Determining Discount Rates Required to Fund Defined Benefit Plans

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AUTHORS John A. Turner
Humberto Godinez-Olivares
David D. McCarthy
Maria del Carmen Boado-Penas

Abstract

Current approaches used by regulators in the United States either base discount rates for determining defined benefit plan funding on bond rates of return or on the expected rate of return on the pension plan's portfolio. In this paper, we present a model that focuses on the probability that the assets based on current contributions will be less than the value of that liability at some point in the future, requiring further contributions. This approach generally results in discount rates that are greater than the risk-free rate advocated in the financial economics literature on the topic, and are less than the portfolio-based rates used by state and local government pension plans in the United States. We find that when the maximum allowable probability of needing to make additional future contributions for past service is less than 50 percent, increases in the expected rate of return reduce the discount rate used to calculate funding. We argue that approaches that focus only on the risk to liabilities or only on the risk to assets do not consider the risk that future contributions will be needed, which involves an analysis of the bivariate distribution of risks of assets and liabilities. With the stochastic funding parameter approach, the discount rate used to determine adequate funding depends on the risk to the assets, the risk to the liabilities, the correlation of those risks, and the duration of the liabilities.

Introduction

In the United States, private sector and public sector defined benefit plans use different approaches for determining the discount rate used in calculating funding ratios. The approach single employer private sector plans use is based on corporate bond rates. The approach state and local government plans use typically is based on the expected rate of return on their portfolios.

Pension funding for defined benefit plans is required by government as a payment mechanism that provides some degree of assurance that benefits promised to workers will be paid. Generally, the contractual obligation of the employer or the pension plan is to pay the promised benefits, with the plan funding serving as a means to assure that they will be paid. Governments regulate funding in order to maintain minimum standards as to the acceptable risk that pension plans will not pay workers full promised future benefits.

Globally, defined benefit plans have liabilities said to be valued at $23 trillion (The Economist 2014). Actuaries, economists and other financial analysts, however, disagree as to how those liabilities should be valued. A separate but related issue is that these groups also disagree as to how the required level of funding should be determined. This paper analyzes the question of the required discount rate by pension regulators for determining adequate funding for defined benefit pension plans. It argues that this discount rate is a different concept than the discount rate used for valuing financial liabilities for the purposes of buying or selling those liabilities.

Defined benefit plans can be thought of as transferring risk from participants to plan sponsors, at least when viewed in comparison to the alternative of a defined contribution plan. Thus, a risk-free pension as viewed from the perspective of a worker as an asset is a risky liability to the plan sponsor, who must bear investment risk, longevity risk and inflation risk. The worker, by contrast, faces default risk.

Financial economists generally argue that defined benefit pension liabilities should be valued by discounting future benefit payments using a yield rate for a bond with comparable duration and risk as the liabilities (Novy-Marx and Rauh 2009). Pension liabilities differ considerably from bond payments because pensions are annuities with mortality risk. However, assuming this approach, and assuming a normal yield curve, with interest rates rising with greater duration, the greater the duration and the greater the risk of the liabilities to the plan sponsor , the higher the discount rate. Risk-free liabilities would be discounted using the rate of return on risk-free bonds. This approach arguably is the correct way to value pension liabilities when determining the value of a company offering a defined benefit plan or in determining the price at which the liabilities could be transferred to another party. The financial economists' approach is the approach to be used by someone considering buying the liability—the greater the risk, the lower the price the person is willing to pay (the higher the discount rate).

Financial economists have used this same reasoning for determining the discount rate that they argue should be used for calculating required contributions for funding liabilities for an ongoing defined benefit plan. We argue that the funding issue is different from the valuation issue.

The determination of the discount rate for that purpose is a regulatory issue rather than purely a financial markets issue. Regulators use that rate to assure a certain degree the security of the pension promises made to workers, rather than to value liabilities for the purpose of buying or selling them. We argue that the approach used for valuing future liabilities has limitations in its use by regulators in determining required contributions for the purpose of funding.

To differentiate between the discount rate conceived by financial economists and the concept we are using for regulatory purposes, we call our rate the "hurdle rate". The hurdle rate is the maximum discount rate that would be permitted under our model, or by a pension regulator, which is established to provide the degree of assurance that the regulator views as appropriate that benefits will be paid.

While the financial economics problem of valuing liabilities for the purpose of buying or selling them only looks at the risk of the liabilities, the regulatory problem also considers the risk of the assets. It considers the risk of the assets as well as the risk of the liabilities because both risks affect the probability that the plan sponsor will need to make additional contributions.

We argue that a higher rate than that conceived by financial economists can be used for discounting liabilities for regulatory purposes because an ongoing plan sponsor has the option, and presumably the ability, of making additional future contributions if needed. A counter argument to this approach relates to the timing of when additional contributions would be needed. Additional required contributions would tend to be needed when the stock market, and presumably the plan sponsor, were doing poorly, which is a bad time to need to make additional contributions. This argument is considerably weakened when it is recognized that regulators generally allow plans to amortize losses over a number of years. The option of additional future contributions provides an implicit funding cushion in that it provides a backup source of funding. If instead the plan liabilities were being settled or defeased, then the financial economists' approach would be appropriate. Thus, we argue that the regulatory issue concerning discount rates is a different problem than that addressed by financial economists. This difference is, in part, because of the possibility for the plan sponsor of accepting the risk of needing to make additional contributions to meet the liability, resulting in a different criteria for determining the appropriate rate. This regulatory standard may be affected in practice by whether there is a pension benefit insurer. Having a pension benefit insurer reduces the risk to worker's pension plans.

This paper investigates the choice of the discount rate (hurdle rate) used for regulatory purposes through modeling and simulations. We explore this problem using a variety of different modeling approaches and assumptions. The paper first discusses the previous literature. Because of the controversial nature of the choice of discount rates, this paper presents the intuition of its approach by starting with simple models to demonstrate the basic points. It then moves to more realistic models that are more complex but also permit a demonstration that the points established in the simple models are robust to more complex analyses. The development of models thus starts with a simple two-period model where either assets or liabilities are risk free, and moves to a more complex multi-period model where both assets and liabilities are risky. The paper ends with a section on policy implications and concluding remarks.

To anticipate the conclusions, we find that approaches that focus only on assets or only on liabilities do not take into consideration the probability that additional contributions will be required and tend to produce discount rates that are either too low (liabilities approach) or too high (assets approach).

1. Four Approaches

We first discuss four different approaches for determining discount rates for funding defined benefit plans. We then use those approaches for organizing our review of the literature.

Market-Based Approach. This approach to determining discount rates for defined benefit plan valuation defines the problem as determining the present value of future pension liabilities. It is an application of the valuation of future payment streams. The riskiness of the liabilities to plan sponsors determines the discount rate that applies for determining their present value. For example, risk-free liabilities are discounted using a risk-free discount rate (Novy-Marx and Rauh 2009, 2013). Risk to plan sponsors concerning the value of pension liabilities arises due to mortality risk, future wage rate risk, and interest rate risk. The choice of discount rates for valuing a liability is independent of the investment portfolio, according to this approach. Novy-Marx and Rauh (2009, 2013), however, note that the question of determining the value of the liabilities is different from determining desired funding.

Mortality risk can be dealt with by making a conservative assumption as to future mortality improvements, meaning that mortality rates can be assumed to decline faster than is the most likely scenario. That approach results in larger liabilities than would be calculated under assumptions viewed as being most likely. Alternatively, the risk associated with the possibility of greater than expected declines in mortality rates can be dealt with through a reduction in the discount rate, which also would result in an increase in measured liabilities.

