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Session 7PD
Applied Modeling Concepts

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Summary: Most financial actuaries are quite familiar with basic modeling applications, as they constitute an integral part of actuaries' responsibilities. This session provides more information on advanced modeling techniques, with an emphasis on current issues, particularly stochastic modeling for variable benefits. Topics include, but are not limited to: advanced liability and asset modeling issues; model distinctions between types of variable annuity guarantees; stochastic mortality; stochastic-on-stochastic modeling; and alternatives to stochastic modeling. At the conclusion, attendees have a greater understanding of current modeling issues and have gained some valuable insight in today's stochastic world.

MS. REBECCA BURTON: The information we present today is what we've spent much of our time on over the past couple of years. I would like to take a moment for my co-presenters to introduce themselves.

MR. RONALD HARASYM: I work in the Corporate Risk Office of Sun Life Financial in Toronto, Canada. In my role, I deal with capital market risks that emerge from embedded liability options that have been sold over the past number of years. I have worked in Canada, lived and worked in the United Kingdom, and I also have an extensive amount of experience in dealing with risk management issues in the United States.

Note: The chart(s) referred to in the text can be found at the end of the manuscript.

MS. REBECCA GRANT: For the past 10 years, I've done work that involves actuarial modeling. Five years of that was on a consulting basis doing cash-flow testing, asset-adequacy analysis and pricing. Then I moved to the life division of ING and did modeling for pricing as well as stochastic testing. Recently I transferred to a reinsurance company, where I have also been involved in work including actuarial models.

MS. BURTON: Our first topic covers liability modeling issues, largely focusing on variable annuities and variable universal life (VUL), but for you reinsurers out there, we have a little bit of a bonus to offer up.

The hot topic today is variable annuities. It seems that every time I turn around recently, someone is asking me to model a variable annuity or some special or new feature about one. Guaranteed minimum death benefits (GMDBs) have been around for a while, followed by living benefits, especially guaranteed minimum income benefits (GMIBs). Also many companies are offering guaranteed minimum accumulation benefits (GMABs) and entering more and more into the picture are guaranteed minimum withdrawal benefits (GMWBs).

With GMDBs, we have a pretty good idea of the pattern of the contingency upon which the benefit is based. There are not a lot of surprises there. More of these are "in the money" than some companies would like, given the movement in the market over the past years. However, for living benefits, some of us probably feel like throwing our hands up, given the difficulty in assessing the probable utilization of these benefits. Not only is it difficult to figure out what will trigger policyholders to act in certain ways, but also there's just very little experience available. The recent market is so unpredictable, resulting in the requirement that we project these annuities across every conceivable scenario.

While in many cases the structure of the product will prove beneficial to the insurer and the policyholder, it does have a catastrophic risk profile. Under a handful of cases, the insurer can lose its shirt.

I've had cause to give a lot of consideration to the development of model points—model points that are necessary given the environment in which we need to be incorporating more and more stochastic modeling. Any good model point development project begins with a thorough analysis of the data and the product types. To the extent that product types have a similar fee structure or the development of the account value is similar, these products can be grouped.

As for ages, you should consider having more granularity at the older ages given that a lot of the benefits either stop increasing, lock in or simply go away at particular older ages. Also the mortality table is steeper at the older ages. However, I do find we have a tendency to model more ages than are necessary. We really need to steer clear of this given that it just develops a larger model for you to keep and maintain, and it will significantly add to your run-time.

As for gender, I suggest blending gender (i.e., placing male and female into the same groups), keeping track of the percentage male either in aggregate across all model points or separately for each model point. A separate mortality assumption could be developed based on that split.

As for asset allocation, I prefer letting it fall out of the mix. I would take your huge number of separate accounts that you have and bring them into groups, perhaps the AG34 funds. Then, group seriatim records into model points, based on age and gender grouping rules and let the asset allocation fall so that each record has an individual fund mix.

In addition, be sure to give some consideration to any reinsurance agreements that you have in place. For example, you may reinsure 85 percent of a particular benefit if it was issued before 1999 and 50 percent if it was issued in 1999 or later, so you need to take care to consider that split.

"In the money" bands might alternatively be called net-amount-at-risk bands. The relationship of the net amount at risk to the account value indicates how "in the money" or "out of the money" the benefit is. You would not want to group a policy that is deeply "in the money" with a policy that is not because you would get an average net amount at risk that is not truly indicative of that model point. Rather, you would want to have more bands around the 0 percent (or 100 percent if you're looking at GMDB versus account value). As you move further away from 0 percent, you could widen your bands since there's less of a chance that a large swing would move you into the opposite position.

MS. GRANT: For VUL, there are several different considerations for developing the model points. One of the main assumptions is the funding level(s), and it's one of the biggest challenges to model—how to generalize the premium that's coming in into certain levels, such as seven-pay premium for a certain number of years, guideline level or minimum payments. It's good to try to group into these different categories, but to do that based on actual data is the challenge. One thing you might want to consider is talking to the pricing people to see what kind of marketing information they obtained at the time of pricing. Find out what a predecessor product might be experiencing as well, if it is similar to the current product. There could be a tendency to get into a lot of detail and try to exactly use what's been paid in the past to determine what will be paid in the future, but that can be a mistake since it's such a big assumption. People can pay anything they want, except for the limitations of tax and minimum payments, of course. But you do want to keep in mind the effects that that big assumption has on modeled items such as your cost-of-insurance (COI) charge that is collected, your mortality and expense (M&E) levels and your lapse assumptions. For instance, if you have a minimum payment, you may want to have a higher lapse scenario for that.

