

ANNUITY LAPSE RATE MODELING: TOBIT OR NOT TOBIT?

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ABSTRACT. We devise an approach, using tobit models for modeling annuity lapse rates. The approach is based on data provided by the Society of Actuaries' Risk Management Task Force. Kim [2005a] models annuity lapse rates using a logit model and the same US data (he also used Korean data separately). We find that the tobit model is a more than suitable approach for all levels of the explanatory variables including interest rates, unemployment rates, and GDP growth rates. Specifically, policyholder behavior in the tail of the distribution of lapse rates is explained as well as it is in the normal range of lapse rates.

1. INTRODUCTION

Single premium deferred annuities (SPDA) issued in the US provide a surrender option to the policyholder at any time during the accumulation period. The policyholder may “put” the policy back to the insurer and take the cash value, less a surrender charge. One motive might be to obtain a higher yield than the insurance company's crediting rate. But there might be other reasons as well — perhaps the policyholder has lost his or her job and needs the cash. Such policyholder behavior is difficult to model. In modeling mortgage backed securities, a similar problem arises in that it is difficult to model borrower behavior as interest rates (and other variables) change. Kim [2005b] provides a good review of the literature in these areas.

SPDA lapse rate models have been developed as a part of larger models, such as asset-liability models. Recently, Kim [2005b] and Kim [2005c] suggest that SPDA lapse rates in the US and Korea can be modeled using a logit model with explanatory economic variables including interest rates, gross domestic product (GDP) growth rates and unemployment rates. We have replicated his results (for the US only) and compared it to another approach, the tobit model.

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We conclude that, for the US data, the tobit model is a more than suitable approach. Kim [2005a]’s work was sponsored by the Society of Actuaries (SOA) Risk Management Task Force and studied the US and Korean annuity lapse rates, focusing on policyholder behavior under extreme conditions.

Finally, we suggest that another approach may be better than others, but it requires individual data rather than aggregate data. This is the count model. We discuss it briefly.

The paper proceeds as follows: In Section 2, we introduce the SOA data by using univariate analysis. In Section 3, we show how to use the tobit regression to model the annuity policy lapse rate. Actual versus predicted full surrender rates are shown in Section 4. We briefly introduce the count model in Section 5. Section 6 is the conclusion.

2. UNIVARIATE DATA ANALYSIS

2.1. Summary statistics. The SOA data has 719 months in which annual full surrenders occurred. Initial written premium is also included for each month. For example, if initial premium is written in March 1995, the data includes the annual full surrender rate for each year ending in February 1996, February 1997 and so on. For each of these we calculated the corresponding annual full surrender amount (`full_surr_dollar`) for each month:

$$\text{full_surr_dollar} = \text{full_surr} \times \text{initial_prem},$$

where `full_surr` is the observed annual full surrender rates by month and `initial_prem` is initial written premium. The SOA data also includes annual partial surrender rate by month (`par_surr`). The sum of annual partial surrender rate by month (`par_surr`) and annual full surrender rates by month (`full_surr`) is denoted as variable `surr`.

The variable `surr_charge` is the surrender charge when the policy was surrendered. For this annuity the surrender charge percentage (`surr_charge`) is initially 7% and decreases with duration to levels 6%, 4%, 2% and zero. In general, the surrender rate increases dramatically in year eight when the surrender charge drops to zero. To account for this, we set a dummy variable `duration8` which equals to 1 if the duration is 8 years and 0 otherwise.

Many factors affect surrender rates. In Section 3, we model annuity lapse rates with a few economic variables following Kim [2005a]. The variable `difftrate` is the annualized five-year Treasury bond rate minus the policy credited rate. The 5-year Treasury bond rates are from the Federal Reserve Board. The variable `unemploy` is the monthly annualized US unemployment rate. The monthly US employment rates are collected from the Bureau of Labor Statistics (US Department of Labor). The variable `gdpgr` is the monthly annualized US GDP growth rate in 2000 dollars. The quarterly US GDP growth rates are obtained from the Bureau of Economic Analysis — an agency of the U.S. Department of Commerce. Our data is summarized in Table 1.

TABLE 1. Summary Statistics from 1993 to 2003

The SOA provided the surrender data and the policy credited rate data. There are 719 observations.