Risk to participants arises due to default risk. The higher the risk of default by the plan sponsor, the greater the risk to the liability from the participant's perspective, and thus the higher discount rate. The Pension Benefit Guaranty Corporation (PBGC) in the United States reduces, but does not eliminate, the default risk to participants.

The Expected Return-Based Approach. Under this approach, the expected rate of return on the portfolio determines the discount rate to be used in order to determine how much assets are needed today to pay for future pension liabilities. This approach is predominantly used by U.S. state and local government defined benefit pension plans and by U.S. multiemployer pension plans.

The two approaches are answering different questions. While the market-based approach is answering the question, *what is the appropriate discount rate for valuing future liabilities?* the expected rate of return approach is answering the question, *what is the appropriate discount rate for assuring that assets will be sufficient to fund future liabilities with a given probability?*

Day Approach. A third approach has been presented by Day (2004). When assets do not exist that perfectly correlate with changes in the plan's liabilities, which is the general situation, he argues that in comparison to a risk-free liability, "In very general terms, the uncertainty of the cash flows makes for a worse liability than before. Thus the liability increases in value rather than decreases as it does for the asset side." Because the liability would increase in value relative to a risk-free liability, that implies that a discount rate lower than the risk-free interest rate would be used for valuing the liability, which is the opposite of the market-based approach.

Probability of Ruin Approach. The approach we take can be characterized as the probability of ruin approach, as discussed later.

2. Review of the Literature

We organize our review of the literature into these approaches.

Market-Based Approach. Nijman et al. (2013) analyze the valuation of pension liabilities under the system of risk sharing in the Dutch pension system. Their paper and Bovenberg et al. (2014) both assume the only risk is financial market risk. Bovenberg et al. (2014) argue that in a complete market where all pension cash flows can be replicated, the discount rate is the rate of return on an investment portfolio that completely replicates (or hedges) the cash flows of the liabilities. The market consistent valuation of the liability can be determined using asset pricing theory.

Brown and Pennacchi (2016) argue that default risk should be taken into consideration when determining the market value of liabilities, but that for measuring pension funding a default-free interest rate should be used regardless of whether the liabilities are default-free. Their approach thus differs from that taken previously by other financial economists, where risky liabilities are discounted using a discount rate that takes into account the level of risk. The default-free interest rate would determine the amount necessary to set aside to have 100 percent assurance that benefits would be paid. They argue that the default-free discount rate is the rate that would be used to determine the amount that would be paid to an acquirer of the liabilities.

Novy-Marx (2015) argues that "the appropriate discount rate for a pension fund's liabilities is the expected rate of return on an optimal "hedge portfolio," where this is the portfolio that would be held under a liability-driven investment policy (i.e., the portfolio of traded assets that has cash flows that most closely approximates the fund's expected future benefit payments). Thus, he argues that it does not depend on the actual portfolio of investments but this alternative portfolio. He also argues that the valuation depends on "whether the payments being valued are the promised payments, or the payments that are actually expected to be made." He notes concerning valuing liabilities based on corporate bond rates that "Corporate bond rates reflect the possibility firms may default on their debts. These rates thus account for the fact that expected payments are smaller than promised payments (because of the possibility of default). They also include a risk premium that arises because defaults co-vary with priced risks (i.e., because defaults are more likely in bad times, when extra dollars are particularly valuable)."

If there is a positive correlation between equity returns and pension liabilities in the long run, investing in equities can serve as a partial hedge against pension liabilities (Black 1989, Lucas and Zeldes 2006). Such a relationship appears to hold over the long term due to a positive correlation between labor earnings growth and stock returns. Thus, equities can be used as a hedge against pension liabilities that are linked to wage growth. An implication is that obligations to older workers and retirees are like bonds and can be valued and hedged that way, while future pension benefits for younger workers have risk and return characteristics that are more like stocks (Lucas and Zeldes 2006).

Expected Return-Based Approach. McCaulay (2010) compares the market-based approach with the expected return-based approach. He argues that the expected return-based approach is superior because the market-based approach overstates the value of liabilities due to the low discount rate it uses. He also argues that the market-based approach results in greater volatility in liability values than does the expected return-based approach because the market-based approach is based on market bond rates while the expected return-based approach is based on long-run expected rates of return on pension investments, which is less volatile.

The Day Approach. The Day approach is consistent with labor market analysis as to the value of the pension to the pension participants. In the labor market, employers do not provide pensions for free. In determining competitive wages, the labor market requires a tradeoff between higher wages versus higher pension benefits for a particular worker. According to the traditional view of the labor market, workers value the insurance aspect of an annuity provided by a defined benefit plan, so that they are willing to forgo more in wages than the expected present value of the payment. Thus, a lower discount rate than the risk free rate would be required. Because the pension insures against risk for workers, it is worth more to workers than its expected present value and costs more as a liability to employers and annuity providers. However, to the extent that the pension promise to workers is risky, workers would be willing to forgo less in wages.

One approach for determining discount rates is that defined benefit liabilities should be valued consistently with the valuation of group annuities. Thus, the literature on valuation of annuities is relevant to this discussion. Cannon and Tonks (2013), in their analysis of annuities, note that the greater the risk of the liability, the greater the reserves needed by the life insurance company to ensure that the liability can be met. Thus, their argument is similar to that of Day (2004).

Jong (2008) presents an approach that takes into account the riskiness of liabilities and the availability of assets to hedge those risks. He considers issues relating to the determination of the expected present value of pension liabilities when those liabilities are based on unknown future labor earnings. For the approach of valuing pension liabilities at market prices (the market-based approach), he notes the problem that typically pension liabilities are not marketed assets. The long maturity of the claims and their indexation to wages for workers accruing benefits make it impossible to find market instruments with similar characteristics. Future wages cannot be hedged perfectly with existing financial market instruments.

In the literature on financial asset pricing, the situation where the payoff pattern of an asset or payment stream cannot be perfectly replicated is referred to as an incomplete market. The holder of the pension liability, who is the plan sponsor or the firm shareholders, assumes an unhedgeable risk, which affects the probability distribution of his consumption and final wealth, and thus his utility. The certainty equivalent wealth of the expected utility is the cost to the pension plan sponsor for the liability. Finding a value for the pension liability amounts to finding a value for the unhedgeable risk. To achieve the same certainty equivalent wealth in the incomplete market, the plan sponsor needs to invest more in the pension plan than in the complete markets case with perfect hedging. Jong advises that in comparison to a risk free discount rate, the discount rate should be adjusted downward for the extent of unhedgeable risk. Thus, Jong has a similar conclusion to the Day approach.

Related literature examines the effect of an increase in risk on precautionary savings by households. That literature demonstrates that risk averse individuals or households increase precautionary savings when a mean-preserving increase in risk of future liabilities occurs (for example, Apps et al. 2014). This result is consistent with the pricing of annuities by life insurance companies and with the argument that the discount rate for valuing a risky liability should be lower than the risk free rate.

The Probability of Ruin Approach. In this paper, we develop a generalization of the expected return-based approach for when it is required that current contributions be sufficient to fund future liabilities more than X percent of the time. This methodology is called ruin probabilities (Milevsky 2016). With ruin probabilities, a random process is subject to random additions (contributions and investment returns) and withdrawals (benefit payments). The outcome of interest is whether the stochastic process will hit a barrier (e.g., zero assets or insufficient funding) before the terminal date of the process.