You may or may not want to reflect substandard policies in your model. The decision might be based on what level of profitability is expected from them. For

instance, if you have a product with substandard adjustments as a flat percentage increase in the COI charge, you may have additional profitability from that because your mortality expectation is probably not as high as the rating increase. If that is the case, it might significantly affect your model results if you include substandard policies. If you do include them, there might be some additional considerations; for example, does the rating drop off after a certain number of years? After 20 years, does the insured gets a standard rating either for the policy issued, or for the reinsurance rating, or both? If that happens, does your mortality expectation go along with it? Do you expect mortality to actually have the rating disappear after a certain number of years? You may want to talk to your underwriting experts to see if that is a real expectation.

Another consideration is the table-shave programs that are coming out these days, where someone might actually be rated Table A through D, and they're issued as standard, for instance. That is something that you would want to reflect in your model if you have that kind of program and are reflecting substandard policies. You should consider whether your reinsurance rating is reflecting their actual rating or the shaved rating. There might be an additional cost from the reinsurance for the table shave program.

The UL Commissioner's Reserve Valuation Method (CRVM) reserves can be significantly different depending on whether you reflect the substandard rating in the calculations, so you should test that separately.

The COI rates are another consideration for substandard that might affect your guarantees and your current rates. Your target premium may also be affected.

For commission options, these days there seem to be multiple options, such as level or semi-heaped. You may want to consider having separate lapse scenarios for those different commission options.

As for blending with riders, I'm specifically referring to the type of term rider that doesn't have a target premium. When you blend with that, it affects the commission that is paid for your product. That can significantly affect your profitability, especially if your expense allocation scenario is a specified amount up to target and then a different amount of excess. You may want to reflect a certain amount of blending, but take into account the fact that your expense allocation may change and whether it is really reasonable to have a different amount of expense coverage, whether it has a term rider or not, even if it's otherwise the same kind of policy.

For product charges, you may want to group certain items in order to save on run-time. But you want to keep in mind what charges you would actually reflect in the model. Would it be the charges for the business with the most inforce, or would you use a blended rate or charge? The problem with using a blended rate is that if you're blending it in a spreadsheet before putting it in the model, that's just one

more thing outside of the model to keep up with. But it would be the more accurate approach.

For reinsurance, you do have different types of reinsurance by different issue years, so that might already be reflected in your model. But you could also have different changes that might not be as obvious, such as annual accounting versus monthly accounting. The effects from this type of assumption on the reserve credits might be significant, so you would need to make sure to have good communication between your modeling team and your reinsurance administration team to make sure you have any updates reflected.

Regarding years of projection and pricing horizon, it depends mostly on two main things: the accounting method being used (U.S. GAAP versus statutory), and the type of product. For instance, if you have a product that has long-term guarantees where you have significant benefits after, say, 30 years when you would normally project for only 30 years, you would want to consider modeling past the 30-year time period so that you can see how those significant benefits affect your total results. But that's usually for U.S. statutory analysis. For U.S. GAAP, a 30-year projection is considered to be adequate, but you want to make sure to communicate with your actuarial valuation people so that you know if they decided to change that.

MS. BURTON: Reinsurers should be familiar with this issue. More often than not the data you seek for the modeling project you have at hand is given to you much too late to complete your project. I'll discuss a few examples of solutions to this problem other than getting the ceding companies to give you your data sooner.

Let's say you need a projection at 12/31, but all you have is 9/30 data. Complete a projection of the inforce and new business as of 9/30, where the new business records would represent actual volume if you know this, expected volume if you don't, but not necessarily the true distribution of the new business. Calculate the present value at 12/31, thus lopping off three months of results. This should be a good estimate of value if your assumptions are similar to what's actually going on. However, it obviously doesn't capture actual experience in that last quarter.

As a second more simplistic example, using the same 9/30 inforce projection, forget about new business. Rather, ratio this value by the change in volume or the ratio of volume at 12/31 over the volume at 9/30. I say volume, but you could use reserve or premium if either is a more reasonable measure for the particular product type. The thing I don't really like about this method, though, is it doesn't really capture big differences in the mix of business or profitability. For example, you may have 10 product types, and the last quarter of business is really just the one product type. However, you've assigned that last quarter of business the same profitability as all of your other inforce business.

As another example, say the profitability of your older business is fairly high, and the profitability of your newer business is fairly low. Again, by using this measure you've assigned the newer business a weighted average profitability of all of your business. If you're able to discern something like this, split up the business in the actual projections, and use only the appropriate piece for the gross up.

In a third example, take the inforce file at 9/30 and, to the extent that you're able, using the volume at 12/31, ratio the 9/30 records to the volume at 12/31, and pretend that this is your 12/31 inforce file.

MR. HARASYM: I would now like to talk about asset modeling issues. We are experiencing an ever-increasing complex array of the assets that are available in the marketplace. On the liability side, we are also experiencing an increased complexity, not only from a product development side with the inclusion of liabilities with embedded options, but also from a reserving and capital perspective where stochastic modeling is being introduced. As actuaries, we tend to pay less attention to the asset side while the investment people tend to pay less attention to the liability side—at least, that is my experience. Certainly, the two sides have to be brought together because you cannot build a high-end liability model without having a robust asset model. Both have to be communicating and talking with one another, and they both have to be structurally consistent. Otherwise, all the post-processing information that emerges out of the overall modeling process is only as credible as the weakest link.

One modeling solution is to buy an external asset-liability software package. The software is likely to already be tested, validated, and documented. Support is often available from either the software vendor or consultants. However, beware of proprietary systems that can be a black box. That is, the software vendor will not provide much detail on the underlying modeling concepts being used.