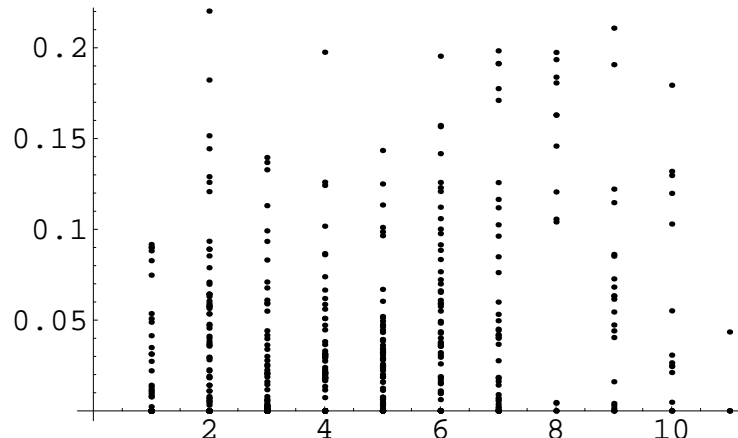
Variable	Mean	Stan. Dev.	Minimum	Maximum
<code>full_surr</code>	5.71%	12.31%	0%	100.00%
<code>par_surr</code>	0.29%	1.05%	0%	19.46%
<code>surr</code>	6.00%	12.33%	0%	100.00%
<code>initial_prem</code>	\$ 1,223,935	1,612,449	3,637	7,272,274
<code>full_surr_dollar</code>	\$ 76,446	201,522	0	2,207,307
<code>surr_charge</code>	5.34%	2.53%	0%	7.00%
<code>duration8</code>	5.84%	23.47%	0	1
<code>difftrate</code>	-0.60%	1.13%	-3.47%	1.96%
<code>gdpgr</code>	3.09%	1.44%	0.22%	4.85%
<code>unemploy</code>	5.11%	0.73%	3.97%	6.63%

2.2. Surrender rate versus policy duration. In addition to showing the summary statistics, this section describes some of the characteristics of this data set. First we look at the surrender rates and the duration of the policy at the time of surrender.

Corresponding to each surrender we also have the policy duration in years. Figure 1 shows the scatter plot of points (x, y) where y is an observed surrender rate and x is the corresponding policy duration.

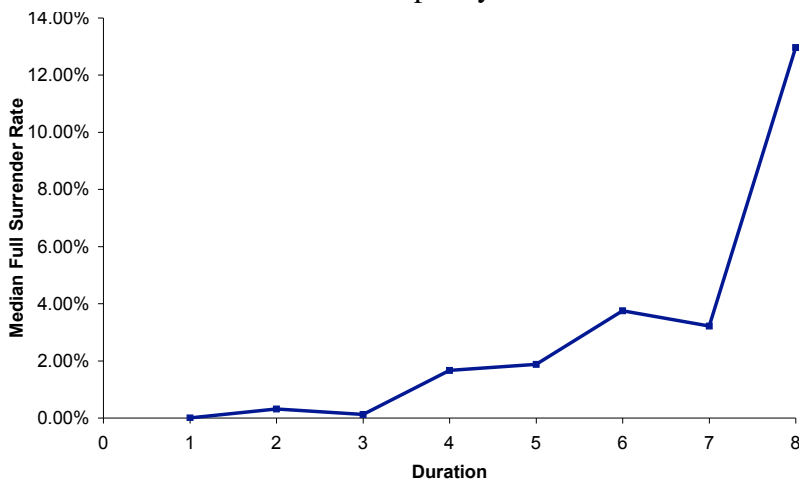
At each duration there is a sub-sample of surrender rates, all with the same duration. In order to get a better picture of the relationship we plotted the median of each of the sub-samples as a function of duration. This is shown in Figure 2. At each duration, half of the observed surrender rates

FIGURE 1. Surrender rate versus policy duration



are at least as high as the plotted values indicate. We see more clearly that there is a tendency for the rate to increase with duration. The rate reaches the peak in duration 8 for which the surrender charge has decreased to zero.