Haberman et al. (2003) present the case that pension plans should be evaluated using stochastic rather than deterministic models. With a deterministic approach, users have taken account of risk by incorporating margins in the valuation. For example, mortality risk can be taken into account by assuming lower mortality rates than the rates considered to be most likely. With a stochastic approach, mortality and other risks can be analyzed directly. They further argue that the measure of risk should take into account the expected size of a shortfall, as well as the probability that a shortfall occurs. Their approach, which is similar in many ways to the approach we take, analyzes funding and investment decisions together. It focuses on contribution rates rather than on the choice of discount rates, but the two parameters are closely related. In their model, contribution rates are affected by investment decisions, while in our paper discount rates are affected by investment decisions. Rather than focusing on the determination of the present value of liabilities, which requires a choice of discount rates, they focus on stochastic projections of future benefit payments.

3. Analysis of Approaches to Choice of Discount Rates

This section contains further analysis of the three basic approaches that we call the market-based approach, the expected-return approach and the Day approach. In later sections, we consider the probability of ruin approach. We start by using a simple two-period model. The following section considers simple scenarios where $1 is to be either received or paid one year from now, and the value of the payment is either known for certain or has a normal distribution. The assumption of a normal distribution is used for convenience. In reality, a log normal distribution may be a more accurate representation of the distribution of plan rates of return. This section explores in a general framework for an individual the difference in the analysis between the valuation of risky liabilities and risky assets.

The Market-Based Approach. This scenario is an exercise in determining the expected present value of a risky future payment to be received. The expected value of the payment valued at the future date of the payment (the mean of the distribution of payments) is 1. Assume the person is risk averse. Because the person is risk averse, the person values the risky future payment to be worth less than a certain payment of $1.

In determining the amount the person is willing to pay for the future asset, the person uses a discount rate r_p that incorporates a risk premium p ($p > 0$), and is thus higher than the risk-free interest rate ($r_p = r_f + p > r_f$), The amount the person is willing to pay to receive the risky payment, which can be denoted as $PV(1, r_f + p)$, is thus less than the amount he would be willing to invest to receive the risk free payment with the same expected future value.

$$PV = PV(1, r_f + p) \qquad (3.1)$$

This is essentially the market-based approach, which uses the analysis of valuing future income receivable for valuing a future liability. This is the amount someone would pay to receive a risky payment in the future. In actuality, those who use this approach assume that the liabilities are risk free. This approach does not recognize the argument of Day that risky assets and risky liabilities would be valued differently.

The Expected Return-Based Approach. Now consider the same scenario except the person (plan sponsor) must make a payment in one year (a liability) rather than receive a payment (an asset). Thus, this scenario directly relates to the payment of pension benefits. It is assumed that no asset is available for hedging the liability. The expected rate of return on the person's portfolio, which incorporates the risk of the portfolio is r_e. In this case, the present value of the future payment, as determined by the expected return approach, is determined by discounting by the expected rate of return on the portfolio.

$$PV = PV(1, r_e) \qquad (3.2)$$

Thus, the two approaches use different methodologies for determining the discount rate.

The Day Approach. Because the plan sponsor is risk averse, and perhaps also because of regulatory reasons, the plan sponsor is more concerned about the outcome of having saved too little than about having saved too much. For that reason, the plan sponsor engages in precautionary saving to try to assure having adequate resources set aside to make the payment. Thus, the plan sponsor needs to save more money in anticipation of the risky payment than the plan sponsor does for a risk-free payment.

Because the plan sponsor facing a risky payment needs precautionary savings beyond what the plan sponsor would save for a risk free liability, the plan sponsor uses a lower discount rate than the risk free discount rate. The plan sponsor subtracts a factor d ($d > 0$) from the risk free rate, so that the rate used for discounting risky liabilities is $r_d = r_f - d$. The magnitude of the discount d is determined by the amount of precautionary savings the plan sponsor needs, given the sponsor's degree of risk aversion, and would be greater the greater the degree of risk aversion. The degree of risk aversion affects the sponsor's target probability c of not having saved sufficient assets to meet the risky liability. The amount the person would set aside to pay for the future risky liability would be

$$PV = PV(1, r_f - d). \quad (3.3)$$

Thus, this result yields a lower discount rate than the other two approaches.

The Risk of Ruin Approach. In our view, all three previous approaches have limitations in that they do not take into account the acceptable probability that plan sponsors will need to make further contributions in the future to pay for liabilities already accrued. That observation can be equivalently framed as ignoring the risk of ruin, with that meaning the probability that future contributions will need to be made.

We thus argue that none of the approaches is asking the right question concerning required contributions for funding defined benefit plan liabilities. The right question needs to take into account the probability of success (or the risk as to whether future contributions will be needed, and also the size of those future contributions).

Thus, we start our analysis by asking this question: What is the discount rate needed for determining contributions to assure that current contributions will be sufficient c percent of the time so that future contributions will not be needed to pay off the liability? More complex versions of this question later in the paper take into account the magnitude of the required future contributions, for example that future contributions not exceeding 10 percent of the current contribution in present value will be sufficient c percent of the time.

For the first three approaches, the expected return-based approach provides the highest discount rate (based on the portfolio expected rate of return), the market-based approach provides an intermediate discount rate (based on corporate bond rates of return), and the Day (2004) approach provides the lowest discount rate (based on a deduction from bond rates to provide a cushion of assets). All three approaches implicitly have a different acceptable probability of success, and thus, it could be said that all three approaches fit within the framework of this paper. However, none of them explicitly take into account the probability of success in selecting the rate to be used to determine required contributions.

The following section provides a simple model presenting the risk of ruin approach, based on the concept that both the risk to assets and the risk to liabilities affect the probability that the plan sponsor will need to make additional contributions in the future.

4. A Model Based on the Probability of Needing to Make Future Contributions for Currently Accruing Liabilities—The Risk of Ruin Approach

In the analysis of the choice of discount rates that follows, we make several assumptions in order to present in a simplified framework the key insights of the approach. We first analyze the scenario where assets are risky but liabilities are risk-free. We then reverse the assumptions and analyze the scenario where liabilities are risky but assets are risk-free. Finally we consider risky assets and liabilities under a two period framework.

We use a measure of liabilities that takes into account liabilities accrued to date based on current wage rates, rather than using projected wage rates. Our approach could also be applied in situations where wages are projected.

Modeling. In analyzing the variability in underfunding, we use the following equation for the variance in the difference between assets A and liabilities L.

$$\mathrm{Var}(A\text{-}L) = \mathrm{Var}(A) + \mathrm{Var}(L) - 2\mathrm{Cov}(A,L) \tag{4.1}$$

It is well known that if only assets are variable or only liabilities are variable, then Var(A-L) equals either Var(A) or Var(L). If perfect hedging were available, the covariance term would equal the sum of the variance terms, and Var(A-L) would equal zero.

For clarity, we distinguish between the discount rate used to determine the expected present value of future liabilities, versus the interest rate used to determine required contributions. We call the latter interest rate the “hurdle” rate. The hurdle rate is the rate of return used for discounting the value of the liability for the purposes of determining required contributions. It is the maximum discount rate that can be used to assure that when the plan is initially fully funded using that rate that future contributions will be needed no more frequently than a given percentage of the time.

Two-Period Model: Risky Assets, Risk-Free Liabilities.

Following the principle of working from simple to more complex analyses, we start with a two-period model. In the first period, the plan determines the amount of contributions it needs to make in order to pay for its second period liabilities. We distinguish between the mean or expected rate of return and the discount rate r needed to assure adequate funding (the hurdle rate).