When you're dealing with complex models, the risk of misspecification can be high, and you have to exercise extreme caution. Constantly perform reality checks; use judgment. One certainly needs to adopt a commonsense approach.

MS. GRANT: One particular asset modeling issue where a commonsense approach may be applied involves the negative cash-flow strategy assumption that is used in investment models. When you use the assumption of purchasing negative assets, which is usually used as a proxy for borrowing from other portfolios or other lines of business, strange results can sometimes be produced. Therefore, you should make sure to consider doing sensitivity testing it with an alternative assumption, such as borrowing at a specified Treasury rate plus the spread.

Now, we will move on to hedging.

MR. HARASYM: A key modeling consideration to hedging is whether you take a deterministic approach or a stochastic approach. The decision rests upon a number

of considerations, some of which are: what your capabilities are; what your expertise is; the nature of your exposure; and whether or not your liabilities are path-dependent. There could also be regulatory reasons for performing stochastic modeling. From an asset-liability activity perspective, you can take either a static approach or a dynamic approach. A static approach more or less is similar to a buy and hold, with very infrequent rebalancing, while a more dynamic approach is one whereby you are looking at rebalancing several times a day or as frequently as your policy or business plan would dictate. Certain policyholder behavior assumptions, such as lapse or annuitization for variable annuities, may be more meaningful when specified on a dynamic basis.

Implementation of a dynamic investment strategy can be a challenge. What will differ are the types of hedging vehicles you use. For a static hedge, you might be using over-the-counter dealer-purchased derivatives, whereas for dynamic hedging you will probably be using exchange-traded derivatives. Your strategy depends upon the nature of the liability that you're trying to hedge. The hedge strategy will often be a part of investor and analyst presentation packages. It's always glorious to implement a hedge strategy. However, people tend to spend not enough time on monitoring both the effectiveness and efficiency of hedging activity.

Model testing, validation and analysis are truly ongoing processes with a constant feedback loop. Feasibility tests are where one asks the question whether the range of simulated results is feasible or sensible. This is not necessarily a test of model calibration, but more or less just a "gut feel" test. There are validity tests where one asks the question whether the outputs reflect what went in. As expert user tests, you can survey a number of individuals, but of course, experts will never agree. That's why they're experts.

Another thing I'd like to focus on is hedge effectiveness versus hedge efficiency. Hedge effectiveness is "doing the right thing," while hedge efficiency is "doing things right." For example, let's say that you have a variable annuity risk and you back the risk with $1 billion of assets. You could say that you hedged the risk by brute force, but your return on capital employed would certainly suffer. However, if you had hedged the risk efficiently, you could have done it with, say, $100 million or $200 million. The point is that hedge effectiveness is not hedge efficiency; it's good to separate the two.

Hedge monitoring also is very important in the sense that you can gain experience via the information loop. By monitoring hedge effectiveness and efficiency in a consistent and structured fashion, you can see where things went right or went wrong and modify your strategy accordingly.

Let's look at hedging equity-indexed annuities (EIAs). One issue here is that EIAs have a short time frame; it makes the use of over-the-counter derivatives a possibility, but they can be expensive. Accordingly, a good argument can be made for dynamically hedging EIAs. It's typically cheaper to hedge dynamically in-house

using exchange-traded options. When you are performing dynamic hedging for an EIA, you're typically managing the various sensitivities to key modeling parameters, such as the interest rate sensitivity, the time sensitivity and the volatility sensitivity. One must not forget about managing expectations. If you're hedging using only the equity market delta, or the first order change in the liability to the equity market, people may think you have put in place an all-encompassing hedge. However, when there is a material equity market shift and you get "hit" by a second order impact, or what investment people call gamma, you could have some explaining to do. Letting people know what they could expect can save you trouble down the road.

I would now like to talk about enterprise risk management and earnings-at-risk. The objective of earnings-at-risk is to try to quantify the frequency and severity of earnings to future economic scenarios. It's a forward-looking process. What we're trying to do is identify, through deterministic stress testing, where our risk factor exposures are. This can ultimately lead to the development of an early warning system. The greatest benefit Sun Life Financial has obtained from its earnings-at-risk framework is the quantification of international, as well as intranational, risk diversification. We have products in the United Kingdom that provide some risk diversification to products in the United States. We can't obviously take regulatory benefit for that. However, we have a better understanding overall of the financial dynamics underlying the macrobusiness of operating multinationally.

Chart 1 is a sample earnings-at-risk report at the international company level. The measure that was used here was deviation from planned (or expected) income. The measure is conditional tail expectation (CTE) at the 95 percent level—that is, the average of the 5 percent of the least favorable results. Along country A, the uncorrelated total at the 95th CTE (a simple addition of the interest rate and equity risk in isolation) is equal to $112. But when you take into account correlation within country A, the aggregate risk level drops to $90. Along the interest rate risk dimension of country A, uncorrelated interest rate risk measured at CTE 95 percent is equal to $211. Total interest rate risk, taking into account correlation (or lack of perfect correlation) between countries, is significantly less at $71. The concept is fairly straightforward; however, this process requires an enormous amount of modeling with heavy computational demands.

MS. BURTON: Now, we'll move on to assumptions.

MR. HARASYM: We would now like to talk about critical risk factors, which depend on what products you are looking at. Sensitivity testing, knowing your product design and knowing your scenarios are all important.

