Equity-Based Insurance Guarantees Conference

Nov. 6-7, 2017

Baltimore, MD

Incorporating a Liquidity Premium into the Valuation of Insurance Liabilities

John Manistre

Sponsored by

MOODY'S ANALYTICS

Incorporating a Liquidity Premium into the Valuation of Insurance Liabilities

2017 EBIG Conference – Baltimore Md
Session 3B: 1530 – 1700 hours

B. John Manistre, Dariush Akhtari

November 6, 2017

Agenda

1. **Introduction – What's the problem?**
   - Some ideas from Solvency II
2. **Methods that depend on the assets you own – Dariush Akhtari**
3. **Methods that depend on assets you could own – John Manistre**
4. **Methods that depend only on Liability characteristics – John M**
5. **A debate on pros and cons**

- Should liability values depend on your asset values Yes/No?
- Open vs. Closed System Approaches to Risk Management

1

Introduction: What is the Problem?

Introduction – What is the Problem?

- Pricing actuaries have been incorporating a liquidity premium into their models for years
  - Good Example: Pricing SPIA contracts off long commercial mortgage rates, "liquidity match"
  - Bad example: In 1980's priced Group Pension GICs off shorter commercial mortgage rates, "liquidity mismatch"
- Traditional US accounting models, Stat & GAAP, swept the issue under the rug – until it was too late
- "Market Consistent" reporting models won't let you sweep the issue under the rug
- Solvency II tries to deal with the issue with two complex tools a) regulatory liquidity premium & b) matching adjustment
- IFRS just tells you to figure it out

2

Methods that depend on the assets you own

Dariush Akhtari FSA, FCIA

Blank Slide For Dariush

3

Methods that depend on assets you could own

John Manistre – FSA, FCIA, CERA

Moody's Analytics

An Optimization Problem – Static Control

1. **Start with a suitably large set of N economic scenarios $\mathcal{S}$, assume these are real world (P measure) for now**
2. **Project *risk adjusted* liability cash flows (LCF) over each scenario and time point t. Result is an array LCF_{tA}, $A \epsilon \mathcal{S}, t = 1, \dots, T$**
3. **Choose a set of linearly independent hedge instruments $\mathcal{H}$ such as bonds, swaps, options etc. Project *risk adjusted* cash flows for each hedge instrument. Result is an array $HCF^{\alpha}{}_{tA}$ for each $\alpha \epsilon \mathcal{H}$**
4. **Let Z^{α} be the observed market price of hedge instrument α**
5. **Choose an asset to act as numeraire – returns on this asset will be used for discounting. Examples - bank account, stock index, bond fund etc.. Let v_{tA} be the discount factor from time t to the valuation date on scenario A**
6. **Choose a CTE level a e.g. $a = 20\%$**

An Optimization Problem - Outputs

Discount liability and hedge asset cash flows using numeraire

$$L_A = \sum_t v_{tA}\, LCF_{tA},\ H^{\alpha}{}_A = \sum_t v_{tA}\, HCF^{\alpha}{}_{tA}$$

Consider a hedge portfolio where we buy b_α units of each hedge instrument and form

$$W(LCF, \boldsymbol{Z}, \boldsymbol{b}) = \sum_{\alpha} b_\alpha Z^{\alpha} + CTE_a\{L_A - \sum_{\alpha} b_\alpha\, H^{\alpha}{}_A\}$$

Intuition: first term is cost of buying hedge cash flows, second term is putting a value on unhedged liability cash flow

Now choose the hedge portfolio weights b_α to minimize the total liability value

» **A convex optimization problem (has nice properties)**

» **Let $b^*{}_\alpha$ be the optimizing portfolio weights**

Set optimal value $V(LCF, \boldsymbol{Z}) = W(LCF, \boldsymbol{Z}, \boldsymbol{b}^*)$

Static Control – Nice Properties

$$V(LCF, \boldsymbol{Z}) = \min_{\boldsymbol{b}} \left[\sum_{\alpha} b_\alpha Z^\alpha + CTE_\alpha \{ L_A - \sum_{\alpha} b_\alpha \, H^\alpha{}_A \} \right]$$