Modeling. In all the scenarios, we assume that the plan is required to make contributions sufficient so that at least c percent of the time no further contributions are needed in the second period. This approach is a budgeting approach, relating to the sponsor's need to budget for future contributions. The parameter c defines the success rate (the complement of the probability of ruin). Monte Carlo simulations generally provide results in terms of probability of success (Pfau 2014), and that concept applies in the case of assuring adequate pension funding. The success rate assumption c is key in determining how much is needed to be contributed. The value of the parameter would presumably depend on the risk that the plan sponsor would be unable (or unwilling) to make the required future contributions. Default risk thus can be taken into account through this parameter. The parameter in principle would be set by pension regulators, and could vary between state government and private sector plans. We discuss in a later section how this parameter might be implemented in policy.

Assume that liabilities are riskless but that assets are risky with normal distribution and known mean and variance. This approach is similar to the expected return-based approach in that it treats the risk of liabilities as irrelevant. Given this distribution, the goal is to determine what level of contribution in the first period is needed to assure that assets in the second period will exceed liabilities in the second period at least c percent of the time, i.e., what level of assets in the first period is needed to assure that prob($A_2 \geq L_2$) = c.

Data. Table 1 provides information about the mean and standard deviation of rate of returns considered in this section. It is derived from the efficient frontier for target date funds that are part of the Thrift Saving Plan, which is the 401(k)-type plan for U.S. federal government workers (Thrift Savings Plan 2013). It shows the relationship between the geometric mean rate of return, standard deviation of returns, and the rate of return at different points on the distribution of rates of return corresponding to those parameters for a set of portfolios. These historical data are used for providing an illustration of the approach being analyzed. Historical data may not indicate future experience.

Results. The L Funds in the Thrift Savings Plan are the target date funds for different target dates. The Income Fund is the final fund for workers who have reached their target date. For these funds, as expected, as the mean rate of return increases, the standard deviation of returns also increases.

Table 1. L Funds and the Efficient Frontier

TSP Fund	Return Mean	Return Std. Dev.	Annual Return for Various Return Percentiles							
			50	45	42	40	30	20	10	5
Income	5.8	4.3	5.8	5.3	4.9	4.7	3.5	2.2	0.3	-1.3
L-2020	7.2	11.0	7.2	5.8	5.0	4.4	1.4	-2.1	-6.9	-10.9
2030	7.6	13.0	7.6	6.0	5.0	4.3	0.8	-3.3	-9.1	-13.8
2040	8.0	15.5	8.0	6.1	4.9	4.1	-0.1	-5.0	-11.9	-17.5
2050	8.3	17.5	8.3	6.1	4.8	3.9	-0.9	-6.4	-14.1	-20.5

Source: author's calculation

Table 1 is calculated for the unrealistic situation of a two-period model where every period lasts one year. It is calculated as the percentile of a normal distribution. The values in the table can be obtained by calculating the inverse of the normal distribution. For the tenth percentile, some of the rates of return are negative. Later sections will investigate whether the results hold under more realistic assumptions.

If the acceptable level of certainty c that future additional contributions will not be needed is 50 percent, the table shows that the mean rate of return on the portfolio can be used for discounting future liabilities when liabilities are riskless, confirming the expected return-based approach, but under limited circumstances. However, for any higher level of likelihood c that future contributions will not be needed, corresponding to a lower point on the probability distribution of rates of return, the rate of return required for the discount rate assumption is lower than the mean rate of return. For example, if the mean expected return on the portfolio is 5.8 percent and the acceptable level of certainty c that the plan will have sufficient assets for the target benefit level is 0.6 (corresponding to 0.4 in the table), the amount of contributions should be based on an assumed discount rate (hurdle rate) of 4.7 percent. In other words, for a 60 percent success rate we need to set aside assets based on the assumption that the rate of return will be at least 4.7 percent. If the mean expected return is 7.2 percent with the same acceptable level of certainty (60 percent) as to having sufficient assets for the target benefit level, the amount of contributions should be based on an assumed rate of return (hurdle rate) of 4.4 percent.

Thus, for a higher mean rate of return (and thus risk) and a success rate greater than 50 percent, the required discount rate would be lower to compensate for the higher risk, counter to the expected return-based approach and the relationship assumed in funding state and local government pension plans and multiemployer plans in the United States. Table 1 demonstrates a key point. When the acceptable level of success c is greater than 58 percent (corresponding to 42 percent in the table), the higher is the expected return for the investment portfolio, and thus the risk, the lower the allowable assumed rate of return for calculating required contributions. This finding does not hold at lower acceptable levels of success. When c is 55%, the table shows discount rates increasing from 5.3% to 6.1% as risk increases. The pattern of inverted expected returns only holds at higher acceptable levels of success. This result is driven by the increase in the standard deviation of returns with higher mean returns. To anticipate findings later in this paper, this finding is robust to a number of variations in modeling assumptions.

Two-Period Model, Risky Liabilities, Risky Assets. We now switch the focus and investigate the situation of risky liabilities and risky assets.

Modeling. For simplicity, the only risk we focus on as affecting the value of liabilities is mortality risk. In the short run, mortality is unpredictable for individuals, but mortality rates are highly predictable for large groups. In the long run, even the rates are uncertain and everything about the uncertainty is uncertain – its growth rate, its variance, its distribution, and its impact on liabilities. To gain quantitative traction, we assume that possible magnitudes of future liabilities are normally distributed with known mean and variance. In a later section of the paper, we explore other distributional assumptions. Although a major factor affecting uncertainty in liabilities is uncertainty in mortality, which spreads liabilities over many years, we separate the issues of uncertain duration of liabilities from uncertain magnitude of liabilities by presenting a simple two-period model – the present funding period and the future payment period D years from present. Early retirement also affects both the duration and magnitude of liabilities, but is not considered here. Subscripts p and f distinguish present from future, and we denote the mean and standard deviation[1] of future liabilities by μ_f and $\sigma_{f.,}$ respectively. We assume that rates of return on assets are constant at a known rate.

[1] Although this assumption has implications regarding the distribution of uncertainty in future mortality rates, these need not be explored here. A right-skewed distribution of future mortality may more accurately reflect expert demographic opinion, but we choose to assume a normal distribution because in this paper conceptual clarity is more important than quantitative accuracy and a normal distribution permits us to more easily build upon the familiar reasoning of confidence intervals. The fact that our normal distribution is an assumption rather than a consequence of the central limit theorem does not compromise our ability to draw upon the logic of confidence intervals when deriving required funding cushions.

The parameters L_f and A_f, represent liabilities and assets D years into the future. The stochastic funding requirement for all the scenarios is that future liabilities L_f be less than future assets A_f, with probability c:

$$P(\,L_f \leq A_f\,) = c,\;\; \text{where } 0 < c < 1 \qquad (4.2)$$

Rescaling liabilities to a standard normal distribution (the normal distribution with mean zero and standard deviation of 1), that requirement can be rewritten as:

$$P\left(\frac{L_f - \mu_f}{\sigma_f} \leq \frac{A_f - \mu_f}{\sigma_f}\right) = c \qquad (4.3)$$

Because $(L_f - \mu_f) / \sigma_f$ is standard normal, we can also write:

$$P\left(\frac{L_f - \mu_f}{\sigma_f} \leq \text{probit}(c)\right) = c \qquad (4.4)$$

where $\text{probit}(c)$ is the inverse of the cumulative distribution of the standard normal function. It returns the positive or negative number below which a standard normal variable falls with probability c.[2]

The right hand sides of the two previous inequalities are therefore equal:

$$\frac{A_f - \mu_f}{\sigma_f} = \text{probit}(c) \qquad (4.5)$$

So

$$A_f = \mu_f + \text{probit}(c)\,\sigma_f \;\equiv \text{CFL} \qquad (4.6)$$

[2] Statistics students may recall using probit(.975)=1.96 when constructing two-tailed 95% confidence intervals. In words, a standard normal variable falls below 1.96, 97.5 percent of the time.

