a report out by the Canadian Institute of Actuaries on segregated funds. Their position is that a total of 1,000 scenarios is the minimum and probably more are needed. One of the groups there, as part of that task force, was using the double-mean-reverting process together with the low discrepancy system. It is the same one we're using for the UVS, if we have time. If we use something like low discrepancy, it might mean that we can get by with fewer scenarios than with Monte Carlo simulation, but it doesn't mean we can get by with as few scenarios as we want. So we probably still need 1,000 or more with low discrepancy. In the case of Monte Carlo, we might need 5,000 or 10,000.

There is a webpage by Anargyros Papageorgio, who's a professor at Columbia, and he has done some studies in convergence even for simple things like Black-Scholes type options. He has found that it often takes 5,000 to 10,000 scenarios for the Black-Scholes price to converge on certain types of options like averaging. In our case, we should be thinking of those sorts of numbers and thinking that even with methods like low discrepancy, we're still going to be at 1,000 or more.

MR. ALTSCHULL: Our final speaker is Stephen Britt. Stephen is an asset consultant with Tillinghast–Towers Perrin in the Hartford office. He specializes in improving the investment performance of financial institutions through long-term asset allocation advice supported by stochastic modeling and optimization techniques, and other asset related areas.

Prior to joining Tillinghast in 1994, Mr. Britt worked as an actuary in the superannuation area working for a large insurance company. He qualified as an actuary in 1989 and achieved his CFA destination in 1998. Mr. Britt has had extensive involvement in most aspects of institutional asset management, including consideration of strategic asset allocation, whether investment management should be outsourced, and in the establishment of appropriate investment performance monitoring systems.

MR. STEPHEN BRITT: The topic of my talk relates to some work that we're doing. We are working for a multinational company that has global operations and wishes to allocate capital on a global basis. To do this, the company requires a globally consistent model that can be applied to most of the countries where it does business.

I'll quickly go over the case for why you might need a global system. I'll briefly talk about the architecture of the system and the challenges because there are more challenges than rewards. I'll also talk about some of the strategies so that if you're ever in the same boat, you can learn from somebody who has almost drowned a few times.

The case for the economic scenario system. If you're a multi-national insurer, and if you're wanting to be systematic about the allocation of your capital, you need to have a system that can assess, on a systematic basis, the risks to writing lines of business both in the U.S. and in other parts of the world. If you're going to do this and use a simulation approach rather than a rule of thumb or dumb luck, you are going to want an economic scenario system that is consistently generating scenarios across the world. When you're testing the capital of a line of business written in the U.K. against a line of business written here, you will be testing both of those lines of business against the same economic scenarios at the same time.

If you're at a consulting firm, such as my own or any of the larger firms, you're going to find that you're going to be doing this sort of business around the world. I can tell you it's much easier for us to develop one model and use it all around the world than to have four different people in four different parts of the world build their own models and then try to explain, in cross-country work, why the results are different and why people are talking a different language.

A U.S. insurance company is not restricted to investing only in the United States. Once it does that, you need to model all of the risks that are inherent in that investment. So there is a need for these models.

Chart 4 is an overview of a global generator, like Global CAP:Link. The top part has the base variables and we have things such as the risk-free yield curve for each country. We model gross domestic product (GDP) in the country. We do that not so much because GDP is used very much, but it's good glue that can link other parts of the model and make sure that they are consistent with the modeled price inflation.

This stuff is used in pension work and also in property/casualty work where inflation is a key determinate of claims. It models currencies as a function of these variables. After we make sure that we've generated those, we move down in the cascade, if you like, and we model earnings yield and earnings growth, which allows us to model stock returns. We will model investment grade credit spreads and noninvestment *grade* credit spreads. We are just finalizing these additions to our model.

Once you have those variables, you can then model the investables, the stocks, the bonds, the Treasuries, cash and the corporate bonds. We have this model for each country, and then we need to combine them into a global model.

Assets run in two modes. If you're not interested in the rest of the world, then you can run a single country model. We found this useful in some countries where the currency is so complex and arbitrary because it's pegged for six months and it's not pegged, and it's re-pegged, and so forth. We find it is easier to not fit these countries into a global framework. We just model that particular country. Under those circumstances, the risks of off-shore investments are not fully captured. The people who use the model recognize that.

We usually model six countries simultaneously because that usually keeps everybody happy. Our model is calibrated for about ten countries at the moment. We use it for pensions, for multinationals, for life companies, and for non-life companies.

