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DEFINITIONS FOR COMPOUND AND SIMPLE INTEREST

by James D. Broffitt and Stuart Klugman

In the cases of compound and simple interest, the accumulation function, $a(t)$, is easily defined for integer values of t. We address the question of how to extend these definitions to include noninteger values of t. Our definitions for compound and simple interest are motivated by a reinvestment example which embodies the basic notion that interest earns interest under compound interest but not under simple interest. From these definitions we obtain $a(t) = (1 + i)^t$ and $a(t) = 1 + it$, for all t, under compound and simple interest respectively.

The fact that compound interest demands $a(t) = (1 + i)^t$ for all t does not automatically follow from knowing $a(t) = (1 + i)^t$ for integer t. This result must depend on some statement about the behavior of $a(t)$ for noninteger t. We suggest the following definition for compound interest.

Definition 1: Interest is said to be compounded at annual rate i if (1) $a(1) = 1 + i$ and (2) $a(t + s) = a(t)a(s)$ for all real s and t.

The second statement may be explained as follows: A \$1 investment accumulates to $a(t + s)$ after $t + s$ years. If, however, the accumulated value is withdrawn after just t years and immediately reinvested, the investment will grow to $a(t)a(s)$ after s additional years. The definition requires that the final accumulated value be unaffected by the intermediate transaction. Clearly compounding is occurring since interest earned during the first t years, earns interest during the final s years. The appropriate theorem is:

Theorem 1: If interest is compounded at rate i and $a(t)$ is differentiable for all t, then $a(t) = (1 + i)^t$.

Proof: $$a'(t) = \lim_{s\to 0} \frac{a(t+s)-a(t)}{s} = \lim_{s\to 0} \frac{a(t)(a(s)-1)}{s} = a(t)a'(0)$$

Therefore $$\frac{a'(t)}{a(t)} = a'(0) \text{ and so } \frac{d}{dt} \ln a(t) = a'(0),$$

which implies $\ln a(t) = a'(0)t + c$. From $a(0) = 1$ and $a(1) = 1 + i$, we obtain $c = 0$ and $a'(0) = \ln(1 + i)$. Consequently $a(t) = (1 + i)^t$.

We also note that simple interest may be developed in a similar manner.

Definition 2: Interest is said to be simple at annual rate i if (1) $a(1) = 1+i$ and (2) $a(t+s) = a(t)+a(s)-1$ for all real s and t.

The motivation for (2) is provided by the same reinvestment example. The value after t years is $a(t) = 1 + [a(t) - 1]$, which has been separated into principal and interest components. Since we want only the principal to earn interest, the final value is $a(s) + [a(t) - 1]$.

Theorem 2: If interest is simple at rate i and $a(t)$ is differentiable for all t, then $a(t) = 1 + it$.

The proof is analogous to that of Theorem 1. In this case $a(t) = a'(0)t+c$ and the constants are determined from $a(0) = 1$ and $a(1) = 1 + i$. □

"PRELIMINARY ACTUARIAL EXAMS"

The 1982 edition is now available gratis from the Society office in Chicago.

In addition to current information on the first two examinations, it contains 44 pages of sample Part 1 and Part 2 questions from the November 1981 and May 1982 exams. Sample Examination booklets will no longer be furnished separately.

Suzanne L. Hunziker

NEW SOCIETY APPLICATION FORM

The Society's APPLICATION FOR ADMISSION AS ASSOCIATE has been revised—mainly to remove the nomination requirement made obsolete by 1982 vote of the Fellows.

Please destroy copies of the old form that your organization may have on hand, and request copies of the new form from the Society's Education and Examination Department.

Suzanne L. Hunziker

THE 1976 AND 1981 RESTRUCTURINGS OF OUR FELLOWSHIP EXAMINATIONS

by Linden N. Cole

In the light of the increasing pace of change in our society, it is not surprising that there have been changes in the Society of Actuaries' education and examination system. There were, in fact, two such changes in only five years.

The 1976 Restructuring

The objective of the 1976 restructuring was to have each examination cover a major subject area applicable to all specialties.

- Part 6 • covered "Assumption of the Risk," including a description of coverages, selection of risks, and marketing.
- Part 7 • covered the "Balance Sheet," both the valuation of liabilities and of assets.
- Part 8 • covered "Paying for the Risk," such as gross premiums and expense analysis.
- Part 9 • covered "The Outside World," including law, taxation, social insurance, and the Annual Statement.

The idea was that the principles involved in each examination could be extended to any product line, helping to make the actuary a very flexible person. Our students would not simply study how past generations had calculated gross premiums for life insurance; they would study "Pricing".

Once implemented, this system proved to have its problems. First of all, the subject of Pensions could not be forced into the structure of the Fellowship examinations. Paying for pensions turned out to be inseparable from valuing the pension liabilities. Thus, the initial concept broke down in a crucial area. Secondly, the new system proved to be relatively inflexible. As there were changes in the law and in the environment, the system could not be adjusted. Finally, most of the changes were occurring in the outside world, and Part 9 was getting longer and longer.

The 1981 Restructuring

The next restructuring occurred in 1981, only five years after the previous one. The new structure was designed

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