Asset level A_f in equation 4.6 will be at least equal to liabilities with probability c. We call that asset level the confident funding level CFL. Note that if c=.5 (corresponding to the median (and mean) of the standard normal distribution of liabilities), probit(c) = 0. This result shows that if a 50 percent probability of success is sufficient, having future assets equal to the mean of the liability distribution, with no need for an asset cushion, is sufficient. If a higher probability of success is required, then a higher level of assets must be set aside. Thus, again it is seen that a required standard of success greater than 50 percent implies using a lower discount rate (hurdle rate) than the expected rate of return. An alternative approach used in practice is to build in margins in mortality and other assumptions to provide a cushion.

In the previous equation, the target level of assets A_f is expressed in absolute terms. It is useful to translate this equation into relative terms by subtracting μ_f from both sides of the CFL definitional equation and then dividing by μ_f, which yields

$$\frac{CFL_f - \mu_f}{\mu_f} = \text{probit(c)} \left(\frac{\sigma_f}{\mu_f}\right) \qquad (4.7)$$

The left-hand side is the fraction by which CFL exceeds the expected value of future liabilities, and we call it the funding cushion FC_f. It is the funding cushion in the second period. A consequence of this definition, which we use below, is that $1+FC_f = CFL_f / \mu_f$. Note that σ_f / μ_f is the coefficient of variation CV_f for liability uncertainty. Using this terminology, confident funding is achieved when the future funding cushion satisfies:

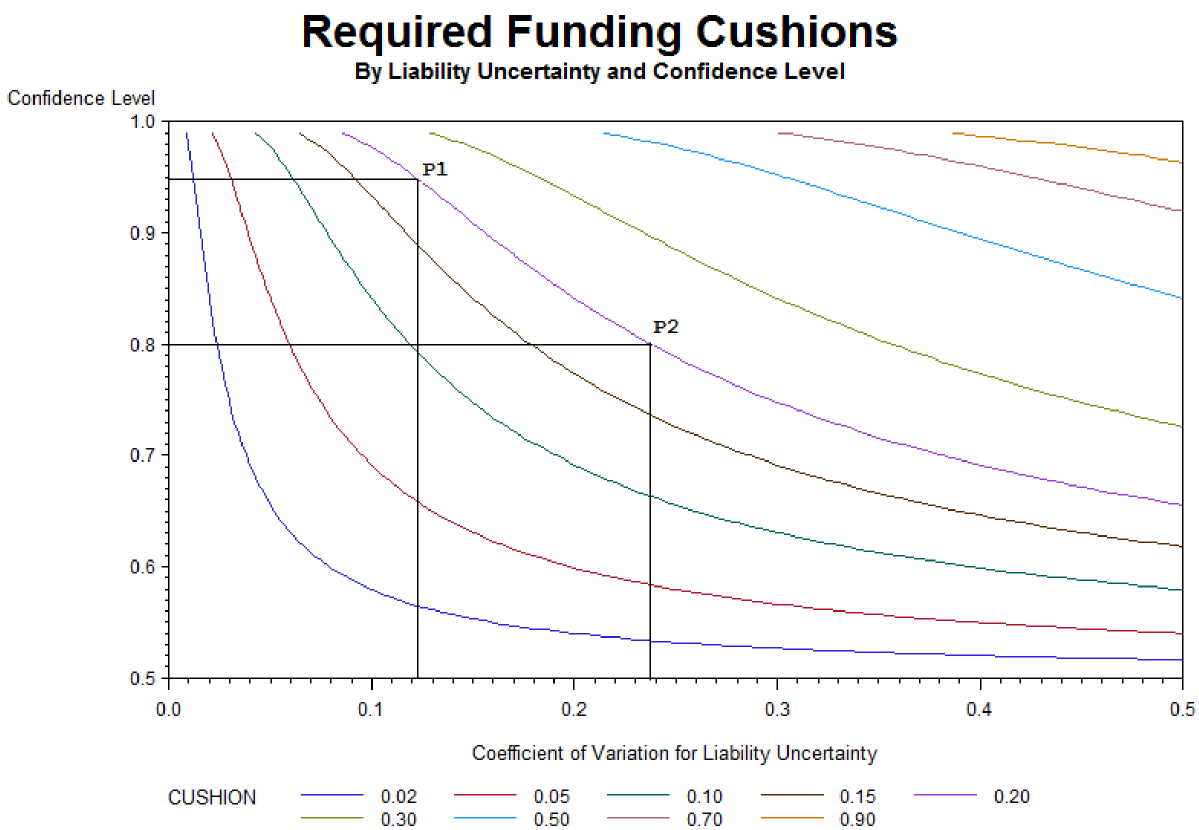
$$FC_f = \text{probit(c)}\ CV_f \qquad (4.8)$$

Thus, the required funding cushion is greater, and the hurdle rate for determining funding lower, the greater the required probability of success c and the greater is the variance of liabilities relative to its mean. And again, no funding cushion is required if the required probability of success is 50 percent or lower.

Data. Goldman Sachs (2007) in a study of liability-driven investing uses 9.8 percent as a typical coefficient of variation for defined benefit plan liabilities with respect to bond prices. Over the period 1970-2011, the quarterly correlation between stocks and bonds was 0.07, but varied considerably over different subperiods (Swedroe 2012).

Results. Figure 1 presents the normal distribution in a way that makes the point for risky liabilities that has been made for risky assets. The required funding cushion increases, and thus the hurdle rate decreases, with both the level of confidence required as to the need for future contributions and the level of risk, here measured by the coefficient of variation for liability uncertainty. Points P1 and P2 show the tradeoff between the liability uncertainty (measured by the coefficient of variation) and the level of confidence required that future contributions will not be needed. Points P1 and P2 show in this example that a 20 percent funding cushion can provide 95 percent confidence that liabilities will not exceed assets if the CV of liability does not exceed 12 percent, and 80 percent confidence if the CV of liabilities does not exceed 24 percent. For a confidence level of 0.58 (58 percent) and a coefficient of variation of about 0.24, a funding cushion of 5 percent relative to full funding valued at the mean rate of return is needed. These results follow from the assumption of a normal distribution for liabilities.

Figure 1.



Source: authors' calculations

Modeling. Because our focus is on the required downward adjustment to the mean rate of return on the asset portfolio to assure adequate funding in the face of risk, we now explore the relationship between the magnitude of the funding cushion and magnitude of the downward adjustment needed to determine the hurdle rate.