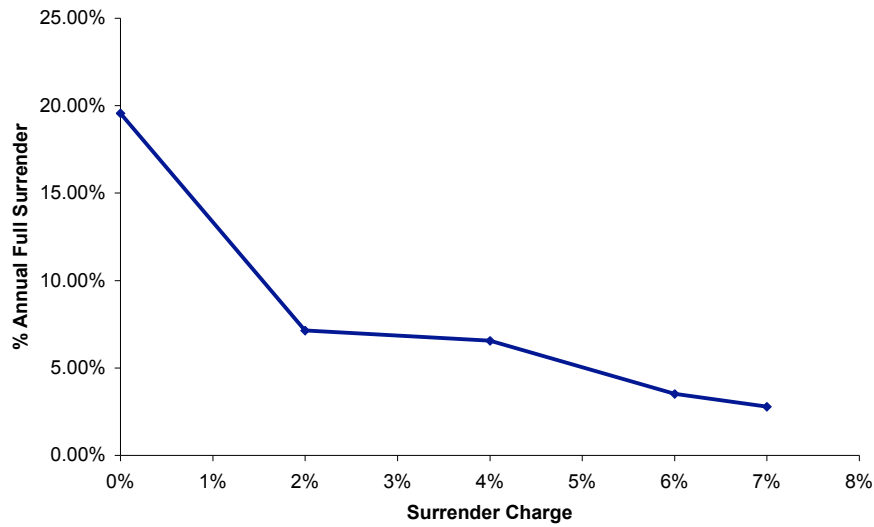
FIGURE 2. Median surrender rate versus policy duration



2.3. Surrender rate versus surrender charge. In the same way, for each observed surrender rate we have the corresponding surrender charge. At each surrender charge level we have a sub-sample of surrender rates. In Figure 3 we see the plot of the sub-sample medians versus the surrender rate. We see an inverse relationship, as one would expect.

2.4. Correlations. We also ran a correlation for six variables, as indicated in Table 2. As we expect, surrender rates (`full_surr`) are strongly correlated with the eighth year duration (`duration8`).

FIGURE 3. Median surrender rate versus surrender charge



Once the surrender charge wears off, surrender rates increase. Surrender rates are negatively correlated with the surrender charge (`surr_charge`) strongly and GDP growth rates (`gdpgr`). Most of the correlations are significant at the 1% level.

TABLE 2. Pearson Correlation results (1993-2001)

	full_surr	surr_charge	duration8	diffrate	gdpgr	unemploy
full_surr	1	-0.4211 (<.0001)	0.5303 (<.0001)	-0.0448 (0.2303)	-0.1402 (0.0002)	-0.0010 (0.9781)
surr_charge	-0.42111 (<.0001)	1	-0.52616 (<.0001)	0.26716 (<.0001)	0.12989 (0.0005)	-0.25638 (<.0001)
duration8	0.5303 (<.0001)	-0.5262 (<.0001)	1	-0.1494 (<.0001)	-0.1303 (0.0005)	0.1292 (0.0005)
diffrate	-0.0448 (0.2303)	0.2672 (<.0001)	-0.1494 (<.0001)	1	0.1730 (<.0001)	-0.6223 (<.0001)
gdpgr	-0.1402 (0.0002)	0.1299 (0.0005)	-0.1303 (0.0005)	0.1730 (<.0001)	1	0.0485 (0.1940)
unemploy	-0.0010 (0.9781)	-0.2564 (<.0001)	0.1292 (0.0005)	-0.6223 (<.0001)	0.0485 (0.1940)	1

Note: # of observations: 719;

p-values are presented below the correlations.

3. DETERMINANTS OF SURRENDER RATES - TOBIT REGRESSION

This section shows how to use tobit to model the US annuity lapse rate. The reason why we propose the tobit model is that the dependent variable `full_surr` is zero for a significant fraction of the observations (about 43%). Conventional regression models fail to account for the qualitative

difference between *limit* observations and *non-limit* (continuous) observations. In our model the dependent variable, the surrender rate, is observed, but, according to this approach, there is an unobserved “latent” surrender rate always present but not always observed. We can think that there is always a surrender rate, but only under certain conditions (for example, a low surrender charge applies or there is an attractive competitive rate) do we observe it.

When the dependent variable is censored, values in a certain range are all transformed to a single value¹. In our case, the dependent variable `full_surr` is censored at 0. If y^* denotes the latent surrender rate, then we observe the variable $y = y^* | y^* > 0$.