The challenge is to build a model that is parsimonious, but that reflects the investment opportunities worldwide. If you're doing a capital allocation for a multinational insurer, the next real challenge is to formulate a set of assumptions, which is accepted by everybody as being right for the global model, but also accepted by each individual subsidiary as being right for their country. If you go to different countries, you'll find assumptions about the real return on cash, the term spread, and so forth. In different countries, people have very different views about this.

If you take everybody's own view on paper and put the model together, you end up with a real mess.

Modeling currency is certainly challenging. Forecasting currency is nearly impossible. The one redeeming feature is that it's highly volatile. As long as you have that, you've captured one of the key essences. Some economies just operate very, very differently in a developed country such as the U.S. or the U.K. A formula with a strong link between interest rates and equity markets doesn't work very well where the sovereign doesn't issue debt past about six months. These links can break down.

The main strategy is to use as few links as you possibly can and study those links carefully. The fewer links you build, the more likely it is that your model will make sense moving forward. I don't think that anybody here would be surprised to learn that a parsimonious model is more robust than any other sort of model.

If you maximize the symmetry of the model, that also helps. If you build the same equations to describe interest rates in the U.K. and in the U.S., then all you need to do is calibrate the model; that leads to considerable savings in the development effort. If you have a system to calibrate in one country, it should work in the other and so forth. That means that you're, first of all, building models for individual countries and then linking them rather than building a global model.

The links can be one of two forms. One is to have a formulaic link. The change in the earnings yield for the Canadian stock market could be some factor of the change in the earnings yield on the S&P 500. By doing that, you can build strong models that way. But if for some reason you don't want to run the U.S., then your model for Canada doesn't work anymore. So those formulaic links are useful, but they do come at a cost to flexibility.

The other way is to link the error terms, and this does make sense. If you simulate a random component being the change in the earnings yield, which will drive the stock market, and you correlate that variable for the U.S. and for Canada, you're likely to get a model that is easy to implement. If you're not running Canada, you just simply remove that part from the model and your system will still work. There are economic justifications that the linkages across the country tend to be driven on a day-to-day basis. If I'm in Australia, and if I'm finding that U.S. markets have gone into a tailspin, it's most likely that my market is going to be going into a tailspin regardless of how strong the Australian market is. So changes in valuation, such as earnings yield, is best covered that way.

We've always run three economies when we run our model. This gives us firm anchor points, and it makes it much easier to add other countries. We model U.S., Europe and Japan as being the major components of economic activity in the world with the largest financial markets.

We have our individual countries, and we're looking at combining links via currencies. We want our stock markets to be linked and we want bond yields to be linked.

There are various ways you can do this and the first approach, which is the bad approach, is to look at all the random variables in your model and to somehow work out a correlation matrix for them. You might be dealing with a large system, such as we have here. There are six countries, each with eleven variables. This model would have 245 correlations, and you really don't have a clue about the vast majority of them. So you just don't do this.

You can say that there are certain variables where you expect there to be linkages. For example, take earnings yield, which is the valuation indicator on a stock market. Let's say that the random error terms are linked across countries, and you have only six variables there giving 15 correlations to estimate. If you also add inflation, you're saying that inflation in the U.K. could be affecting U.S. equity markets. Then you've got 12 variables giving 66 correlations. Maybe the long interest rates can also be a source of the link. Maybe you now have 18 variables. So you're building an eighteen by eighteen matrix with 153 correlations, which is still pretty horrible. So you would still want to reduce the links that you have here.

One way to deal with this problem further is to say we will model a global inflation rate or a global perturbation to inflation rates. Then we will link each individual stock market and each individual inflation to this one variable. Or we'll say that in this scenario, equity markets, as a whole, will tend to fall. Then we will link each individual country to that one random shot. These are very useful because they mean that the correlations are linear and not increasing by *n* squared or thereabouts. If you use terms like this, you can build fewer and fewer links between your countries and still maintain a credible model.

Having developed the formulaic structure for your model, you have to develop the consistent assumptions. You have issues with different countries, which will have different views about what the equity risk premium is. Let's say you do this and you try to allocate capital. You have one country saying that you get a return of 8% above bonds from investing in stocks, and another country says you will get 2%. Then your results are going to be invalid because your assumptions are also invalid.

Having said that, you need to be careful. I come from a country where our risk-free debt is rated AA, as in any other country. I consider risk-free debt to be, at best, a AA-rated debt. I think that I should get a higher return from investing in the Australian debt market than in the U.S. market. This is valid. Actually incorporating that into a model becomes very, very difficult. I need to forecast how often my country's debt will default, even though it never has. So these are some of the issues with setting the assumptions.