Let D represent the duration[3] of the liabilities, while μ_P and CFL_P represent the present values of μ_f and CFL; that is

$$\mu_f = \mu_P \times (1+r)^D \qquad (4.9)$$

and

$$CFL_f = CFL_P \times (1+r)^D \qquad (4.10)$$

where r is the mean of the distribution of rates of return. Assume also that we wish to adjust the discount rate (hurdle rate) r downward to a level r' that will yield the higher present value CFL:

$$\mu_f = CFL_p \times (1+r')^D \qquad (4.11)$$

Equating these two expressions for μ_f:

$$\mu_P \times (1+r)^D = CFL_p \times (1+r')^D \qquad (4.12)$$

So

$$\left(\frac{1+r}{1+r\prime}\right)^D = \left(\frac{CFL_p}{\mu_p}\right) \qquad (4.13)$$

Solving for r' yields:

[3] Readers may be more familiar with the concept of duration in the context of bonds where duration is the approximate change in price resulting from a 100 basis point change in interest rate. Like a bond, an annuity is a series of cash flows having a present value that is sensitive to the discount rate in the same way that a bond price is sensitive to interest rates, and, just as for bonds, that sensitivity can be gauged using the concept of duration. Of the three common forms of duration, "modified duration" is the precise concept we use here because this concept is applicable to bonds having expected cash flows that do not change when the yield changes and payments from the annuity we are discussing do not vary with the discount rate.

$$r' = \frac{1+r}{\left(\frac{CFL_p}{\mu_p}\right)^{(1/D)}} - 1 \qquad (4.14)$$

As noted, $1+FC_f = CFL_f / \mu_f$, and similarly $1+FC_p = CFL_p / \mu_p$, so we can rewrite this equation as:

$$r' = \frac{1+r}{\left(1+\mathrm{FC_p}\right)^{(1/D)}} - 1 \qquad (4.15)$$

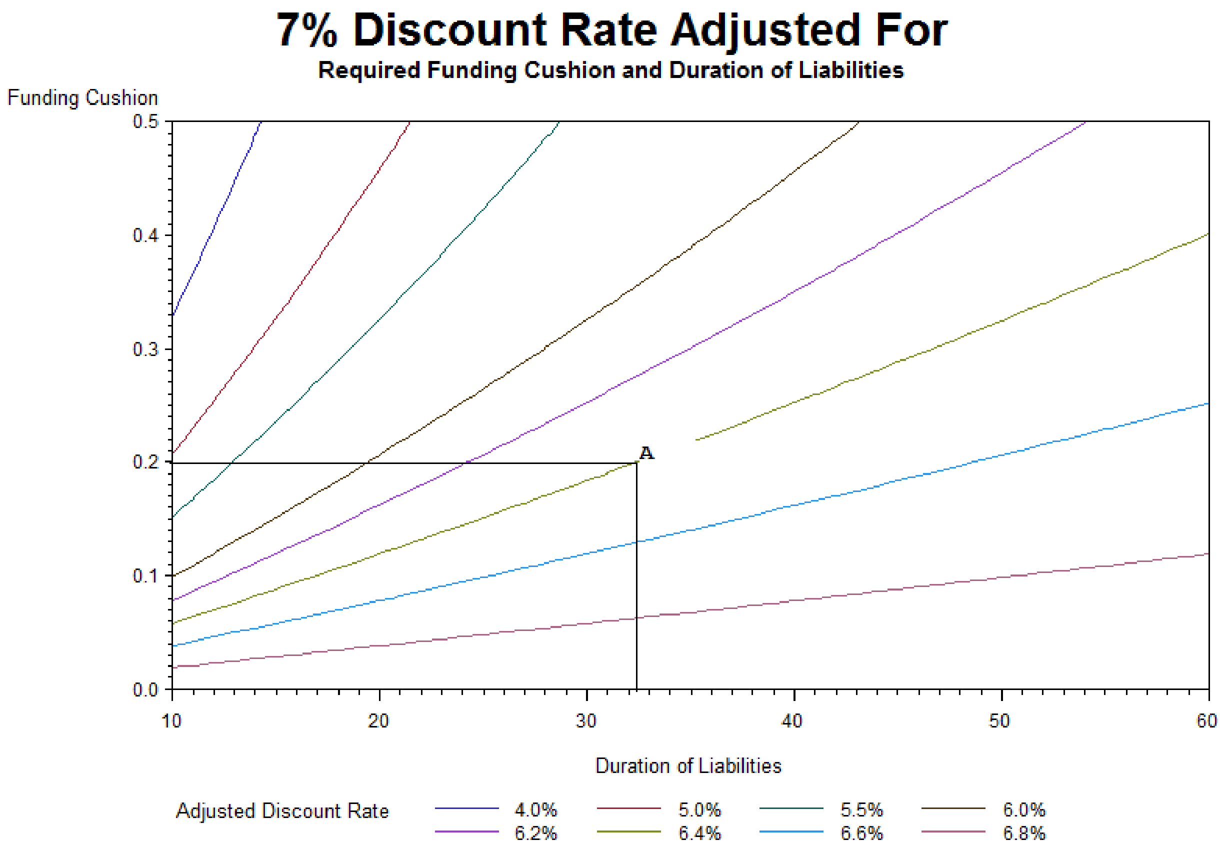
Thus, the greater the value of the funding cushion needed, the greater the reduction in the discount rate.

We can use equation (4.8) to express this adjustment as a function of the coefficient of variation for future liabilities CV_f and the required probability c of funding sufficiency.

$$r' = \frac{1+r}{(1+\mathrm{probit}(c)\ \mathrm{CV_f})^{(1/D)}} - 1 \qquad (4.16)$$

Results. Figure 2 provides a contour plot for equation 4.15. In Figure 2, the hurdle rates (discount rates) at the bottom of the figure from left to right correspond to the lines going from top to bottom in the figure. In Figure 2, point A, for example, shows that a new plan where the duration of liabilities is 32.4 years can build in a funding cushion of 20 percent by lowering the discount rate from 7 percent to 6.4 percent, while a plan with a duration of liabilities of 20 years with a hurdle rate of 6 percent would have a funding cushion of approximately 10 percent. The figure shows durations of up to 60 years, but for most plans the durations would presumably be in the range of 7 to 30 years. A target funding cushion of 5 percent requires that hurdle rate be adjusted below the expected rate of return to 6.6 percent.

Figure 2.



Note: The adjusted discount rate is the hurdle rate. It is calculated for a mean rate of return of 7 percent, the assumption used throughout the paper. The empirically relevant part of the figure is contained in the box having point A as the upper right-hand corner.

Source: authors' calculations

The chart shows the combinations of liability uncertainty and confidence level that this cushion could provide. As expected, controlling for the duration of liabilities, a higher discount rate provides a lower funding cushion. Also as expected, the longer the duration of liabilities, the higher the discount rate that can achieve a given funding cushion. The assumed 7 percent rate of return was used to determine the liability duration.

5. The Extended Model

Modeling. This section extends the modeling of the previous sections by involving 75 cohorts over a simulation period of 75 years. We assume that each year a new cohort enters the plan at age 65 and starts receiving benefits. Over the 75-year projection period, 75 cohorts enter the plan. Each cohort consists of one person who faces a probability[4] of death up to the final age of 110. In this model, we are not concerned with the person until they reach age 65, when they are endowed with a pension benefit of $1 a year.

Still, the model is a substantial simplification from a real-world defined benefit plan but confirms the qualitative results of the previous simpler models.

The analysis in this paper assumes that the funding ratio at the beginning of the analysis is equal to 1 (assets=liabilities). The discount rate for determining the value of the initial liability is selected so that the plan sponsor using that rate would not have more than an X percent probability of needing to make further contributions of Y percent of the liability at the time of those additional contributions.