MS. BURTON: Potential benefits resulting from GMIBs are a function of net amount at risk and annuitization rate. In addition, lapsation can be thought of as a function of the current net amount at risk. For GMIB modeling, a huge question is in regard to reasonable estimates of annuitization and lapse rates. Unfortunately, we have

very limited experience. Certainly, the relationship of the guarantee amount to the account value affects policyholder behavior, but for GMIBs, the actual relationship to consider is that between the current and guaranteed payment—where the current payment is based on the current account value and the current purchase rates, while the guaranteed payment is based on the guaranteed amount and guaranteed purchase rate. We also might wonder to what extent the policyholder really understands that.

MS. GRANT: Dynamic policyholder behavior assumptions include lapses, annuitization, partial withdrawals, loans and funding persistency. For all these types of assumptions, it is difficult to obtain experience data. Also, the data that you have might not even be relevant. For instance, if you have a lapse study, it would be based on the results of the actual interest rate scenarios that have been experienced in the past. It might not have any relevance to the future and the scenarios that you are modeling now. You also need to make sure to consider any changes in management or the way they react to certain economic conditions. For dynamic assumptions, you need to weigh the effort of developing an assumption with the amount of expected effect that it has on your overall model. For instance, loan utilization—if your product has a preferred loan where the credited rate is the same as the charged rate after a certain number of years, then that might result in a larger effect on the loan utilization rate.

Regarding lapse rates, you should consider having several different ranges of possibilities to choose from depending on the product characteristics, since some products will be more sensitive than others. For instance, corporate-owned life insurance (COLI) may be much more sensitive to changes in the rates of the market than a non-COLI product, or a joint life product may be expected to be less sensitive than a non-joint product. For each main product type, there would be a different threshold (the difference between the product's credited rate and the competitor's credited rate) where products that are more sensitive have a smaller threshold, and vice versa.

MS. BURTON: For variable annuity products, the question is how lapse rates are impacted by the presence of guarantees, particularly if the latter is significantly "in the money." This is especially relevant if the policy is within the surrender charge period. I've come to think that there are a few schools of thought regarding lapse rates for variable annuities. First of all, as the guaranteed amount increases relative to the account value, policyholders might be less prone to surrender because they have such a rich benefit. Alternatively, if the account value is very high relative to the guaranteed amount, policyholders might be less prone to surrender because performance is so good. Under this school of thought, we would then expect that as the account value decreases more funds might be moved out of variable annuity products.

It is tricky to develop a reasonable dynamic assumption. I'm not aware of a particular experience study, and it is difficult to say which of these has the more

important impact or influence. There are certainly other factors that affect the inclination or reluctance to lapse.

For annuitization, the dynamic assumption should be a function of the relationship of the guaranteed payment to the current payment. As the guaranteed payment increases relative to the current payment, so should the assumed annuitization rate. The question, of course, is by how much. And this is likely an exponential increase rather than a linear increase.

For partial withdrawals, and I'm talking again from the variable annuity perspective, there are a couple of reasons why we might need a dynamic assumption. For policies that allow dollar-for-dollar reduction upon withdrawal, the dynamic assumption could be geared off the relationship between the guaranteed amount and the account value. For policies with a GMWB, there is a question regarding what a reasonable assumption would be. The drivers behind the utilization of the GMWB, in my opinion, are time to lapse after issue and account value performance. But there are many questions regarding the assumptions to make about policyholder behavior.

MS. GRANT: Regarding the assumption for maintenance expense inflation, you should consider whether your assumption is based on per unit, per policy or both. For the funding level, you may want to test a range of sensitivities since that is such a difficult thing to model specifically. You could consider modeling 50 percent higher funding and then 50 percent lower funding, but you should keep in mind the applicable limitations, such as tax limits based on the guideline level or seven-pay tests and minimum premium amounts.

Premium financing is a methodology where wealthy individuals who don't want to liquidate their assets to pay the premiums for a life insurance policy in cash take out a loan from a bank or other financial institution and pay the premiums with the funds from the loan. The insurance policy is held as collateral for that loan, which has an interest rate that is usually tied to the London Interbank-Offered Rate (LIBOR) or some other index plus a spread—resulting in a loan rate that can change quarterly or annually. It is important to reflect in the dynamic modeling of these policies the expected effect of that changing loan rate compared to your credited rate. For instance, if your credited rate is going down at the same time as their loan rate is going up, you might have a lot of excess lapses realized from that type of result.

MR. HARASYM: One might question: why would we want to perform stochastic modeling when deterministic modeling has served us so well for so long? The word stochastic is a Greek word meaning, in essence, "to guess at." However, this is a meaning you may not want to share with senior management. Stochastic modeling is preferred when you're dealing with complex guarantees. It's also preferred when you want to model tail or extreme events. Often there are real economic incentives to perform stochastic modeling. For example, in Canada, if you do not perform

stochastic modeling to calculate your capital requirements on segregated funds with investment performance guarantees, then you have to perform a prescribed factor approach which will likely result in a higher capital requirement.

MS. GRANT: An important application of stochastic modeling is to determine a cost for the volatility of interest rates so that the cost can be reflected in a deterministic model. The challenge is to develop a single amount to represent the cost, when the stochastic results are a distribution of results rather than a single number.

So how do you transform the stochastic result from the distribution to a single result that's meaningful? One thing that must be considered is the weighting of all the scenarios. Do you weight them equally? Or do you weight them according to some type of distribution assumption? The challenge is that there's no obvious weighting formula to be used if you develop the scenarios randomly. As a result, they usually just get weighted evenly. To determine the stochastic cost, you need to compare the weighted average result to the result produced when the level scenario assumption is used in the same model.

