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DEFINITIONS FOR COMPOUND AND SIMPLE INTEREST

by James D. Broffitt and Stuart Klugman

In the cases of compound and simple interest, the accumulation function, $a(t)$, is easily defined for integer values of t. We address the question of how to extend these definitions to include noninteger values of t. Our definitions for compound and simple interest are motivated by a reinvestment example which embodies the basic notion that interest earns interest under compound interest but not under simple interest. From these definitions we obtain $a(t) = (1 + i)^t$ and $a(t) = 1 + it$, for all t, under compound and simple interest respectively.

The fact that compound interest demands $a(t) = (1 + i)^t$ for all t does not automatically follow from knowing $a(t) = (1 + i)^t$ for integer t. This result must depend on some statement about the behavior of $a(t)$ for noninteger t. We suggest the following definition for compound interest.

Definition 1: Interest is said to be compounded at annual rate i if (1) $a(1) = 1 + i$ and (2) $a(t + s) = a(t)a(s)$ for all real s and t.

The second statement may be explained as follows: A \$1 investment accumulates to $a(t + s)$ after $t + s$ years. If, however, the accumulated value is withdrawn after just t years and immediately reinvested, the investment will grow to $a(t)a(s)$ after s additional years. The definition requires that the final accumulated value be unaffected by the intermediate transaction. Clearly compounding is occurring since interest earned during the first t years, earns interest during the final s years. The appropriate theorem is:

Theorem 1: If interest is compounded at rate i and $a(t)$ is differentiable for all t, then $a(t) = (1 + i)^t$.

Proof: $$a'(t) = \lim_{s\to 0} \frac{a(t+s)-a(t)}{s} = \lim_{s\to 0} \frac{a(t)(a(s)-1)}{s} = a(t)a'(0)$$

Therefore $$\frac{a'(t)}{a(t)} = a'(0) \text{ and so } \frac{d}{dt} \ln a(t) = a'(0),$$

which implies $\ln a(t) = a'(0)t + c$. From $a(0) = 1$ and $a(1) = 1 + i$, we obtain $c = 0$ and $a'(0) = \ln(1 + i)$. Consequently $a(t) = (1 + i)^t$.

We also note that simple interest may be developed in a similar manner.

Definition 2: Interest is said to be simple at annual rate i if (1) $a(1) = 1+i$ and (2) $a(t+s) = a(t)+a(s)-1$ for all real s and t.

The motivation for (2) is provided by the same reinvestment example. The value after t years is $a(t) = 1 + [a(t) - 1]$, which has been separated into principal and interest components. Since we want only the principal to earn interest, the final value is $a(s) + [a(t) - 1]$.

Theorem 2: If interest is simple at rate i and $a(t)$ is differentiable for all t, then $a(t) = 1 + it$.

The proof is analogous to that of Theorem 1. In this case $a(t) = a'(0)t+c$ and the constants are determined from $a(0) = 1$ and $a(1) = 1 + i$. □

"PRELIMINARY ACTUARIAL EXAMS"

The 1982 edition is now available gratis from the Society office in Chicago.

In addition to current information on the first two examinations, it contains 44 pages of sample Part 1 and Part 2 questions from the November 1981 and May 1982 exams. Sample Examination booklets will no longer be furnished separately.

Suzanne L. Hunziker

NEW SOCIETY APPLICATION FORM

The Society's APPLICATION FOR ADMISSION AS ASSOCIATE has been revised—mainly to remove the nomination requirement made obsolete by 1982 vote of the Fellows.

Please destroy copies of the old form that your organization may have on hand, and request copies of the new form from the Society's Education and Examination Department.

Suzanne L. Hunziker

THE 1976 AND 1981 RESTRUCTURINGS OF OUR FELLOWSHIP EXAMINATIONS

by Linden N. Cole

In the light of the increasing pace of change in our society, it is not surprising that there have been changes in the Society of Actuaries' education and examination system. There were, in fact, two such changes in only five years.

The 1976 Restructuring

The objective of the 1976 restructuring was to have each examination cover a major subject area applicable to all specialties.

- Part 6 • covered "Assumption of the Risk," including a description of coverages, selection of risks, and marketing.
- Part 7 • covered the "Balance Sheet," both the valuation of liabilities and of assets.
- Part 8 • covered "Paying for the Risk," such as gross premiums and expense analysis.
- Part 9 • covered "The Outside World," including law, taxation, social insurance, and the Annual Statement.

The idea was that the principles involved in each examination could be extended to any product line, helping to make the actuary a very flexible person. Our students would not simply study how past generations had calculated gross premiums for life insurance; they would study "Pricing".

Once implemented, this system proved to have its problems. First of all, the subject of Pensions could not be forced into the structure of the Fellowship examinations. Paying for pensions turned out to be inseparable from valuing the pension liabilities. Thus, the initial concept broke down in a crucial area. Secondly, the new system proved to be relatively inflexible. As there were changes in the law and in the environment, the system could not be adjusted. Finally, most of the changes were occurring in the outside world, and Part 9 was getting longer and longer.