To be specific, we set as an example the requirement that the plan sponsor has no more than a 10 percent probability of needing to make additional future contributions exceeding a fixed percent, such as 10 percent of the value of liabilities at the time when the additional contribution is needed. Therefore, the funding ratio-threshold at which additional contributions are required-is 90%. This requirement takes into account both the probability of needing to make future contributions and the amount of those future contributions. The plan sponsor must make additional contributions if the funding ratio falls below a fixed percent. While different parameters can be chosen and arguments can be made for different standards, we choose this standard to provide an example of the approach we propose in this paper for choice of a discount rate for determining the regulator's requirement for contributions by the plan sponsor.

Calculation of the Liabilities

Modeling. In this model, there is no difference between accrued benefits and projected benefits because we value benefits at the point of retirement. Based on Queisser and Whitehouse (2006),[5] the parameters involved are as follows:

1) The present value of an annuity depends on the probability of being alive to claim the pension in each period, as measured by the survival function s_x. That is: $s_x = \prod_{i=x}^{w-1}(1-q_i)$ where q_i is the mortality rate at age i and w is the latest age at which it is possible to survive.

To take into account likely future reductions in mortality rates, we use the projected mortality tables for the U.S. (obtained from Life Tables for the United States Social Security Area

[4] To take into account likely future reductions in mortality rates, we use the projected mortality tables for the U.S obtained from *Life Tables for the United States Security Area 1900-2100*.
[5] Queisser, M. and Whitehouse, E. (2006) "Neutral or Fair? Actuarial Concepts and Pensions-Systems Designs". OECD Social, Employment and Migration Working Papers. Paris, France.

1900-2100[6]). We calculate the survival probabilities from age 65 (the retirement age) to age 110 (the maximum survival age).

2) The calculation of the present value of future benefit payments is done as follows. Let Z be the present value random variable of a pure endowment benefit of 1 payable at an uncertain time T. Then $Z = v^T = (1+i)^{-T}$.

3) Finally, account must also be taken of the adjustment of pensions in payment to reflect changes in costs or standards of living, called the "indexation" (U.S.) or "uprating" (U.K.) policy. The value of the adjustment to pensions in real terms is shown below as λ.

So, the present value of 1 future monetary unit is denoted by:

$$PV_{Liabilities} = \frac{s_x(1+\lambda)}{(1+i)^{-x}} \qquad (5.1)$$

Following Queisser and Whitehouse (2006), the value of the pension payment received in a future period is the initial payment, discounted, adjusted through price indexing procedures, and multiplied by the probability that the participant is still alive to receive the benefit. Summing these present values of flows gives the present value of the pension. This is the annuity factor,AF:

$$AF = \sum_{t=x^{(r)}}^{w} \frac{s_t(1+\lambda)^t}{(1+i_t)^t} \qquad (5.2)$$

where $x^{(r)}$ is the age of retirement. Each year a new contribution is made based on the calculation of the liability for the new cohort and whether the plan is underfunded based on past liabilities. Payments are made annually in advance. The value of s65 is 1.00, and survival rates for subsequent years vary by cohort.

Assuming an indexation of benefits in payment of 0 percent and a discount rate of 7 percent, the annuity factor assuming a 75-years horizon is $144.90. That is, a lump sum of $144.90 would be needed to pay a pension of $1 a year for all one-person cohorts during 75 years, according to the mortality projections and an expected rate of return of 7 percent. We assume the pension for each new cohort is initially fully funded based on the discount rate assumption selected.

Empirical Assumptions. We assume the return of the portfolio of assets has a stochastic rate of return that is mean reverting, using an Ornstein-Uhlenbeck process[7]. This assumption is superior to the assumption of a normal distribution made earlier because it recognizes the mean reversion

[6] Bell, F.C. and Miller, M.L (2005) Life Tables for the United States Social Security Area 1900-2100. Actuarial Study No. 120. 2005 Annual Report of the Old-Age, Survivors, and Disability Insurance (OASDI).

[7] The Ornstein-Uhlenbeck process is a stochastic process that has been used in financial mathematics to model currency exchanges rates, commodity prices and interest rates. The process, over time, tends to revert to its long-term mean.

process in rates of return over time. The mean reversion process means that each year there is a tendency for the rate of return that year to revert to the mean of the distribution of rates of return, compared to the rate of return the previous year. The tendency to revert to the mean is greater the farther away from the mean was the previous year's rate of return.

While we assume that mortality rates decline over time, the riskiness of the liabilities in our model arises due to the stochastic aspect of the decrease in the mortality rates over time. This risk is modeled using a beta distribution. The beta distribution is always positive and between 0 and 1, and it does not have heavy tails. This assumption is superior to the assumption of a normal distribution, used in earlier modeling because the normal distribution assumes that there is a risk that mortality rates for the population will increase over time. Thus, the survival probabilities are modeled as follows:

$$s'_x = \prod_{x=0}^{t} \{1 - [q_{x*} Beta(\alpha, \beta)]\} \quad (5.3)$$

Using mortality tables for the United States[8], we calculate unisex survival probabilities from age 65 to 110. Life expectancy at age 65 is 18.22 years for the first cohort. It is 22.10 years for the 75th cohort.

As the risk from assets and liabilities come from different sources (longevity risk and market risk, respectively), we assume that the risks for assets and liabilities are uncorrelated. For simplicity, we don´t considered changes in interest rates, which can affect both the value of assets and liabilities and the effects could not be measured.

Simulations

Using Monte Carlo simulation, we calculate dynamically the value of the liabilities in each period assuming a mean rate of return of 7 percent. We compare this value with the total assets of the system in the same year. If the funding ratio falls below 90 percent, an additional contribution, with an upper bound of 10 percent of the initial level of contribution discounting with the same rate as for the liabilities (in order to make this quantities comparable), is needed. If the probability of this contribution is higher than 10 percent, an initial funding cushion (and lower hurdle rate) is needed in order to satisfy the restriction of no more than 10 percent of the time.

Results. Confirming the earlier results in our simpler models, the higher the risk to liabilities and to assets (the higher the standard deviation of portfolio rates of return), the lower is the hurdle rate (Table 2). However, the hurdle rate is substantially higher than for earlier simulations we calculated which assumed only a single cohort. In the robustness analyses reported later, we find that variations in the degree of mean reversion have relatively little effect on the results.

[8] Bell, F.C. and Miller, M.L (2005) Life Tables for the United States Social Security Area 1900-2100. Actuarial Study No. 120. 2005 Annual Report of the Old-Age, Survivors, and Disability Insurance (OASDI).

While future rates of return are stochastic, we assume that the distribution of future rates of return is known. Thus, we do not deal with the problem of the actual distribution of rates of return differing from the expected distribution. An additional simplifying assumption is that the plan sponsor will be able to make the required future contributions. The impact of this assumption is mitigated, however, by the assumption that the hurdle rate is set taking into account the risk that the plan sponsor will not be able to make future contributions. That risk is not reduced to zero, however, and in the case of bankruptcy of the plan sponsor, we assume that either the pension benefit insurer will pay the promised benefits or that the beneficiaries will only receive the amount of benefits that can be paid.

Table 2. Hurdle rate r(7% rate of return, funding ratio 90%, 10% of the initial level of contribution, no more than 10% of the time A std, L std) for portfolio with expected rate of return of 7 percent and 90 percent funding requirement.[9]

Standard Deviation of Assets	Standard Deviation of Liabilities			
	0%	1%	11%	12%
0%	7.00	6.92	6.89	6.73
3%	6.97	6.88	6.80	6.69
6%	6.83	6.73	6.68	6.55
9%	6.70	6.59	6.51	6.34
12%	6.20	6.08	6.08	5.90

Note: The hurdle rates in this table depend on the expected rate of return (7 percent), the standard deviation of returns, the standard deviation of liabilities and the 90 percent funding requirement.