3.1. Tobit regression model. The tobit regression model is based on the idea that for each observation, there is a latent variable y^* following an ordinary regression model:

$$y_i^* = \beta' \mathbf{x}_i + \varepsilon_i$$

$$(1) \quad \text{full_surr}_i = \beta_1 \text{surr_charge}_i + \beta_2 \text{duration}_i + \beta_3 \text{diffrate}_i + \beta_4 \text{gdpgr}_i + \beta_5 \text{unemploy}_i + \varepsilon_i$$

where the error terms ε_i are independent normally distributed and have common standard deviation σ . As usual with ordinary regression, the latent surrender rate y^* is normal with mean $E(y_i^* | \mathbf{x}_i) = \beta' \mathbf{x}_i$ and variance $\text{Var}(y_i) = \sigma^2$.

Now we model the censored observations y_i as follows:

$$(2) \quad y_i = \begin{cases} y_i^* & \text{if } y_i^* > 0 \\ 0 & \text{if } y_i^* \leq 0 \end{cases}$$

We have been following the convenient abuse of notation in which y_i denotes both a random variable and its observed value. We have to be more careful here. Let $f_i(x)$ be the density of y_i and $f_i^*(x)$ the density of y_i^* . The distribution of y_i is a mixed discrete–continuous distribution with

¹The section is based on [Greene, 2000, section 22.3.2]

a probability mass at 0 and the density of y_i^* at $x > 0$. Here it is for $x > 0$:

$$(3) \quad \begin{aligned} f_i(x) &= f_i^*(x) \\ &= \phi\left(\frac{x - \beta' \mathbf{x}_i}{\sigma}\right) \end{aligned}$$

The mass at 0 is

$$F_{y_i^*}^*(0) = \Pr(y_i^* \leq 0) = \Phi\left(\frac{-\beta' \mathbf{x}_i}{\sigma}\right) = 1 - \Phi\left(\frac{\beta' \mathbf{x}_i}{\sigma}\right).$$

The likelihood function for the observed values (including exactly k zeros) y_1, y_2, \dots, y_k is

$$\begin{aligned} L &= F_i^*(0)^k \prod_{y_i > 0} f_i(y_i) \\ &= \left[1 - \Phi\left(\frac{\beta' \mathbf{x}_i}{\sigma}\right)\right]^k \prod_{y_i > 0} \phi\left(\frac{y_i - \beta' \mathbf{x}_i}{\sigma}\right) \end{aligned}$$

The log-likelihood for the censored regression model is

$$(4) \quad \begin{aligned} \log L &= k \log \left[1 - \Phi\left(\frac{\beta' \mathbf{x}_i}{\sigma}\right)\right] + \sum_{y_i > 0} \log \left[\frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x - \beta' \mathbf{x}_i)^2}{2\sigma^2}} \right] \\ &= -\frac{1}{2} \sum_{y_i > 0} \left[\log(2\pi) + \log \sigma^2 + \frac{(y_i - \beta' \mathbf{x}_i)^2}{\sigma^2} \right] + k \log \left[1 - \Phi\left(\frac{\beta' \mathbf{x}_i}{\sigma}\right)\right]. \end{aligned}$$

The first part in equation (4) corresponds to the classical regression for the non-limit observations and the second part adjusts for the limit observations. This likelihood is a nonstandard type, since it combines continuous and discrete distributions.

Marginal effects provide economic meaning for the impact of changes in explanatory variables on the dependent variable. Specifically, it measures the percentage change in a dependent variable caused by a one percentage change in an independent variable while holding other independent variables constant. Marginal effects of independent variables in the OLS are equal to the coefficient estimates. However, marginal effects of the tobit model are not equal to the regression coefficients. For the standard case with censoring at zero and normally distributed disturbances, the marginal

effect for the tobit model specializes to

$$(5) \quad \frac{\partial E[y_i | \mathbf{x}_i]}{\partial \mathbf{x}_i} = \beta' \Phi \left(\frac{\beta' \mathbf{x}_i}{\sigma} \right).$$

3.2. Estimation results. In the tobit model, our dependent variable is the full surrender rate `full_surr` censored at zero. The independent variables are `surr_charge`, `duration8`, `difftrate` and `gdpgr`.