Other challenges are modeling currency pegs and emerging markets. If you put purchasing power parity into your model, you generally end up with a model that has very good, strong characteristics because it adjusts a high inflation economy (if there are any left) with a low one. The currency tends to adjust for these effects, and that's very useful. If you use uncovered interest rate parity, then you also have very useful characteristics because you don't get apparent free lunches from investing in riskless debt across various countries. The problem is that you can't have a model that is driven solely by purchasing power parity and solely driven by uncovered interest rate parity. There is immediately some sort of a compromise there. You need to build models where the currency is behaving adequately and correctly, and adjusting for different inflation rates.

It is very difficult to work for a country that has currency pegs. The solution I've always used is to not model the sovereign debt of a country that has a currency peg. I have been able to do that so far because the countries that I've done work for didn't have any sovereign debt to speak of. That raises the question of how do you model a binomial event, such as a break in a currency peg, which sends the whole economy into complete chaos? How do you model how frequently that should occur? You can't. You can't actually sample from experience because they are so dependent on the environment at the time. In some countries where you do business, you'll actually find that you can't do business if you even suggest that the peg is going to break. That makes that one easier.

Emerging markets work differently. A model that works for the U.K. and for the U.S., France, and Germany might not work for North Korea. I don't know what you do, but you do the best that you can. In those cases, keep it simple because anything sophisticated is not going to work.

I did have a summary. The three points are this stuff is challenging, but certainly doable. The trick is, first of all, to build your links, but make them as few as possible. The third is, be very careful when you are setting assumptions.

MR. ALTSCHULL: At this point, we'd like to take any questions that you all might have. I have one question that I'd just pose to the whole panel. Stephen Britt talked in depth about developing correlation assumptions. I know that volatility is another assumption that's pretty key in developing economic scenarios. I'd just be curious about how any of you on the panel go about developing that volatility assumption.

MR. SONLIN: The models we are building were used to manage upwards of $80 billion dollars in the real world. We want to come up with practical models that will do what we need and do them in a timely fashion. When we build our models, we have a set of roughly 550 different types of facts and conditions that we're looking at and evaluating all models against. It's a lot of information, and it's a lot of data.

Most of that data are based on a pretty thorough historical review, some of the issues that Mark had addressed. We're looking at various historical periods. What historical periods are you going to look at? How much are you going to weight different historical periods? If you're looking at interest rates, in particular, the interest rate environment has changed over time. Interest rates used to be regulated.

Volatility assumptions prior to the deregulation are going to be different than volatility assumptions. There's a lot of judgment in applying those rules, and there's a lot of assumptions that go into them. There are a lot of conditions that you have to take into account when you're building these things. When you have that many, you're not going to hit on any one of them exactly.

We're trying to come as close to all these different criteria as we can to produce reasonable simulations that we can use for practical purposes in our managing of insurance company money. It's very judgmental, and there is a lot of historical analysis, and a lot data collection. It's hard to say that there's any one right answer in any modeling approach.

MR. TENNEY: I agree with that. There's no one right answer. The tendency is, of course, to always think that the last few years are what is normal and whatever has happened beyond that is abnormal. Of course, as I pointed out earlier, that means most of our history is abnormal. There's a tendency to say rates mean-revert rapidly to whatever our current level is, and there's a small volatility around that. It's a trade-off between that sort of an approach, which might be what's reflective of pricing—not simply in emerging markets, but even pricing in insurance liabilities versus a longer term picture.

If you're looking at a 30-year horizon for solvency, you probably need to have some reasonably high level of volatility or low mean reversion. Let's say you are looking at a shorter period of time. You're looking at a profit analysis and you're looking at your risk premium for unit of volatility. For that sort of work, you could use a lower volatility. You might use different measures for different things, like the profitability analysis versus solvency and capital adequacy.

MR. BRITT: We found that the further you go back, the less reliable data are, and the approach we tend to take is to fit our equations to recent data. You might use ten years worth of data and then take a look at what that means. Then you look at the longer history of the data, and we'll be quite happy to adjust the results of our curve fitting to capture some of the broader characteristics that we've seen in longer time periods.

The key example of that is I think interest rates. You saw interest rates that were very stable up until about 1979, and then they went completely crazy for about four years. The volatility has been falling consistently ever since. You might capture the essence of those four years and say, "Are we going to reflect that in our model, or are we going to average it? Are we going to ignore it?" That's just a judgment call, but you can't fit complicated sophisticated models to the data that was created in the 1920s, the 1930s and the 1940s because of the errors that occurred in the recording of the data. I mean you're fitting a model to noise.