MS. BURTON: Another metric that we can use to study results is the CTE, where something like CTE60 would be the average of your worst 40 scenarios.

MS. GRANT: A byproduct of stochastic modeling is that it can be used to test a company's crediting strategy. By "crediting strategy," I'm referring to whether you would keep the modeling pricing spread steady, or you would decrease that spread in the decreasing rate scenario in order to keep the business on the books and attempt to minimize excess lapses. Fixed annuities, especially, because they're so spread-driven, are significantly affected by this assumption. Recently some companies have had to decrease the spread or drop the rates for their fixed annuities.

Regarding reflecting a credited rate in the model, one of the main challenges is for portfolio-rate products where the credited rate is based on the portfolio rate minus the spread. In the model for a new product, the new money coming in is invested in the modeled assets at new money rates. So how do you model your credited rate and make it reasonable versus your new money rate? One way to do it is to have separate coding for the credited rate so it's based on your portfolio rate minus the spread and then have different asset earnings. Another way would be to do it in a spreadsheet and have liability model results combined with new business model results in a spreadsheet to come up with your portfolio rate and credited rate.

MS. BURTON: Another example or another use of stochastic modeling is the development of capital based on C3 RBC phase II requirements. The purpose of this regulation is to address interest rate and equity risk associated with variable

products. The stochastic part, the scenario testing, is required for living benefits, but note that it can be used for death claims as well.

While assuming initial assets equal liabilities, statutory cash flows in surplus are projected for each scenario over a particular horizon. The maximum present value over this horizon of the deficit positions is captured for each scenario and ranked, and a CTE calculation is performed to derive the required capital.

We do not discuss the important issue of the scenario generation, but note that the calibration and generation of these scenarios is important for this regulation. Also note that the calculations are done in aggregate across all model points where the results are compared at the scenario level.

Let's move on to stochastic-on-stochastic modeling. Some of you may wonder what is meant by stochastic-on-stochastic modeling. A plain stochastic model consists of three dimensions—model points, scenarios and time.

A fourth dimension is introduced that creates the "on-stochastic" environment. This fourth dimension could be a second set of scenarios. Think of a horizontal line as a base projection across time. For every model point and at every future specified point in time, you would stop and run a separate set of scenarios—the fourth dimension. Or this dimension could be another parameter, perhaps mortality. Again, you would stop and run these side calculations to develop whatever value it is that you're seeking before you continue on with your base projection.

For each scenario, you would collect aggregate results across model points, maybe rank them and perform statistical analyses in order to develop the number you're seeking before you continue on with the rest of your base projections.

In the case of 100 model points and 100 scenarios, we should technically have 10,000 lines on this page. Since that's not realistic, we have it represented by the downward ellipsis. The side calculations that would be performed are represented by these spider legs.

An example use of stochastic-on-stochastic involves Actuarial Guideline 39, which requires reserves for variable annuities with living benefits be calculated based on an asset-adequacy testing approach. For each node at which you desire to calculate reserves per AG39, you're required to complete a separate projection of benefits and premiums across hundreds or thousands of scenarios. Current reserves combined with the present value of your future premiums are compared to the present value of benefits. The excess of benefits over the sum of current reserve and present value of premiums equals the additional piece of reserve. You would determine this additional piece of reserve for each of the scenarios. You would then aggregate across all of the model points, rank them and then pick something like your 85^{th} or 90^{th} percentile to determine what increase you need to make to your

reserves. The latter selection would be made by someone like the valuation actuary.

MS. GRANT: Obviously this stochastic-on-stochastic analysis can have significant implications on the projection's run-time, Therefore, you should consider whether it's even necessary for all intervals of your projection, or if you can just use it for a subset of your projections, such as the first five years, especially for a product like GMIB where the significant cash flows really only occur in the first few years. You could consider different things like doing stochastic-on-stochastic annually and then interpolating for the in-between year values, or you could use it on a quarterly basis for the first five years or on a monthly basis for one year and annually for four years. You could also consider if the model could be simplified in any other way—if there is something else that can be simplified to decrease your run-time. If you can control the amount of output that your model produces and the reports that are produced, you may want to consider limiting those, thereby reducing your run-time.

MS. BURTON: The next topic is on stochastic modeling of mortality.

Traditionally, mortality assumptions have been modeled deterministically by pulling the mortality rate from a specific table for a specific individual, say, a 45-year-old, male, standard, nonsmoker. Then perhaps a multiple is applied. Mortality tables have been developed based on tons of data across many years, and these specific multiples are developed based on years of company-specific experience.

Using the deterministic approach, the following aspects are ignored: mortality volatility risk and mis-estimation risk.

Stochastic modeling of mortality becomes more necessary when your analysis has a limited number of lives or when there's a low probability of an event, but high severity.

A commonly used method for generating mortality scenarios is Monte Carlo simulation. For Monte Carlo simulation, you need a random number generator and a parameterized probability distribution, as well as a real-life interpretation of what your random number tells you. As an example, let's say you have N policies. You would generate N random numbers, let's call them X. Using the Bernoulli distribution around Qx, if your X is greater than Qx, then the insured survives. If your X is less than Qx, then the insured dies and that is your real-life interpretation.

Alternatively two parameterization techniques can be used. I'm going to skip the first one because I'm less familiar with it, and it seems to be a bit time-consuming and mathematically intensive. The second technique is easier to implement and for me to understand. Mortality is a function of a tabular rate and random deviations and shocks. The deviations would have a mean of one, and the shocks would have

a mean of zero, so that the average mortality rate using this formula for a very large number of trials would be the tabular rate.