The correlation matrix in Table 2 shows that the unemployment rate `unemploy` has a significantly high negative correlation with the interest rate spread `difftrate` (-0.62). The highly correlated variables are redundant, which causes a ‘‘collinearity problem.’’ To solve the collinearity problem, we drop the unemployment variable.

We report the coefficients (also marginal effects) of ordinary least squares (OLS) regression to compare with the marginal effects of tobit regression in Table 3. The sign and significance of the estimates are similar in these two models but the tobit model gives us robust standard errors reflecting the effect of censoring the data.

TABLE 3. Ordinary Least Squares and Tobit Regression Results

Independent variables	OLS		Tobit	
	Marginal Effect	Standard Error	Marginal Effect ^a	Standard Error
Intercept	0.124	0.014	0.077	0.011
<code>surr_charge</code>	-1.038	0.182	-1.139	0.141
<code>duration8</code>	0.221	0.019	0.111	0.014
<code>difftrate</code>	0.952	0.351	1.784	0.289
<code>gdpgr</code>	-0.621	0.270	-0.418	0.209
Adj. R-square	31.59%			
Max-Likelihood			26.90	

All of the standard errors indicate that the coefficient estimates are significant at the 0.001 level, except for GDP which is significant at the 0.05 level in both models.

^a Marginal effects are estimated in the mean.

4. ACTUAL VERSUS PREDICTED FULL SURRENDER RATES

Now we check actual versus predicted values for the tobit models. Recall that for each observed value of the surrender rate, we have concurrent values of the variables `surr_charge`,

duration8, diffrate, unemploy and gdpgr. We put these values into the tobit model regression equation (1) and call the resulting computed value the model's prediction of the expected surrender rate. For each duration, there are sub-samples of actual and predicted values, for which we calculate the actual and predicted mean and median. This gives us a graphical representation of the predictive power of the tobit model.

Table 4 and Figure 4 show the prediction power of the tobit model in mean. The column \hat{y}_{tobit} in Table 4 shows the predicted full surrender rates from the tobit model averaged over all the observations at the given duration.

Similarly, Table 5 and Figure 5 show the predictive power of the tobit model evaluated at the median of the independent variables by each duration.

TABLE 4. In sample test of predictive power at the mean

Duration	No. of observations	Mean	
		full_surr	\hat{y}_{tobit}
1	125	0.011	0.024
2	113	0.036	0.031
3	101	0.044	0.029
4	89	0.022	0.028
5	78	0.035	0.038
6	66	0.066	0.059
7	54	0.071	0.079
8	42	0.319	0.319
9	29	0.103	0.091
10	17	0.089	0.089
11	5	0.058	0.099

Based on the above results, we conclude that the tobit model has adequate goodness of fit for all of durations, when the predicted full surrender rate is evaluated at the mean of independent variables.

When we look at the fit at the medians in Table 5 and Figure 5, the tobit model has good predictive power for policy duration between 4 and 9 years, based on the median value. And we can see that the tobit model overestimates the full surrender rate in the median (its graph is always

FIGURE 4. Actual and predicted average surrender rate by duration. “Full_surr” is the actual average full lapse rate. “yhat_TOBIT” is the predicted full surrender rates from the tobit model evaluated at the mean.