FROM THE FLOOR: A question for Steve, I think. I think I missed something on the risk-neutral-scenario discussion. The risk-neutral-scenarios assume that all asset classes have the same return.

MR. SONLIN: Right.

FROM THE FLOOR: Are they also arbitrage-free?

MR. SONLIN: Correct.

FROM THE FLOOR: So how do you replicate the observed prices in a set of scenarios if you assume all these asset classes have the same expected return?

MR. SONLIN: That's the magic of the process. Actually, it's more of a mathematical technique. The key differentiation between risk-analysis models and pricing models is, in fact, this. The simplicity of the whole thing is that, in the real world, we don't know what the discount rate is. We can predict out the cash flows, but we don't know what rate to discount those cash flows at. It will depend on the risk premium you associate with the riskiness of those cash flows.

The pricing models get into some pretty complex mathematics. You can translate real world scenarios and real world probabilities into risk-neutral probabilities by changing some growth assumptions. I don't know all the detailed mathematics. Mark would probably be better at answering this than I would be. Once we translate everything into a risk-neutral world, we do know what to discount the cash flows at because there's only one rate in that world, and that's the risk-free rate of return. So we know what the cash flows are. We know what the rates to discount are, and we can come up with the correct price. That's because the growth assumption also has to be adjusted in those risk-neutral world scenarios.

As Mark said, you're going to change the rate you're reverting to. Many of the calibration parameters in risk-neutral modeling will change so that the sophisticated term is, "it all comes out in the wash." You project out at a certain growth rate, and you can discount back at the risk-free rate in this risk-neutral world. You'll actually come up with the correct price. You might want to add a little more mathematical rigor to my description there.

MR. TENNEY: The random value of that state variable determines the prices of all the securities, which is conditional on whatever parameters you have for the risk-neutral probabilities. The rates of the meaner version, the target rate, the volatility for the risk-neutral probability are for some fixed numerical values.

Rather than think of the market price as starting with the actual market price, think instead that we're in the model world and we're going to determine what the prices are. We simulate. For any given security, we simulate the short-term rate. We find the cash flow for the security and discount at the short-term rate, where we simulated the movements of the short rate with the probabilities implied by the risk-neutral parameter values. So the rate of the mean reversion is for the risk-neutral world and the target rate. So you generate the cash flows in the scenarios and discount them into the short-term rate. Those are the prices that the securities should have.

We've constructed the prices that way by discounting it at the short-term rate. If we project forward in the risk-neutral world, every security earns the short-term rate because you've taken the equally weighted average of the different scenarios by discounting it to the short-term rate to get the price. By projecting forward, they'll grow at the short-term rate on average for each security.

Then, with the real world probabilities, you have different values for the rate of mean reversion of the target rate, but the same value for the volatility. That would determine the frequency of rate reversions.

The final issue comes up when your starting market prices are different than the prices you just calculated with your risk-neutral rate. You calculated a three-month bond yield, a six-month bond price, and then the yield at one year, two years, three years, five years, ten years, and thirty years. Then you compare them to the actual yield curve, and you find that they're totally different. That then involves a calibration step.

If you have a couple of random factors, like a short rate and a target rate, you can try to calibrate those two factors to give you the closest fit to the actual observed yield curve. There's some residual spread there for each bond that you can model as an option-adjusted spread. You can decide to put some of the parameters of your risk-neutral model, like the forward rate parameters, where they're a function of time, to fit your initial curve.

When you do that, and if you keep your real probability the same and recalibrate the risk-neutral probability, you've recalibrated your risk premiums without realizing it. This offset requires thinking with the model. The use of it for something like UVS or for managing a company requires understanding what drives the results and how those drivers are changed by calibration decisions. If you change your risk-neutral parameters each time you calibrate to the yield curve or option prices, then you have changed your expected rates of return for bearing risks for having longer asset duration than your liability. If you've priced too long, it's a business and development.

The whole strategy is for your company based on simulation models and thinking about strategies using a model as a tool. If every three months when the yield curve changes, your risk-neutral parameters change as a result of calibration, then that throws out all the work the company has done to come up with an understanding of its profitability and risk-taking. So the issue of what's going on with risk-neutral is really when you have to understand very clearly the use of these models. The key characteristics are changing because no one realizes that those are the characteristics to pay attention to. It's changing the answer whether the company makes money, whether it's making the right bets, and whether it has the right capital.