MS. GRANT: Regarding alternative approaches for reflecting a stochastic cost in your deterministic model, there are two main ways for doing it. One is decreasing the pricing spread, which decreases the profitability as a result. The main problem with that approach is that if you look at your model and it is creating cash values based on a credited rate that is different from what you would actually expect because of this decrease in the spread, it might cause some confusion. An alternative would be to model it as a separate expense, so that it's a percentage of account value. This expense would be easier to isolate, but may cause some problems in your expense accounting analysis, so you want to make sure to remember that that stochastic cost is coming out of your model as a separate expense.

Also, the effects on competitiveness from your stochastic cost that you've included in your model might be considerable because they're decreasing your profitability, but not increasing the policy performance or increasing the commissions. So it makes it even more important to make sure that the cost is reasonable.

Regarding sensitivity analysis, it is always important to identify and direct effort on a key assumption and what the effects of changing that assumption are. For variable account performance, usually you use an average rate for all of the policies, but you might want to test some extreme changes, such as a big drop one year and increases thereafter, or low levels generally versus high levels and the effects on COIs and M&Es.

Regarding behavioral changes, the main assumptions are lapses, loans, partial withdrawals, annuitization and funding levels. Those all need to be tested to see the effects of different assumptions. For example, if the product is "in the money," you might expect to have lower lapses, but then again the policyholders are not always rational, and you need to keep that in mind as well. There are in general two main types of policyholders. One is the active kind. They always watch their policies and react to all the changes. The other is those who don't change anything at all. Sometimes it might be a mistake to model the average of those two. You might want to consider those separately.

For funding levels, you should keep in mind different things that you may have modeled for your baseline scenario that may not apply in your sensitivity testing. For instance, if you have a rolling target assumption, and in your baseline model you've paid target premium all years or higher, then you wouldn't need to model that rolling target. But if your sensitivity testing tests a lower funding rate, then you would need to make sure that that is reflected in your model.

As for regulatory changes, that would include reflecting possible change to statutory accounting, GAAP accounting or solvency requirements and their expected resulting effects.

MS. BURTON: As an alternative to actual stochastic-on-stochastic modeling, you could use distributions coming from your base projections to approximate an array of results in the future. Lloyd Foster, at Transamerica Re, who I think has about 15 degrees in statistics, has come up with an approach that he calls the generalized lambda technique. It's a technique that follows along with developing a means of approximating an array of future results based on distribution of base projections.

Another alternative approach would be scenario reduction and various sampling techniques, which Ron will discuss.

MR. HARASYM: We all know that stochastic modeling is time-consuming from a run-time perspective. Variance reduction techniques are available, but they tend to focus or converge on the mean very quickly while distorting the overall risk distribution and in particular, the tail. Low discrepancy or quasi-Monte Carlo methodologies have also been shown to be less effective in modeling tail risks when trying to perform reduced sampling sizes. That is, not when you're trying to perform more runs, but when you're trying to shrink the number of runs down to a more manageable number.

I would now like to talk about scenario reduction techniques. Let's say we had 1,000 scenarios. What we'd like to do is come up with a representative set of scenarios that would mimic the same risk profile that would emerge out of the complete set of 1,000 scenarios. Yvonne Chueh in the July 2002 edition of the *North American Actuarial Journal* wrote an article that discusses interest-rate sampling algorithms. Alastair Longley-Cook of Tillinghast at the CIA-SOA Joint Stochastic Symposium presented a paper on modeling of GMIBs and GMDBs using some of the techniques that Yvonne Chueh mentions. We've also performed some scenario reduction work in our own company, and I want to share our GMIB example with you.

First of all, you start with 1,000 (or some other large number) scenarios. There are several different methodologies or algorithms that have been proposed for reducing it down from 1,000 to, say, 50 or 100 scenarios.

If you're dealing with a GMIB, the majority of cash flows might be in the first five years. The beauty of the whole process is that it becomes more of an art than a science. When you are selecting scenarios, if you understand the risk profile of your product and you understand the sampling method or algorithm, then you can customize the process to achieve a better fit. In essence, there is no carved-in-stone answer as to how to perform scenario reduction.

In Chart 2, we used 1,000 scenarios and performed several scenario reduction techniques: the significance method, which basically picks 50 scenarios, assigning equal probability to each; and on the right-hand side, scenarios that are denoted D1 and D2, which are various distance measures as defined in Chueh's article. We found that if you're going to pick a mean measure or a lower CTE level, it seems that you're able to converge using a fewer number of scenarios fairly quickly. If you're going to a tail measure, the results haven't been as promising.

There are some other quirky things that we noted, too, which means that this concept of scenario reduction is not the end-all solution. With some of the algorithms, you have to select an initial scenario and then keep working your way through the scenarios. Depending upon which scenario you select as your starting scenario, you could get better or worse results. Your 50 ideal scenarios would also change. Furthermore, from one product to another, the best 50 would not necessarily be the same. So it becomes very hard to explain to management that the ideal 50 scenarios this month may not be the same ideal 50 scenarios from last month. A key caveat with scenario reduction is that you really have to know the underlying risk profile for the product you are dealing with as well as have a good understanding of the scenario reduction process.

The other caveat I would offer is that we ran 1,000 scenarios on this model, and this isn't necessarily enough to generate the true risk profile within a high level of confidence. It may take 2,000 to 3,000 scenarios on this model to get a fairly stable tail distribution. It may be preferred to perform the scenario reduction technique on a larger set of scenarios (perhaps from a universe of 10,000 scenarios) and then you have a better representation and perhaps a better fill of the path space.