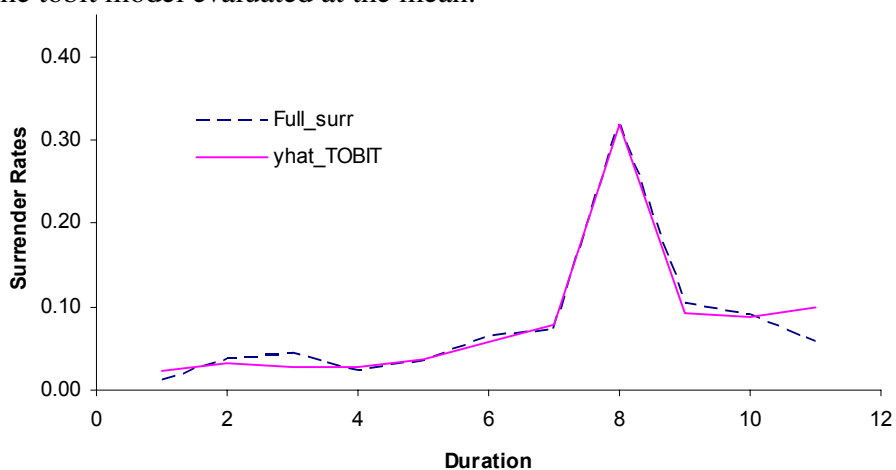


TABLE 5. In sample test of predictive power at the median

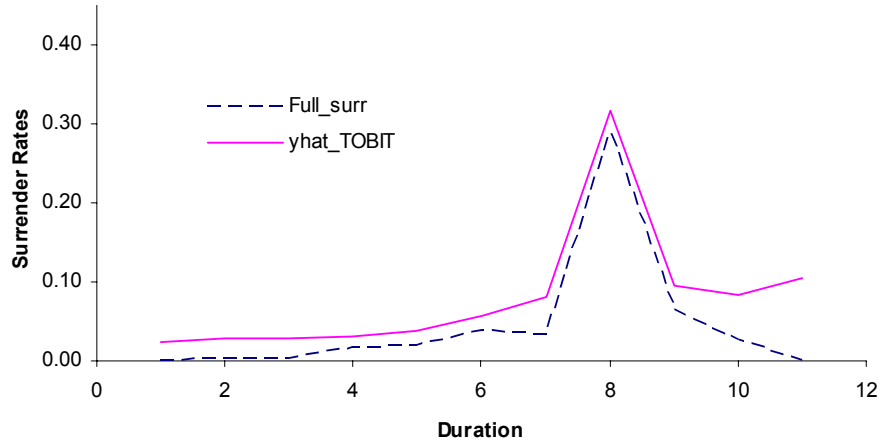
Duration	No. of observations	Median	
		full_surr	\hat{y}_{tobit}
1	125	0.000	0.024
2	113	0.003	0.029
3	101	0.001	0.028
4	89	0.017	0.030
5	78	0.019	0.039
6	66	0.038	0.058
7	54	0.032	0.081
8	42	0.291	0.317
9	29	0.063	0.095
10	17	0.027	0.083
11	5	0.000	0.104

above the observed value graph). Therefore, insurers should be cautious when they use the model to predict the median surrender rates by duration.

5. DETERMINANTS OF SURRENDER RATES - COUNT MODELS

If we had policyholder counts, we could use a more appropriate model, called the count model. The count model is a nonlinear regression with discrete dependent counting variables. Data for

FIGURE 5. Actual and predicted median surrender rate by duration. “Full_surr” is the median actual lapse rate. “yhat_TOBIT” is the predicted full surrender rates from the tobit model evaluated at the median.



the number of policies surrendered are typical of count data. The Poisson regression model and negative binomial model have been widely used to study such data. The Poisson regression model specifies that each y_i is drawn from a Poisson distribution with parameter λ_i , which is related to the regressor \mathbf{x}_i [Greene, 2000]. The primary equation of the model is

$$(6) \quad \Pr(Y_i = y_i) = \frac{e^{-\lambda_i} \lambda_i^{y_i}}{y_i!}, y_i = 0, 1, 2, \dots$$

The parameter λ_i is usually modeled as a log-linear model:

$$(7) \quad \log \lambda_i = \beta' \mathbf{x}_i.$$

The expected number of events per period is given by

$$(8) \quad E[y_i | \mathbf{x}_i] = \text{Var}[y_i | \mathbf{x}_i] = \lambda_i = e^{\beta' \mathbf{x}_i}.$$

We can estimate the parameters with maximum likelihood method. The log-likelihood function is

$$(9) \quad \log L = \sum_{i=1}^n [-\lambda_i + y_i \beta' \mathbf{x}_i - \log y_i!].$$

The Poisson regression is criticized by assuming equality of the conditional mean and variance. If the data indicates that the conditional variance is larger than the conditional mean, we can use a negative binomial count model in very much the same way.

6. CONCLUSION

We propose the tobit model to predict the US annuity surrender rates. The tobit model has adequate fit and prediction power and is a more than suitable approach. Finally, we briefly discuss the count model as another way to model annuity lapse rates. With additional data, the count model could be a more appropriate model.

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