Source: authors' simulations.

Note: The regulatory requirement is that additional contributions must be made if the funding ratio falls below 90 percent. In addition, the discount rate (hurdle rate) must be set so as to assure that additional contributions of more than 10 percent of liabilities at the time will not be required at any time during more than 10 percent of the stochastic trials.

Our earlier calculations (Table 1) suggest that a portfolio with an expected rate of return of around 7 percent would have a standard deviation of around 11 percent, making the final row of Table 1 the most relevant. The assumption of mean reversion does not affect the calculation of the standard deviation. Depending on the risk to liabilities, over the range simulated the hurdle rate would vary from 5.90 percent to 6.20 percent for a portfolio with an expected rate of return of 7 percent (Table 2). Thus, these simulations are qualitatively similar to our earlier, simpler simulations that found that the hurdle rate is above the rate of return on bonds but below the expected rate of return on the portfolio.

[9] The "standard deviation of assets" is the standard deviation of annual rates of return, while the standard deviation of liabilities is the variation in the emerging liability for each new cohort.

Robustness Testing

For further robustness testing, we change some of the assumptions.

Assumptions. Table 3 redoes Table 2 but with the requirement that additional contributions must be made if the funding ratio falls below 99 percent.

Results. It is seen that with the more stringent requirement as to when additional contributions must be made a slightly higher hurdle rate can be used.

Table 3. Hurdle rate r(7% rate of return, 99% funding ratio, 10% of the initial level of contribution, no more than 10% of the time, A std, L std) of portfolio with expected rate of return of 7 percent and 99 percent funding requirement

Standard Deviation of Assets	Standard Deviation of Liabilities			
	0%	1%	11%	12%
0%	7.00	6.83	6.80	6.76
3%	6.88	6.75	6.72	6.70
6%	6.80	6.67	6.66	6.60
9%	6.65	6.56	6.39	6.32
12%	6.16	6.11	6.09	6.05

The hurdle rates in this table depend on the expected rate of return (7 percent), the standard deviation of rates of return, the standard deviation of liabilities and the 99 percent funding requirement.

Source: authors' simulations.

Note: The regulatory requirement is that additional contributions must be made if the funding ratio falls below 99 percent. In addition, the discount rate (hurdle rate) must be set so as to assure that additional contributions of more than 10 percent of liabilities at the time will not be required in more than 10 percent of the trials.

Assumptions. Table 4 redoes Table 2 but with a 3 percent expected rate of return on the portfolio, instead of 7 percent.

Results. The lower expected rate of return proportionately lowers the hurdle rate, thus showing that the hurdle rate is affected by the expected rate of return on the portfolio.

Table 4. Hurdle rate r(3% rate of return, 90% funding ratio, 10% of the initial level of contribution, no more than 10% of the time, A std, L std) for portfolio with expected rate of return of 3 percent and 90 percent funding requirement

Standard Deviation of Assets	Standard Deviation of Liabilities			
	0%	1%	11%	12%
0%	3.00	2.92	2.88	2.84
3%	2.97%	2.90%	2.82%	2.78%
6%	2.90%	2.83%	2.81%	2.67%
9%	2.70%	2.55%	2.54%	2.47%
12%	2.41%	2.30%	2.02%	1.96%

Source: authors' simulations.

Note: The regulatory requirement is that additional contributions must be made if the funding ratio falls below 90 percent. In addition, the discount rate (hurdle rate) must be set so as to assure that additional contributions of more than 10 percent of liabilities at the time will not be required more than 10 percent of the time.

Assumptions. Table 5 assumes a closed population where the simulation ends when everybody dies. By comparison, in Table 4 the simulation ended after 75 years, at which point some people were still left in the plan.

Results. This change has little effect. The results in the two tables are similar.

Table 5. Hurdle rate r(7% rate of return, 90% funding ratio, 10% of the initial level of contribution, no more than 10% of the time, A std, L std) for portfolio with expected rate of return of 7 percent, 90 percent funding requirement and a closed population

Standard Deviation of Assets	Standard Deviation of Liabilities			
	0%	1%	11%	12%
0%	7.00%	6.71%	6.66%	6.61%
3%	6.73%	6.69%	6.64%	6.55%
6%	6.61%	6.61%	6.54%	6.52%
9%	6.52%	6.43%	6.41%	6.19%
12%	6.20%	6.16%	6.00%	5.94%

Source: authors' simulations.

Note: The regulatory requirement is that additional contributions must be made if the funding ratio falls below 90 percent. In addition, the discount rate (hurdle rate) must be set so as to assure that additional contributions of more than 10 percent of liabilities at the time will not be required more than 10 percent of the time.

In addition, for further robustness checking, we redid the results assuming the degree of mean reversion was half what was assumed in the preceding tables. Changing the degree of mean reversion has little effect on the results.

6. Conclusions

Around the world, defined benefit plans have liabilities of more than $23 trillion, but there are different approaches and views as to how these liabilities should be measured for purposes of determining required funding. The analysis in this paper argues that different measures of liabilities are appropriate for different purposes. Using a variety of models, it shows that for risky assets and risky liabilities, a discount rate lower than the expected or mean rate of return on the defined benefit plan portfolio is needed to provide a target funding cushion whenever the target probability of success exceeds 50 percent.

Thus, this analysis shows, for a fully funded plan, that the "market-based approach" of using the risk-free interest rate for discounting risk-free liabilities has limitations for determining required levels of funding. Generally, a higher discount rate is appropriate. Also, this analysis shows that the "expected return-based approach" of using the expected rate of return for discounting liabilities has limitations for determining required levels of funding, and that generally a lower rate is appropriate.

With the stochastic funding parameter or risk of ruin approach, the risk to both the assets and liabilities affects the hurdle rate (discount rate) used for determining required funding. Generally, that rate is less than the expected rate of return on the portfolio, but higher than a rate or rates derived from current bond yields.

The key to this approach is the evaluation of the risk that future contributions will need to be made because the contributions made at the time the benefit liability was incurred turn out to be insufficient. The ultimate risk of concern is the risk that contractual obligations to pension participants concerning their future benefits will not be honored. This risk varies across different types of plan sponsors. Corporate plan sponsors face the risk of bankruptcy. The risk to plan participants varies across different types of government entities, being greater generally at lower levels of government. Governments have the power to tax, which in principle limits the risk to their pension participants. The power to tax implies the possibility of using higher discount rates for funding government-sponsored pension plans than for private sector pension plans. The power to tax, however, is limited by the ability of citizens to leave high-tax jurisdictions and move to lower-tax jurisdictions, called "voting with their feet".

The policy implications of this analysis are clear. Current approaches used in the United States and elsewhere that determine interest rates for funding defined benefit plan liabilities solely by valuing the liabilities based on risk-free or corporate bond rates, or solely based on the expected rate of return on the assets both have limitations. With the stochastic funding parameter approach, while the exact discount rate (hurdle rate) used to determine adequate funding depends on the risk to the assets, the risk to the liabilities, and the duration of the liabilities, a simple rule of thumb can be stated. That rule would be to select a discount rate that is less than the expected rate of return on assets but greater than the risk free rate, with the discount being greater the higher the percentage of the portfolio invested in equity and the longer the duration of the liabilities.

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