The Actuary

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ACTUARIAL EDUCATION

by James J. Murphy

Accompanying this issue is a letter from President Lautzenheiser introducing an attached statement, *Strategic Premise for Actuarial Education*. Developed by Michael J. Cowell, 1981-82 General Chairman of the E & E Committee, its concepts, although not previously assembled into one document, have been guiding the Society's education efforts for decades. It will now provide useful background for consistent and organized changes in our education system.

While it was being written, several Education Task Forces have been preparing recommendations for changes in our syllabus. Articles by two of these Task Forces that have completed their work are printed here, viz.:

Operations Research and Applied Statistics

Mathematical Aspects of Demography

As other Task Forces complete their assignments, we will present similar reports in *The Actuary*. Also, as new Task Forces are established, we will announce them and will call for ideas and volunteers. Keep your eyes open for more news from E & E!

Operations Research and Applied Statistics, (James A. Tilley, Chmn.)

Formed in September 1981, with members drawn from both the academic and business communities, and with representation from the Canadian Institute of Actuaries and the Casualty Actuarial Society, this Task Force began by establishing criteria to guide us to decisions on syllabus and course of reading. Our starting premise was that the scope of actuarial work will continue to broaden to embrace all aspects of the financial products and services business; hence, actuaries should become familiar with

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TWO CENTURIES AGO, THE NORTHAMPTON TABLE

To mark this year's bicentennial of Richard Price's Northampton Table—the earliest mortality table to be constructed especially for life insurance premiums—we show here the contrast of its life expectancies with those of population tables of, respectively, one and two centuries later.

Complete Expectations of Life

Age	*Northampton Table (Data of 1735-1780)*	*English Life Table No. 4 (Data of 1871-1880)*	*Calendar Year 1980 U.S. Life Tables* *
0	25 yrs.**	43 yrs.	74 yrs.
20	35	41	55
35	26	30	41
50	18	20	28
65	11	11	16
80	5	5	8

*from *Actuarial Study No.* 87, Joseph F. Faber, Social Security Administration, Sept. 1982.
**q_0 in the Northampton Table was .258!

Since the two later of the above tables are sex-distinct, these figures show for them the arithmetic means of the $\mathring{e}_x$ values for males and females.

Notes on the Northampton Table

Actuary Price, well experienced by having studied mortality in other English towns and aware that he was aiming to generalize from a tiny base (a single church parish), evidently felt free to take major liberties with his data. The many adjustments he made were not in the values of q_x but in the column of deaths. For example, he showed exactly 75 deaths at every age from 21 to 39, 82 deaths each year from age 51 to 61, and 80 deaths from 64 to 75, in a table whose radix was 11,650 at age 0.

The improvement in longevity between the Northampton Table and English Life Table No. 4 is attributable in part to acknowledged overstatement of mortality in the former, and in part to progress in sanitation and medicine. Vaccination was introduced into England by physician Edward Jenner in 1796.

The Northampton Table, though by no means the earliest—Halley's Breslau table dates from 1693 and Kerrsboom's in Holland from 1738—is bound up with the history of the Equitable Life Assurance Society (of London). Griffith Davies, in his *Treatise on Annuities* (1825) quotes Price's nephew, William Morgan, thus:

> "(T)he Society had computed all their premiums from the (early 18th century) London Table of Observations (but, after seeing their experience from 1768 to 1780) they determined to compute the premiums in future from a table which should give the probability of life *higher* (emphasis in original) than that which they had hitherto used; and for this purpose they adopted one which had been just formed by Dr. Price, from very accurate observations made in the town of Northampton."

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The Actuary

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EDITORIAL

ANNUITANT MORTALITY OVER SIXTY YEARS

The figures in this message may be accepted by interested readers as evidence that your editor (i) is inspired by the recent work of Robert J. Johansen and his Committee, (ii) is intrigued by discovering that benchmarks happen to be available for annuitant mortality in the evenly spaced years 1923, 1943, 1963 and 1983, and (iii) is reckless of the pitfalls in directly comparing the mortality rates in four such widely separated actuarial explorations into this complex territory.

The actuaries in whose papers these figures have been revealed to our profession are:

Robert Henderson (1871-1942): U.S. Annuitants 1918-27, *T.A.S.A.* XXX (1929), 246.
Wilmer A. Jenkins (1901-1976) and *Edward A. Lew*: 1943 Experience Table, *T.S.A.* I (1949), 462-3.
Harold Cherry: 1963 Experience Table, *T.S.A.* XXIII (1971), 490.
Robert J. Johansen: 1983 Basic Table, *T.S.A.* XXXIII (1981), being printed.