The 1981 Restructuring

The next restructuring occurred in 1981, only five years after the previous one. The new structure was designed

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quantitative methods arising from other disciplines to solve new problems and to add new perspectives to old problems. Our educational goal, though, should be to train *generalists*, not *specialists*.

The syllabus should present a wide range of mathematical topics which have, or potentially have, useful applications to practical actuarial problems, or which help actuaries to communicate effectively with those in allied professions. It should develop those topics in a way that emphasizes fundamental principles and concepts, and that reveals the limitations of techniques and the necessity for scrutinizing results for reasonableness and for consistency with the underlying assumptions.

The result is that the following topics will be on the new Part 3 syllabus:

Operations Research: Linear and dynamic programming, decision analysis, queuing theory, project scheduling, simulation.

Applied Statistics: Regression analysis, analysis of variance, time series analysis.

The course of reading will include parts of the text, *Operations Research* (Holden-Day) by Hillier and Lieberman, chapter 4 of the Society's text, *A Study Manual For Operations Research*, Eugene A. Narragon (Ed.), and parts of *Intermediate Business Statistics* (Holt, Rinehart and Winston) by R. Miller and D. Wichern.

These syllabus changes, as well as some in the readings for numerical methods, will take effect for the May 1983 examination, its length becoming four hours.

A study note on simulation is being written, and one on actuarial applications of applied statistics is planned; these will be effective no earlier than November 1983. Also, changes in the 1984 syllabus are being developed by the Task Force on Numerical Methods and Graduation.

Anyone wishing a copy of this Task Force's Report or more particulars on the 1983 Part 3 syllabus, ask Linden N. Cole at the Society office.

Mathematical Aspects of Demography, (Judy A. Faucett, Chmn.)

Demography, a topic long covered on Part 5, was reduced in 1979 to Mathematical Aspects of Demography with the intent that non-mathematical aspects would be placed on a Fellowship exam. Unfortunately this hasn't yet been done, so students are being required to absorb mortality and demographic statistics without guidance on how or when to use them.

Demography is a valuable tool for actuaries; the need to forecast populations has become evident in the health, pension and other financial security fields, and ties in with increasing use of life company corporate models. This Task Force recommended that Demography be split into two sub-topics, viz.:

Survival Models and Data Analysis

- Mathematics of Mortality and Morbidity Measures
- Survival Distributions
- Fundamentals of Life Table Construction
- Studies Based on Incomplete Data
- Comparisons of Mortality Data

Population Forecasting

- Methodology & Sources of Mortality and Morbidity Measures
- Use of Government Statistics
- Mortality and Morbidity Characteristics and Trends
- Forecasting Methods

Survival Models and Data Analysis would replace Mathematical Aspects of Demography on Part 5B. Population Forecasting, the only completely new subject matter, would more appropriately go on the Fellowship syllabus.

As text, the Task Force chose *Survival Models and Data Analysis* by Regina C. Elandt-Johnson and Norman L. Johnson, the latter an F.I.A. This book, using actuarial notation, describes methods for analyzing data and constructing interpretive models, with emphasis on general principles; examples are interspersed, and each chapter offers a lengthy set of problems. One drawback is absence of solutions, but the possibility of preparing sample solutions is being explored; also, there are too many printing errors, requiring us to distribute a formidable errata list.

These Part 5B changes will be effective in 1983. A new Task Force will tackle the Population Forecasting subtopic, likely to take longer to introduce because texts are lacking. Anybody interested in being part of that Task Force or in writing study notes, please let Education Chairman Sam Gutterman know, at his Yearbook address. □

Restructuring

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with flexibility as a major objective. Also, the degree of specialization, which had been increased in the 1976 restructuring, was further increased. In the new Parts 9 and 10, there are now three specialties, and each candidate chooses a major specialty and a minor specialty. The three specialties are Individual Life Insurance and Annuities; Group Insurance and Individual Health Insurance; and Pensions.

The point about increasing specialization is worth some comment. If everyone has to learn about everything, the Education and Examination Committee is faced with some difficult decisions as the world grows more complex. There is, after all, an upper limit to the material we can ask our students to learn. Once that limit is reached, new material can be added only by deleting old material. The ultimate effect is that every subject is gradually cut down, and nothing can be treated in depth. By abandoning the objective of making everyone learn everything about every subject, and requiring pension specialists to learn some things that insurance specialists do not have to learn, and vice versa, it becomes possible once again to treat important subjects in satisfactory depth. The new syllabus requires everyone to learn something about every subject, but not everything.

So far, at least, the new structure is proving to have the flexibility hoped for it. It should last for many years because of its ability to accommodate change. Also, the pension content is stronger, and potentially much stronger. The task remaining is to examine every subject area, to assure that the study material is current and of high quality. Perhaps that will be the major task in the 1980s. □