MR. BRITT: The risk-neutral world is a strange one. I'll leave it as a puzzle for the audience to work out whether people look before they leap into in a risk-neutral world.

MR. SONLIN: It does change when you're valuing an option in a risk-neutral world. The cash flows themselves are different and the discount rate you're using is different than what they would be in the real world. You're really dealing with just a different view of valuing or pricing options, but that world is a very convenient and very nice mathematical world. It creates a lot of conveniences for doing pricing work. Everything is distorted because that's not the world we live in.

But the mathematics actually come back out so that the price we get in the risk-neutral world is identical to the price we get in the real world. It's a very complex mathematical phenomenon to make that happen, but I don't want you going around thinking, "We have the same cash flows in both worlds, and we have different discount rates." How can you get the right price? Everything changes when you move to the risk-neutral world, but mathematically it's a convenient way of coming up with the correct price.

MR. BRITT: If you have a large universe of bonds and you can completely replicate the payoffs of your claim of your insurance contract solely by investing in those bonds, then you know the value of your insurance contract right now. It's the value of the bonds that you have to hold now to completely replicate the portfolio. The trick is to find out what the particular combinations of bonds should be now. What's just as important is deciding how should you trade those into the future to completely replicate the payoff, regardless of what happens in the future. It is a very difficult exercise.

If you're good at linear algebra, you can go through the mathematics. That shows that if you work out an expected value using the risk-neutral probabilities (called a Q measure), you get the right answer. Otherwise, we would have a real tough time pricing.

FROM THE FLOOR: I was wondering what each of your opinions would be on the practical use of representative interest rate scenarios for ALM, other risk management, and those types of things?

MR. TENNEY: Maybe I could comment on that. You take a number of different measures like frequency inversions. How often are three-month rates between 3% and 6%? You have those calculated for some historical period, and then you try to produce a set of scenarios like 50 or 100 that comes as close as possible on those different measures. Then you apply it to calculate something like profitability or capital, or the 98^{th} percentile of surplus at some future date.

It's not formulated as a rigorous theory that says you have certain limits. As you take certain limits, you get certain conversions. It has been formulated as an idea or a hypothesis. There has been some testing of it from an academic point of view. That might be unfair as an ad hoc point of view. But it's not based on a mathematical theorem proved structure. Nonetheless, it has been used in some cases, and it has been beneficial.

In contrast, there are methods like low discrepancy sequences or quasi Monte Carlo that are based on the mathematical theory of convergence. However, when you apply them to small samples, there's no guarantee of what the error bound is going to be for applying it to that small sample. Of course, 100 scenarios or 500 is a small sample. So while you have a theorem in the case of low discrepancy that you're going to get convergence, it is a conditional convergence. With both methods, it's incumbent on the user to test convergence.

In the case of representative scenarios you don't, for example, pick 200 scenarios with a particular random numeracy, and then find 50 random scenarios or 50 selected scenarios that replicate those 200. You would have to find a set of representative scenarios, such that you would have to test against, 10,000, 50,000 or some large number in order to see how well you really are converging within your test space. The problem is you can extrapolate that to something that's costly to calculate for large numbers of scenarios. You don't even have the support of a theorem to tell you whether it is a valid extrapolation or not.

MR. SONLIN: When you're talking about 1,000 scenarios, you can just forget it on some of the applications that we have. Mathematically, that might be the right answer. You might need that many scenarios to converge, but it is practically not going to work. You have to weigh these things out. You want to be as rigorous academically as you can with the work you're doing, but you have to get it done. At least, we have to get it done in our field.

We do the best we can with the smallest number of samples we can. We try and get a good feel for how robust the answers are that we're getting. I think, in a lot of cases, you have to do things that are considered shortcuts—things beyond what the pure right mathematical theoretical answer is. That's unfortunate and maybe technology will advance at a pace where that won't always have to be the case, but the constant pressure and questions is, should I do it in an academically correct way, or should I get it done? My answer is, I have to get it done. My job depends on getting an answer, and it also depends on getting it as right as I can get it. So that's just a trade-off that all practitioners have to make.