The first of these papers was presented at a time when actuaries were becoming painfully aware of the existence of large numbers of immediate and deferred annuities and life income settlement options issued at seriously inadequate rates; the subsequent three depict instalments in the (thus far) successful actuarial campaigns to avoid any more debacles of that kind. The figures here speak eloquently of the necessity for unremitting vigilance.

	Age 60	Age 65	Age 75	Age 85
		1000 q_x — WOMEN		
1923	11.2	18.5	64.1	131.5
1943	9.2	14.9	41.3	114.5
1963	8.4	11.7	31.8	103.0
1983	4.9	8.2	22.4	72.4
1943/1923	.82	.81	.64	.87
1963/1943	.91	.79	.77	.90
1983/1963	.58	.70	.70	.70
1983/1923	.44	.44	.35	.55
		1000 q_x — MEN		
1923	18.6	31.8	73.9	146.2
1943	19.0	27.0	60.2	143.3
1963	15.2	22.0	51.2	122.4
1983	9.3	14.2	39.0	101.3
1943/1923	1.02	.85	.81	.98
1963/1943	.80	.81	.85	.85
1983/1963	.61	.65	.76	.83
1983/1923	.50	.45	.53	.69

E.J.M.

LETTERS

Cost Of A Pension Plan

Sir:

Gerald Richmond (Nov. 1982 issue) undertakes to clear up confusions about the meaning of "cost" as it relates to pension plans. While I fully agree that considerable confusion exists and that its removal is greatly to be desired, I am far from convinced that his proposals contribute to this; confusion may indeed be inevitable because of the very nature of the "cost" of a pension plan.

My doubts focus upon Mr. Richmond's rearrangement of the familiar equation of pension plan cost elements, to wit: C + I = B + E, where C stands for the contributions (from employer and employees), I is investment income, B is plan benefits and E is plan expenses. Mr. Richmond believes that, if we isolate C, which by definition is equivalent to B + E − I, we are getting close to the "actual cost" over the plan's lifetime, and that the annual cost calculations determine an approximation to the current-year value of C which eventually converges to the desired true value.

The difficulty with this is that the values of C and I are strongly affected by the timing of the plan contributions. If the funding program is such that large contributions are made early in the plan's life, the value of I will necessarily be greater than if they were made later; accordingly the sum of the annual elements of I will be greater, and the corresponding sum for C will be less. This is of little significance if C and I are on the same side of the equation but when I is treated as a subtractive element from B and E, then C, the apparent cost to the employer, is highly affected by incidence.

It cannot, I think, be maintained that this is of little importance because of the effects of discounting and the time value of money; the vagaries of the investment rate of return will certainly have a bearing upon the value of I, hence of C.

Another way to view this problem is to recognize that B and E are subject to variations in both incidence and amount. Incidence is basically determined by the plan's underlying experience, but amount reflects both the plan provisions and the impact of inflation. To a greater or lesser

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Letters

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degree, inflation also affects I, but, as experience has shown, the impact upon I is not nearly as direct as it is upon B and E. This is yet another reason why it is best to keep B and E segregated on one side of the equation. (B + E) should be considered as representing the actual cost of the *plan* (as affected by inflation as well as by the plan provisions), while C measures ultimately the actual pre-tax cost to the *employer*.

Not only is it impossible to state in advance the true cost of a pension plan; even after the plan has finally been wound up, the so-called true cost has to be modified by so many different factors —discount for interest, opportunity cost of money, etc.—that I do not believe it can ever be assigned a unique value.

Charles Barry H. Watson

* * * *

Sedentary Admiration

Sir:

As a slothful television watcher of the New York City Marathon, I was pleased to learn from the official demographics that 44* of our compatriots put on short pants and sneakers as live participants. Actuaries comprised one of the smaller identified groups, roughly matching Waiters (44), Filmmakers (43), and Bartenders (47), but easily outswarming Urban Planners (25), Politicians (18), and Security Guards (12).

The groupings are not mutually exclusive, though; perhaps some of our brethren were masquerading, e.g., as Unemployed (46) or Company Chairmen (182).

James B. Ross

*Same number as 1981 (John H. Cook, Jan. 1982 issue)—Ed.