I would now like to talk about scenario generators. I've seen a number of them, and the one thing to keep in mind is that no one single model has all the properties that we may need. Different models would be used for different purposes. The primary objective of the economic scenario generator is to produce future capital market scenarios. There is a wide variety of scenario generators to choose from. One consideration is whether you are concerned about the mean or the tail results. Are you concerned about pricing—coming up with market-consistent pricing? Or are you just concerned with doing a best estimate, roll-forward-type process?

One of the other issues is black box versus transparent. Some of the models offered by outside companies are proprietary, and so it's difficult, if not impossible, to find out what's going on inside. It may be a little harder to explain to management it's the black box that's driving the whole risk process versus a model that is transparent.

Calibration is another important issue. People will always disagree, but it's a function of what the regulations require and the ultimate use as to what you're using the economic scenario generator for.

I'd like to now talk about the use of stochastic price deflators. I first saw these being used in the United Kingdom and in Europe, but I haven't seen the use of stochastic price deflators in North America to a large extent as of yet.

When you're looking at discounting in a stochastic world, the discount factor will be different for each simulation. It's really dependent upon the future state that you're in. The mean discount rate using stochastic price deflators is in essence the risk-free rate, so it is a risk-neutral-type valuation. It gives you the fair market value of a liability. The biggest advantage of it is that if you were trying to model derivatives along the stochastic paths, then market pricing is consistent.

The deflator has two functions. One is to allow for the time value of money, and the other is to discount for systematic risk. The caveat is that the deflators do not always exist. You have to be working with an arbitrage-free model; otherwise the deflator will just not exist. The other thing, too, is that it's probably best when you're generating your stochastic scenarios to calculate the stochastic deflators at the same time, as opposed to going back and regenerating them.

MS. BURTON: Our last topic regarding stochastic modeling is on stochastic deferred acquisition cost (DAC) unlocking. I need to source and attribute my comments to Alastair Longley-Cook at Tillinghast.

Deterministic calculation of DAC amortization is proving or has proven to not adequately address the volatility in actual and future margins, which is in turn caused by the volatility in equity markets.

Stochastic modeling might be a preferred technique. You would use your scenarios to generate an array of unlocked DAC balances. Based on predetermined percentiles, you would build a corridor for yourself. If your current DAC falls within this corridor, you would make no change. If your current DAC falls outside of this corridor, you would have your catch-up that would bring you to the nearest boundary.

MS. LISA LEFKOWITZ: I would like to address this question not just to the panel, but to everyone who is involved with modeling. As long as I can remember in these symposia or other meetings, when dynamic policyholder behavior comes up someone says, "Well, we don't have enough data yet." In the last couple of years I think we've had some data that perhaps you could call New York V or the falling part of New York VI, and maybe in this year or next year we'll get to see New York II or New York III. But we still have to do our cash-flow testing or ALM work every year and assign some kind of dynamic behavior formula. What are people doing?

MS. BURTON: Do you have a specific assumption to comment on?

MS. LEFKOWITZ: Where I am we just came up with some rather standard formulas, say, for lapses, and we said we don't have any data so let's not change

them. It's better than changing it without data, but I don't think it's a very adequate response.

MS. BURTON: I agree, and I think it's unfortunate, but the best that we can do is put our heads together and develop what would seem like a reasonable assumption, maybe multiple assumptions, and test them to see how sensitive the changes in our results are to various types of assumptions.

Does anyone in the audience want to address this particular question, offer up comments as to what you're doing as far as dynamic lapse, annuitization, withdrawals or any other assumptions? Are we all just as perplexed? In my firm there are a few of us who, if we're on a client project, work in conjunction with the client to use what limited data they have to develop some kind of dynamic formula that is largely based on, if it's a variable annuity, the guaranteed amount, the relationship to the account value. We may test it as not just one assumption but rather three—make it harsher than the base and make it less of a movement as the relationship goes back and forth.

MR. HARASYM: One thing I'd like to say about policyholder behavior and why it's such a challenging issue is that from what I've seen, it can vary dramatically between countries. Even within a country it can vary significantly based on product design, geographical location and distribution channel. To some extent it's difficult to qualify, let alone quantify, because there's such a big variation in the experience that we see.

MS. BURTON: I think the answer is going to be to keep seeking the data and, as the months and year unfold, try to look at it in relation to what you are doing and have your dynamic assumptions constantly evolve.

MR. JIM THOMPSON: Somebody mentioned the borrowing assumption, saying that instead of testing negative cash flow, see how sensitive you are to that by testing borrowing. I've seen people use borrowing as the assumption, which means that they're borrowing 90-day cash. They put it in, and they put some spread. If you start relying on that, you probably have to document your proof unless you're just using the borrowing as a sensitivity to your negative cash flow. My other question is, you used the term CTE and that's new terminology to me. Could you explain it?

MR. HARASYM: Using CTE, you would rank the events based upon some sort of exposure measure. For example, at CTE 95 percent, we would average the worst 5 percent of the exposures. CTE is considered to be a much more robust measure than a percentile because percentiles tend to be poor at capturing catastrophic risks. For example, if you look at a GMIB that typically only had a cost at the 98^{th} – 99^{th} percentile, you could look at the 97^{th} percentile and say that the financial consequence is zero. The CTE measure, on the other hand, by capturing the tail

event and averaging it, tends to be more robust and contain a higher level of information content.