CHART 1
Regression Models

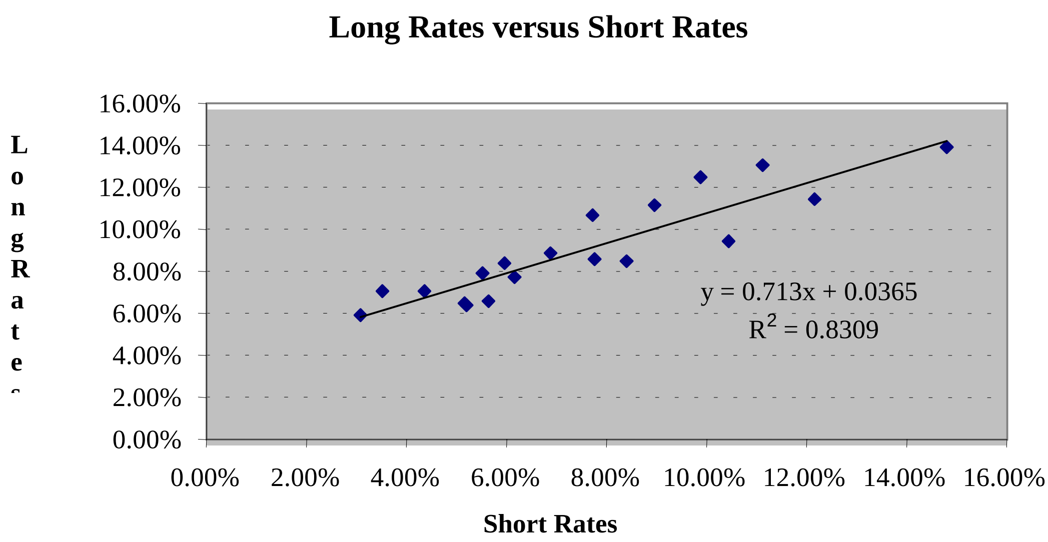


CHART 2
Lattice/Tree Structures

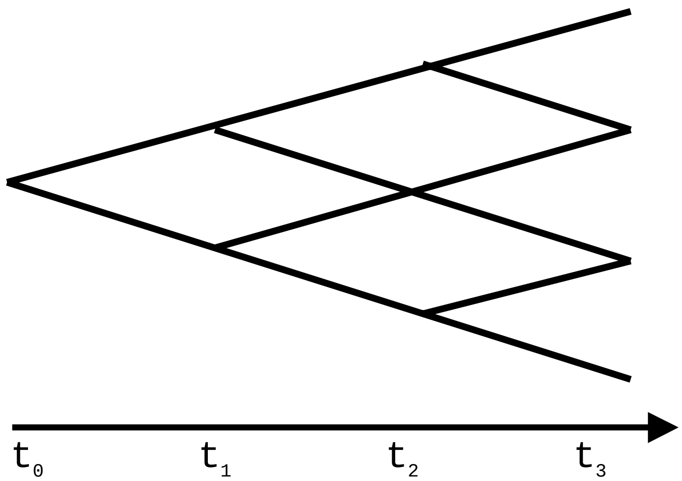
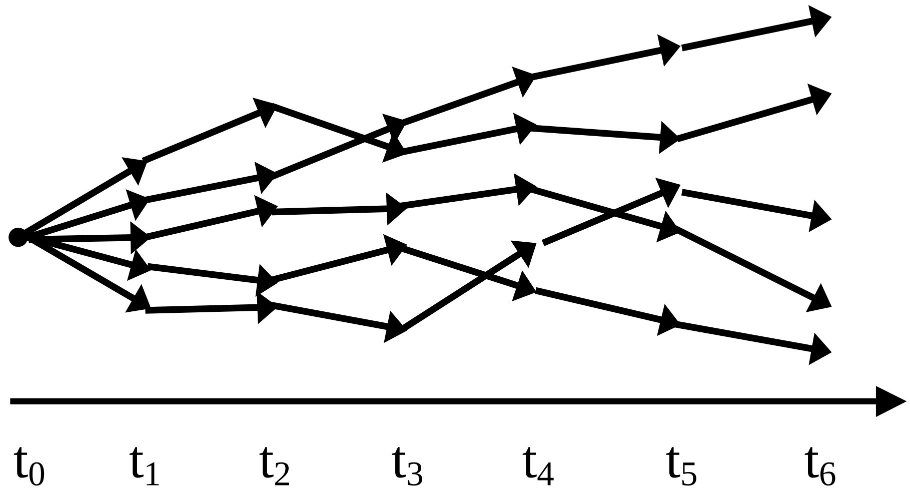


CHART 3
Stochastic Differential Equations



Note: $dx = \kappa(\mu - x)\,dt + \sigma x^{\gamma} dw$ (mean-reverting random walk)

CHART 4
Global CAP:Link