* * * *

GERMA(I)N(E).

Sir:

Who is Hagel (Oct. issue, p. 1)? Is he any relation to Cant?

James B. Germain

Ed. Note: Our apologies to Georg Wilhelm Friedrich Hegel. We have goethe improve our proofreading.

* * * *

Osculation

Sir:

On the quality of an actuary's kiss, Warren A. Wild poses questions but doesn't provide the answers.

My wife and I are both FSA's, and thus are two of a small group of actuaries particularly well qualified to uphold the Society's motto on this matter. We do not support *Savvy's* dictum.

Peter W. Plumley

* * * *

BOOKS WANTED

The Society Library in Chicago seeks to acquire the following books:

Transactions of the International Congress:
- 16th (Brussels, 1960)
- 17th (London and Edinburgh, 1964)

ASTIN Bulletins

The Actuary's Handbook, Crocker, Sarason & Straight

Other books by Harry M. Sarason

If you are willing to part with any of these, please phone James L. Cowen, (312) 236-3833.

EMPLOYMENT INFORMATION SERVICE FOR STUDENTS

To actuarial students who have passed at least one examination, the Society will begin giving, upon request, employment information in the form of a list of potential employers, outlining standards for entry into their actuarial programs, application deadlines and salaries. This list will be arranged geographically, and revised annually.

Chief actuaries have been invited to send particulars for this list; actuarial club officers and others have been asked to help make its coverage complete. Any member who knows of employment opportunities not already submitted, please inform the Society office promptly. Deadline for 1983 list is *March 15th.*

Suzanne L. Hunziker

RUIN PROBABILITIES

Our Query for Actuaries last October—on the meaning and usefulness of calculated ruin probabilities of life insurance companies—sparked only two responses, one each from Canada and the U.S. Both we rate as much to the point; perhaps they will prompt observations from actuaries who have examined this momentous question.

Albert K. Christians said: To the man who has only a hammer, everything looks like a nail; the assiduous probabilist can calculate the probability of anything. Those who endorse promiscuous use of subjective probabilities will allow determination of a probability of even such statements as "The stock market will fall tomorrow."; "Shakespeare wrote *Hamlet*."; "God exists."

But such probabilities are subjective, representing only relative degrees of belief, so their arithmetical values depend on the extent of the ignorance on which they are based. They cannot be estimated by the objective methods of classical statistics.

Probabilists will argue that the instances given above differ only in degree from the problem of determining the probability of an individual's death, for subjective considerations are involved in that case also. In that contention the probabilists are substantially correct, but I do not believe this justifies applying the calculus of probabilities whenever probabilities can be guessed at. Indeed, application of probability models to insurance arrangements has its justification, not in analysis of appropriateness of underlying theory, but because society, satisfied with the results, endorses their use.

We should be cautious about asking society to endorse the use of probability models in new areas where their results are uncertain, such as the case in point. Quoting probabilities to support a given method or rule for reserve determination may mislead. It would be hard to convey, along with such an estimate, a thorough understanding of all the underlying assumptions.

Determination of reserve requirements for insurance companies issuing equity guarantees isn't much different from determination of margin requirements for common stock investors. Regulators of securities markets operate at some distance from actuarial theory, but they

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Ruin Probabilities

(Continued from page 3)

appear to perform reasonably without benefit of detailed probability models; they simply promulgate rules that appear to promote a desired result, i.e., orderly markets. Individuals and firms may base their own decisions on subjective probability estimates if they wish, but like action by government as a matter of public policy should be avoided if reasonable more direct methods of achieving the purpose are available.

James E. Jeffery expressed his views thus: It seems to me of little consequential difference to a life company whether it faces a stock market collapse or a catastrophic epidemic. In either case, acceptance of the risk of ruin is reasonable provided (1) prudent measures are taken to make the likelihood very small, (2) the risk takers are aware of the risk, and (3) they are reasonably compensated.

Although specific arrangements of maturity guarantees on equity products may be improper in terms of these tests, the making of such guarantees by life companies is not in itself improper.

Our thanks to these two contributors for their thought-provoking expressions.

E.J.M.

TAX SITUATIONS UNDER TEFRA

by James P. A. Knight

Passage of the Tax Equity and Fiscal Responsibility Act of 1982 (TEFRA) has further complicated the analysis of a life insurance company's tax situation. In conveying the conceptual impact of the tax law changes to company people already familiar with the principles of the 1959 Act, it is useful to develop a new classification system, based on the amount of Special Deductions allowed under Section 809(f) of the Tax Code.

