Article From:

The Actuary

January 1983 – Volume No. 17, Issue No. 1

ON THE ANALYSIS OF FRACTIONAL PREMIUMS

by James D. Broffitt

Jordan's excellent text on life contingencies teaches a technique for determining a relation between net fractional and net annual premiums, which is based on an analysis of the annual loss of interest and the loss of premium in the year of death. For ordinary life the result is

$$(1) \quad P_x^{(m)} \doteq P_x + \frac{m-1}{2m} P_x^{(m)} d + \frac{m-1}{2m} P_x^{(m)} P_x$$

which coincides with the standard approximation. The terms $\frac{m-1}{2m} P_x^{(m)} d$ and $\frac{m-1}{2m} P_x^{(m)}$ are purported to represent the annual loss of interest, and the loss of premium in the year of death, respectively, experienced by the insurer.

The technique is to start with P_x and add the appropriate adjustments to obtain $P_x^{(m)}$. To logically accomplish this we must consider m^{thly} payments of $\frac{1}{m} P_x$, and determine the resulting losses incurred by the insurer due to spreading P_x over m payments. Then we may add to P_x the additional annual amount, payable by the insured on an m^{thly} basis, needed to bring the annual premium up to $P_x^{(m)}$. Since the annual loss of interest and loss of premium in the year of death are approximately $\frac{m-1}{2m} P_x d$ and $\frac{m-1}{2m} P_x$, the relation obtained is

$$(2) \quad P_x^{(m)} \doteq P_x + \frac{m-1}{2m} P_x d + \frac{m-1}{2m} P_x P_x^{(m)}$$

which disagrees with (1).

The fallacy with Jordan's argument is that when net premiums of $\frac{1}{m} P_x^{(m)}$ are paid m times a year, there is no loss of interest or premium which needs to be made up by the insured, since the insured is making correct premium payments.

Although (2) is reasonable we prefer a logically correct analysis which will produce the standard approximation. This may be accomplished by modifying the previous argument. Rather than starting with P_x, we will start with $P_x^{(m)}$ and make appropriate adjustments to obtain P_x. That is, we will put $P_x^{(m)}$ on an annual payment basis, by assuming $P_x^{(m)}$ is paid to the insurer at the start of each year. This results in an annual overpayment of interest, and an overpayment of premium in the year of death, of approximate amounts $\frac{m-1}{2m} P_x^{(m)} d$ and $\frac{m-1}{2m} P_x^{(m)}$, respectively. To get P_x we must subtract from $P_x^{(m)}$ the amount (payable annually) needed to refund the insured for these overpayments. The result is

$$(3) \quad P_x \doteq P_x^{(m)} - \frac{m-1}{2m} P_x^{(m)} d - \frac{m-1}{2m} P_x^{(m)} P_x$$

which agrees with (1).

Finally we note that the argument used to obtain (3) works for installment and apportionable premiums, and for more complicated cases such as limited pay endowment policies. □

SIGHTINGS

The quantity and general quality of submissions of book and press references to actuaries continue high. We are most grateful to our many correspondents, and will print as many of these fine items here as space will permit.

Frederick R. Rickers sent us a delightful essay from the May 1982 *Contract Bridge Bulletin* marking our colleague Oswald Jacoby's golden wedding as well as his 80th birthday and his 10,000th syndicated bridge column. Its photos of Mr. and Mrs. Jacoby in 1932 and 1982 show them in fine fettle. There is just a brief reference to his life insurance career, from 1919 till 1928, and to his respect for, as well as his spectacular record in, passing the actuarial examinations.

J. Bruce MacDonald in Canada and *Ellis A. Wohlner* in Sweden both sent us an item in *The Economist* containing this comparison:

> "Just as an insurance actuary can tell you the average expected lifetime of a child born in Britain today, so a quantum physicist can tell you what the average lifetime of a group of radioactive particles will be. But, just as the actuary cannot say when any particular person will die, nor can the physicist tell you when one particular radioactive particle will decay."

Robert A. Moreen found a reference in classical literature that we don't recall having been picked up before. In Charles Dickens' mystery "Hunted Down", one of the prominent characters is the young actuary of the Inestimable Insurance Company whose boss describes him as "at once the most profound, the most original, and the most energetic man I have ever known *connected with life insurance*" (emphasis added). Mr. Moreen recommends the story, which is reprinted in the Arbor House Treasury of Mystery and Suspense.

Robert J. Myers showed us that Duke University in North Carolina has come up with a research project entitled "Bio-actuarial estimates and forecasts of health care needs and disability."

Douglas S. Van Dam saw in the Louisville, Ky., Courier-Journal a municipal matter described as "normally the stuff to make even an actuary yawn." □