MS. GRANT: Regarding your first question or comment about the negative cash-flow strategy, I think you're right. It's a documentation issue as well as a communication issue. Where I've seen it is where you have someone who is used to running a liability model with several different lines of business, and to borrow from those other lines might make sense in that kind of model. But then if you do just a new business model on its own, then you don't have any other lines of business to borrow from and you need to make sure to test the alternative assumption of borrowing at a Treasury rate plus a spread.

MR. BOB MEILANDER: In the earnings-at-risk example, what was the time period on it?

MR. HARASYM: Being a Canadian-based company, the metric was one-year Canadian GAAP earnings. Of course, it could be also tailored to be a shorter or longer time frame.

MR. MEILANDER: We've done that same sort of thing. Coming up with an appropriate time period is very interesting. You get very different answers in the relative risk depending on how far out you go.

MR. HARASYM: We were trying to leverage off embedded value and dynamic capital adequacy testing that's already being done. Our embedded value models cover a very long time horizon. The business plan typically covers a three-year horizon while dynamic capital adequacy testing would look at anywhere from one to five years. So as not to replicate any existing work but to create another management tool, we decided on a one-year time period.

MR. MEILANDER: Is anyone doing stochastic-on-stochastic modeling? I see examples of how to do it and that is all well and good, but is anyone actually doing it?

MS. BURTON: Yes, I think so, and I've been asked about it a lot lately.

MR. HARASYM: We were performing stochastic modeling for segregated fund (or separate account) investment guarantees for Canadian reporting purposes as well as for our U.S. variable annuity products.

MR. DON SKOKAN: I have a question with regard to validating models. I spent quite a bit of time looking at working on the GAAP SOP, which focuses on the GMIB risk. I'm somewhat skeptical that you can aggregate contracts and then use those model cells to project out. When you validate your model cells—or model points, I guess your term was—are you doing that against the current inforce, and how well do those cells represent the current inforce? Or are you at any point taking your

entire block of business and projecting it out over one or more of the scenarios and then validating how well your model has performed in the projection period against the entire block of business projected on that same scenario over the projection period?

MS. BURTON: To the first comment, am I validating to the actual inforce style, kind of like a static validation? Yes. To the second comment, and I'm not sure if I'm going to address what you're asking, but it's along the lines of a dynamic validation. Are you saying you would run all records with maybe one scenario and then see how close the projection of our model points? Yes, that would be a definite validation technique I would use.

MR. SKOKAN: I guess what I'm trying to do is clarify how valid my skepticism is that that could be done. I take it you've been able to establish model points then that adequately represent the block of business in the projection period in the scenarios?

MS. BURTON: Yes. We're torn between the seriatim modeling and using your model points. It really depends on your purpose. You enter thousands or tens of thousands of scenarios, and I'm not sure I'm going to term this properly, but you have your model points, and you're running them out across thousands of scenarios. To get some result that is an average or a CTE, you pick a particular percentile. At that point that you're grabbing that number, I would say any kind of accuracy you lost by developing your model points is kind of moot. If you're concerned about the accuracy of your model points as far as reflecting, then move your percentile up. Pick a slightly more conservative result.

MS. GRANT: That's a good point though. A huge issue in modeling in general is how to simplify your model and reduce the number of cells, but not get too general so that it doesn't properly reflect the business.

Chart 1

ERM Earnings-at-Risk Example

Millions

CTE [95%]	Risk Category			Uncorrelated	Correlation	
Risk Originator	**Interest**	**Equity**	**Currency**	**Total**	**Effect**	**Total EaR**
Country A	$88	$24	$0	**$112**	($22)	**$90**
Country B	$23	$68	$14	**$105**	($18)	**$87**
Country C	$44	$42	$17	**$103**	($5)	**$98**
Country D	$56	$9	$55	**$120**	($35)	**$85**
Uncorrelated Total	**$211**	**$143**	**$86**	**$440**	**($80)**	**$360**
Correlation Effect	($140)	($51)	($5)	**($196)**	$37	**($159)**
Total EaR	**$71**	**$92**	**$81**	**$244**	**($43)**	**$200**

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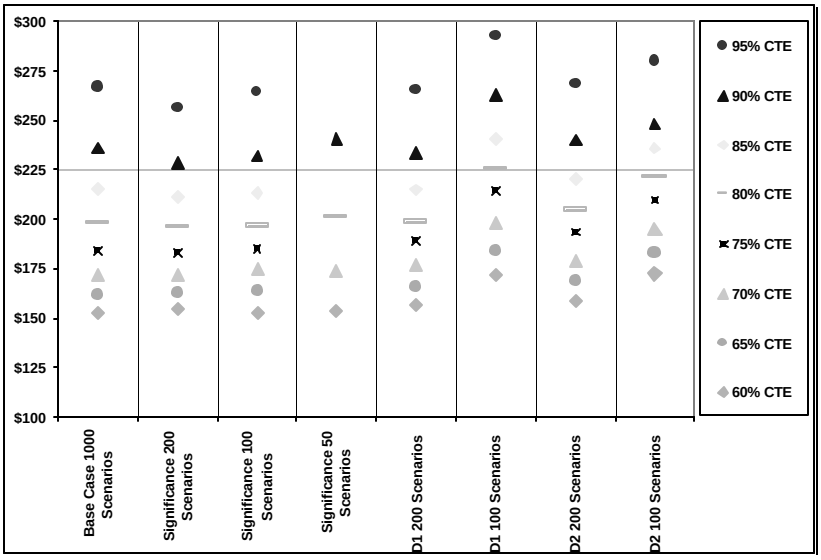
Chart 2

GMIB Scenario Reduction Example (Negative NPV of Cash Flows)



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