For brevity, use the following notation:

T = Taxable Investment Income
G' = Gain From Operations Before Special Deductions
Q = Qualified Pension Plan Policyholder Dividends
P = Policyholder Dividends on Non-Qualified Plans
N = Non-Participating Contract Deductions
H = Group Life and A&H Deductions
$S = Q + P + N + H$ = Maximum Special Deductions
S^* = Allowable Special Deductions Under Section 809(f)
$G = G' - S^*$ = Taxable Gain From Operations
I = Taxable Income

First, note that the calculation of life company taxable income remains unchanged by TEFRA and can be written as:

$$I = \text{the smaller of } T \text{ or } G, \text{ plus } [\tfrac{1}{2}(G - T), \text{ if positive}]$$

However, TEFRA affects the calculation of both T and G. Because all companies are taxed in whole (if $G < T$), or in part (if $G > T$), on Gain From Operations, this note focuses on $G = G' - S^*$.

Before TEFRA, the effect of Section 809(f) was to set $G = T - \$250{,}000$ for many companies. This led directly to the classification system of identifying a company's tax position: a Phase I or Situation B tax was on $G = T - \$250{,}000$; a Phase II− or Situation A tax was on $G < T - \$250{,}000$; a Phase II+ or Situation D tax was on $G > T$.

Section 809(f) places a limit on certain Special Deductions (S) used to calculate the Gain From Operations (G). Shown below are pre-TEFRA and current formulations of the allowable Special Deductions (S^*) under Section 809(f).

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INDEXED-LINKED SECURITIES IN THE U.K.

by Alistair Neill

Should prices of index-linked securities move with interest rates, with common stock prices, a combination of these, or neither? Perhaps there will be a contra-movement compared with fixed interest securities; if interest rates come down, this will probably be at a time of lower inflation—the attractions of the index-link as an inflation hedge would then be reduced—and thus the price will fall.

For much of the time since my last report (May 1982 issue), the expectation of lower inflation seems to have been pulling the price down, i.e., increasing the yield. The 2½% yield which was mentioned increased to about 3%, and there had been relatively little change in the position despite a considerable fall in interest rates in the last few months to about the 10% level and a decline in our price index into single figures. But suddenly, last October, interest in the index-linked stocks revived; prices rose so that they yielded less than 2½%, where they have since hovered.

Why this quick change occurred isn't clear. It may be because government securities and common stock have both had significant increases, the index-linked securities being pulled along as an investors' afterthought; or perhaps investors don't believe that single-digit inflation will be with us for long, so let's buy the index-linked securities before everybody else does; or, it may be something else entirely. □

Death

Ruth Helen Peck, A.S.A. 1979

GOLDEN ANNIVERSARIES

Congratulations to 12 Fellows and 2 Associates who qualified for those categories in 1933:

Fellows

J. Finlay Allen — Leland J. Kalmbach
John C. Archibald — Harold R. Lawson
Lachlan Campbell — A. Earl Loadman
Thomas E. Gill — Leonard H. McVity
Russell O. Hooker — Frederick P. Sloat
James Hunter — Andrew C. Webster

Associates

Gerald M. Grassby — Leona Kuntz

The 1983 cohort of 50-year Fellows has proved itself a relatively hardy group, in that 63% of its 19 originals

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PRE-TEFRA

S* = the smaller of:

(a) S, or

(b) [(G' − T), if positive,] + L.

TEFRA

S* = the smaller of:

(a) S, or

(b) the larger of

(1) [(G' − T), if positive,] + L, or

(2) Q + the smaller of (i) (1+f) * (P+N), or

(ii) L+f * (P+N).

where L is $250,000 pre-TEFRA and is now defined as $1 Million, reduced for S > $4 Million (to 0 when S = $8 Million), allocated proportionately to the number of companies in the affiliated group. And where f = .85 for stock companies and f = .775 for mutual companies.

Thus, a company's tax position can be classified in terms of the amount of Special Deductions allowed under Section 809(f). Assuming increasing levels of S*, the classes for stock companies would be:

Category V	:	S* = L
Category W L > (P+N)	:	S* = Q + 1.85 (P + N)
Category X L < (P+N)	:	S* = Q + L + .85 (P + N)
Category Y	:	S* = G' − T + L
Category Z	:	S* = S

The variable L introduces a factor into the tax calculation that may come from data not included in the company's tax return. Also, a new corridor situation develops when an affiliated group's total special deductions fall in the range from $4 Million to $8 Million. Interesting marginal tax rates develop within this corridor.

Comments are being made indicating a switch of the tax phase for most mutual companies from Phase I to Phase II−. While being basically true, the statement is not fully accurate in that only a few companies will find themselves in the old Phase II-position. It might better be said that the old Phase II-companies, which previously had a $250,000 limit on Special Deductions, will join the old Phase I companies in a new category, both having a variable amount of allowable Special Deductions. □

The Northampton Table

(*Continued from page* 1)

It is right for us today to remember and to acclaim Richard Price's work. James S. Elston, in the second edition of *Sources And Characteristics of the Principal Mortality Tables* (1932) gives this endorsement by an 1823 author:

> "Dr. Price did as much as the nature of his materials would allow. For in those days no census or enumeration of the population had been made; and without (that) . . . an accurate Table of Observations cannot possibly be obtained."

My thanks to Howard W. Johnson, F.I.A. of London's Equitable Society for sending helpful material used in this account.

Wigglesworth's Table (1789)

"The first American table used at all for calculating life contingencies"—these words are quoted from *TASA* VII (1901), 3—made up from records in healthy portions of Massachusetts, was published, by Prof. Edward Wigglesworth of Harvard University, only six years later than was the Nortohampton Table.

E.J.M.

Golden Anniversary

(*Continued from page* 4)

remain. They became Fellows in the year in which the total number of Fellows went past the 400-mark; happily, 90 of those 400 are still with us.

The number of Associates who have 50 or more years as such is now 33.

The Society member who has been one for the longest time is Horace Holmes (F.S.A. 1921); he earned his Associateship in 1913 and is our only living member whose name is in the first published Index to the *Transactions* (1889-1914). Erston Marshall, though, is still our dean among Fellows, dating from 1919. □

THE PROPOSED NOTATION OF ENGELFRIET AND KOOL

by Frank G. Reynolds

(This is Article No. 6 in a series.)

Engelfriet and Kool explored the possibilities of using a linear form involving only the keys found on the standard typewriter keyboard. To replace the superscripts and lower left corner resort was made to an ingenious series of combinations of the special characters. For example, the double quotation symbol replaced the dieresis; $\underline{\underset{\cdot}{-}}$ was used to indicate that annuity payments were deferred for a given period and then continued, and this for a limited period from the end of the deferment period; the apostrophe was used to indicate that the annuity was payable in advance. Thus, $n|m\ \ddot{a}_x{}^{(h)}$ became '"**a** $\underline{\underset{\cdot}{-}}$ (x,n,m,h).

For a compound status an additional letter was added to the stem to indicate last survivor and other conditions. In general, the proposal met its design criterion of being linear, of being readily transformable into programming names, and of using only typewriter characters. The problem was the extensive use of backspacing to create characters such as $\underline{\underset{\cdot}{-}}$ and the use of auxiliary symbols which made it difficult to relate symbols to the present notation. □

ON THE ANALYSIS OF FRACTIONAL PREMIUMS

by James D. Broffitt

Jordan's excellent text on life contingencies teaches a technique for determining a relation between net fractional and net annual premiums, which is based on an analysis of the annual loss of interest and the loss of premium in the year of death. For ordinary life the result is

$$(1) \quad P_x^{(m)} \doteq P_x + \frac{m-1}{2m} P_x^{(m)} d + \frac{m-1}{2m} P_x^{(m)} P_x$$

which coincides with the standard approximation. The terms $\frac{m-1}{2m} P_x^{(m)} d$ and $\frac{m-1}{2m} P_x^{(m)}$ are purported to represent the annual loss of interest, and the loss of premium in the year of death, respectively, experienced by the insurer.

The technique is to start with P_x and add the appropriate adjustments to obtain $P_x^{(m)}$. To logically accomplish this we must consider m^{thly} payments of $\frac{1}{m} P_x$, and determine the resulting losses incurred by the insurer due to spreading P_x over m payments. Then we may add to P_x the additional annual amount, payable by the insured on an m^{thly} basis, needed to bring the annual premium up to $P_x^{(m)}$. Since the annual loss of interest and loss of premium in the year of death are approximately $\frac{m-1}{2m} P_x d$ and $\frac{m-1}{2m} P_x$, the relation obtained is

$$(2) \quad P_x^{(m)} \doteq P_x + \frac{m-1}{2m} P_x d + \frac{m-1}{2m} P_x P_x^{(m)}$$

which disagrees with (1).

The fallacy with Jordan's argument is that when net premiums of $\frac{1}{m} P_x^{(m)}$ are paid m times a year, there is no loss of interest or premium which needs to be made up by the insured, since the insured is making correct premium payments.

Although (2) is reasonable we prefer a logically correct analysis which will produce the standard approximation. This may be accomplished by modifying the previous argument. Rather than starting with P_x, we will start with $P_x^{(m)}$ and make appropriate adjustments to obtain P_x. That is, we will put $P_x^{(m)}$ on an annual payment basis, by assuming $P_x^{(m)}$ is paid to the insurer at the start of each year. This results in an annual overpayment of interest, and an overpayment of premium in the year of death, of approximate amounts $\frac{m-1}{2m} P_x^{(m)} d$ and $\frac{m-1}{2m} P_x^{(m)}$, respectively. To get P_x we must subtract from $P_x^{(m)}$ the amount (payable annually) needed to refund the insured for these overpayments. The result is

$$(3) \quad P_x \doteq P_x^{(m)} - \frac{m-1}{2m} P_x^{(m)} d - \frac{m-1}{2m} P_x^{(m)} P_x$$

which agrees with (1).

Finally we note that the argument used to obtain (3) works for installment and apportionable premiums, and for more complicated cases such as limited pay endowment policies. □

SIGHTINGS

The quantity and general quality of submissions of book and press references to actuaries continue high. We are most grateful to our many correspondents, and will print as many of these fine items here as space will permit.

Frederick R. Rickers sent us a delightful essay from the May 1982 *Contract Bridge Bulletin* marking our colleague Oswald Jacoby's golden wedding as well as his 80th birthday and his 10,000th syndicated bridge column. Its photos of Mr. and Mrs. Jacoby in 1932 and 1982 show them in fine fettle. There is just a brief reference to his life insurance career, from 1919 till 1928, and to his respect for, as well as his spectacular record in, passing the actuarial examinations.

J. Bruce MacDonald in Canada and *Ellis A. Wohlner* in Sweden both sent us an item in *The Economist* containing this comparison:

> "Just as an insurance actuary can tell you the average expected lifetime of a child born in Britain today, so a quantum physicist can tell you what the average lifetime of a group of radioactive particles will be. But, just as the actuary cannot say when any particular person will die, nor can the physicist tell you when one particular radioactive particle will decay."

Robert A. Moreen found a reference in classical literature that we don't recall having been picked up before. In Charles Dickens' mystery "Hunted Down", one of the prominent characters is the young actuary of the Inestimable Insurance Company whose boss describes him as "at once the most profound, the most original, and the most energetic man I have ever known *connected with life insurance*" (emphasis added). Mr. Moreen recommends the story, which is reprinted in the Arbor House Treasury of Mystery and Suspense.

Robert J. Myers showed us that Duke University in North Carolina has come up with a research project entitled "Bio-actuarial estimates and forecasts of health care needs and disability."

Douglas S. Van Dam saw in the Louisville, Ky., Courier-Journal a municipal matter described as "normally the stuff to make even an actuary yawn." □

DEFINITIONS FOR COMPOUND AND SIMPLE INTEREST

by James D. Broffitt and Stuart Klugman

In the cases of compound and simple interest, the accumulation function, $a(t)$, is easily defined for integer values of t. We address the question of how to extend these definitions to include noninteger values of t. Our definitions for compound and simple interest are motivated by a reinvestment example which embodies the basic notion that interest earns interest under compound interest but not under simple interest. From these definitions we obtain $a(t) = (1 + i)^t$ and $a(t) = 1 + it$, for all t, under compound and simple interest respectively.

The fact that compound interest demands $a(t) = (1 + i)^t$ for all t does not automatically follow from knowing $a(t) = (1 + i)^t$ for integer t. This result must depend on some statement about the behavior of $a(t)$ for noninteger t. We suggest the following definition for compound interest.

Definition 1: Interest is said to be compounded at annual rate i if (1) $a(1) = 1 + i$ and (2) $a(t + s) = a(t)a(s)$ for all real s and t.

The second statement may be explained as follows: A $1 investment accumulates to $a(t + s)$ after $t + s$ years. If, however, the accumulated value is withdrawn after just t years and immediately reinvested, the investment will grow to $a(t)a(s)$ after s additional years. The definition requires that the final accumulated value be unaffected by the intermediate transaction. Clearly compounding is occurring since interest earned during the first t years, earns interest during the final s years. The appropriate theorem is:

Theorem 1: If interest is compounded at rate i and $a(t)$ is differentiable for all t, then $a(t) = (1 + i)^t$.

Proof: $$a'(t) = \lim_{s\to 0}\frac{a(t+s)-a(t)}{s} = \lim_{s\to 0}\frac{a(t)(a(s)-1)}{s} = a(t)a'(0)$$

Therefore $\frac{a'(t)}{a(t)} = a'(0)$ and so $\frac{d}{dt}\ln a(t) = a'(0)$, which implies $\ln a(t) = a'(0)t + c$. From $a(0) = 1$ and $a(1) = 1 + i$, we obtain $c = 0$ and $a'(0) = \ln(1 + i)$. Consequently $a(t) = (1 + i)^t$.

We also note that simple interest may be developed in a similar manner.

Definition 2: Interest is said to be simple at annual rate i if (1) $a(1) = 1+i$ and (2) $a(t+s) = a(t)+a(s)-1$ for all real s and t.

The motivation for (2) is provided by the same reinvestment example. The value after t years is $a(t) = 1 + [a(t) - 1]$, which has been separated into principal and interest components. Since we want only the principal to earn interest, the final value is $a(s) + [a(t) - 1]$.

Theorem 2: If interest is simple at rate i and $a(t)$ is differentiable for all t, then $a(t) = 1 + it$.

The proof is analogous to that of Theorem 1. In this case $a(t) = a'(0)t+c$ and the constants are determined from $a(0) = 1$ and $a(1) = 1 + i$. □

"PRELIMINARY ACTUARIAL EXAMS"

The 1982 edition is now available gratis from the Society office in Chicago.

In addition to current information on the first two examinations, it contains 44 pages of sample Part 1 and Part 2 questions from the November 1981 and May 1982 exams. Sample Examination booklets will no longer be furnished separately.

Suzanne L. Hunziker

NEW SOCIETY APPLICATION FORM

The Society's APPLICATION FOR ADMISSION AS ASSOCIATE has been revised—mainly to remove the nomination requirement made obsolete by 1982 vote of the Fellows.

Please destroy copies of the old form that your organization may have on hand, and request copies of the new form from the Society's Education and Examination Department.

Suzanne L. Hunziker

THE 1976 AND 1981 RESTRUCTURINGS OF OUR FELLOWSHIP EXAMINATIONS

by Linden N. Cole

In the light of the increasing pace of change in our society, it is not surprising that there have been changes in the Society of Actuaries' education and examination system. There were, in fact, two such changes in only five years.

The 1976 Restructuring

The objective of the 1976 restructuring was to have each examination cover a major subject area applicable to all specialties.

- Part 6 • covered "Assumption of the Risk," including a description of coverages, selection of risks, and marketing.
- Part 7 • covered the "Balance Sheet," both the valuation of liabilities and of assets.
- Part 8 • covered "Paying for the Risk," such as gross premiums and expense analysis.
- Part 9 • covered "The Outside World," including law, taxation, social insurance, and the Annual Statement.

The idea was that the principles involved in each examination could be extended to any product line, helping to make the actuary a very flexible person. Our students would not simply study how past generations had calculated gross premiums for life insurance; they would study "Pricing".

Once implemented, this system proved to have its problems. First of all, the subject of Pensions could not be forced into the structure of the Fellowship examinations. Paying for pensions turned out to be inseparable from valuing the pension liabilities. Thus, the initial concept broke down in a crucial area. Secondly, the new system proved to be relatively inflexible. As there were changes in the law and in the environment, the system could not be adjusted. Finally, most of the changes were occurring in the outside world, and Part 9 was getting longer and longer.

The 1981 Restructuring

The next restructuring occurred in 1981, only five years after the previous one. The new structure was designed

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Actuarial Education

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quantitative methods arising from other disciplines to solve new problems and to add new perspectives to old problems. Our educational goal, though, should be to train *generalists,* not *specialists.*

The syllabus should present a wide range of mathematical topics which have, or potentially have, useful applications to practical actuarial problems, or which help actuaries to communicate effectively with those in allied professions. It should develop those topics in a way that emphasizes fundamental principles and concepts, and that reveals the limitations of techniques and the necessity for scrutinizing results for reasonableness and for consistency with the underlying assumptions.

The result is that the following topics will be on the new Part 3 syllabus:

Operations Research: Linear and dynamic programming, decision analysis, queuing theory, project scheduling, simulation.

Applied Statistics: Regression analysis, analysis of variance, time series analysis.

The course of reading will include parts of the text, *Operations Research* (Holden-Day) by Hillier and Lieberman, chapter 4 of the Society's text, *A Study Manual For Operations Research*, Eugene A. Narragon (Ed.), and parts of *Intermediate Business Statistics* (Holt, Rinehart and Winston) by R. Miller and D. Wichern.

These syllabus changes, as well as some in the readings for numerical methods, will take effect for the May 1983 examination, its length becoming four hours.

A study note on simulation is being written, and one on actuarial applications of applied statistics is planned; these will be effective no earlier than November 1983. Also, changes in the 1984 syllabus are being developed by the Task Force on Numerical Methods and Graduation.

Anyone wishing a copy of this Task Force's Report or more particulars on the 1983 Part 3 syllabus, ask Linden N. Cole at the Society office.

Mathematical Aspects of Demography, (Judy A. Faucett, Chmn.)

Demography, a topic long covered on Part 5, was reduced in 1979 to Mathematical Aspects of Demography with the intent that non-mathematical aspects would be placed on a Fellowship exam. Unfortunately this hasn't yet been done, so students are being required to absorb mortality and demographic statistics without guidance on how or when to use them.

Demography is a valuable tool for actuaries; the need to forecast populations has become evident in the health, pension and other financial security fields, and ties in with increasing use of life company corporate models. This Task Force recommended that Demography be split into two sub-topics, viz.:

Survival Models and Data Analysis

- Mathematics of Mortality and Morbidity Measures
- Survival Distributions
- Fundamentals of Life Table Construction
- Studies Based on Incomplete Data
- Comparisons of Mortality Data

Population Forecasting

- Methodology & Sources of Mortality and Morbidity Measures
- Use of Government Statistics
- Mortality and Morbidity Characteristics and Trends
- Forecasting Methods

Survival Models and Data Analysis would replace Mathematical Aspects of Demography on Part 5B. Population Forecasting, the only completely new subject matter, would more appropriately go on the Fellowship syllabus.

As text, the Task Force chose *Survival Models and Data Analysis* by Regina C. Elandt-Johnson and Norman L. Johnson, the latter an F.I.A. This book, using actuarial notation, describes methods for analyzing data and constructing interpretive models, with emphasis on general principles; examples are interspersed, and each chapter offers a lengthy set of problems. One drawback is absence of solutions, but the possibility of preparing sample solutions is being explored; also, there are too many printing errors, requiring us to distribute a formidable errata list.

These Part 5B changes will be effective in 1983. A new Task Force will tackle the Population Forecasting subtopic, likely to take longer to introduce because texts are lacking. Anybody interested in being part of that Task Force or in writing study notes, please let Education Chairman Sam Gutterman know, at his Yearbook address. □

Restructuring

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with flexibility as a major objective. Also, the degree of specialization, which had been increased in the 1976 restructuring, was further increased. In the new Parts 9 and 10, there are now three specialties, and each candidate chooses a major specialty and a minor specialty. The three specialties are Individual Life Insurance and Annuities; Group Insurance and Individual Health Insurance; and Pensions.

The point about increasing specialization is worth some comment. If everyone has to learn about everything, the Education and Examination Committee is faced with some difficult decisions as the world grows more complex. There is, after all, an upper limit to the material we can ask our students to learn. Once that limit is reached, new material can be added only by deleting old material. The ultimate effect is that every subject is gradually cut down, and nothing can be treated in depth. By abandoning the objective of making everyone learn everything about every subject, and requiring pension specialists to learn some things that insurance specialists do not have to learn, and vice versa, it becomes possible once again to treat important subjects in satisfactory depth. The new syllabus requires everyone to learn something about every subject, but not everything.

So far, at least, the new structure is proving to have the flexibility hoped for it. It should last for many years because of its ability to accommodate change. Also, the pension content is stronger, and potentially much stronger. The task remaining is to examine every subject area, to assure that the study material is current and of high quality. Perhaps that will be the major task in the 1980s. □