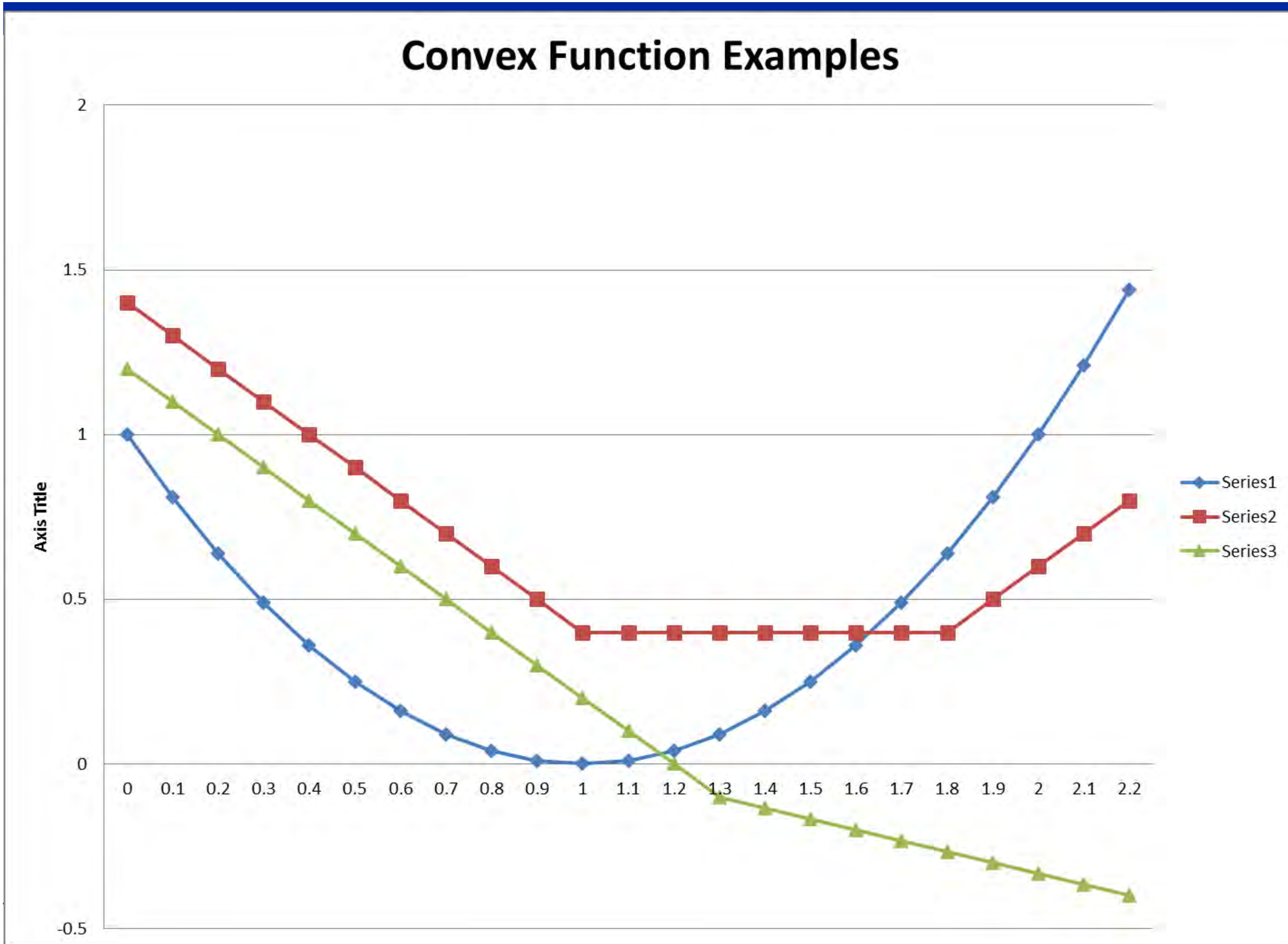
Looks complicated? This is actually a linear program in disguise! (Uraysev's Theorem)

Relatively easy to work with as a result. Optimization works (bounded) if the CTE level α is large enough

Optimal value $V(LCF, \boldsymbol{Z})$ satisfies $\frac{\partial V}{\partial Z^\alpha} = b_\alpha$ candidate for replicating portfolio

Valuation process is sub-additive

Value reflects any liquidity premium built into the hedge instruments and their projected cash flows


Convex Function Examples
2
1.5
1
0.5
0
-0.5
Axis Title
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2 2.1 2.2
Series1
Series2
Series3

Static Control – Dual Version

$$V(LCF,\boldsymbol{Z}) = \min_{\boldsymbol{b}}\left[\sum_{\alpha} b_\alpha Z^\alpha + CTE_a\{L_A - \sum_{\alpha} b_\alpha\, H^\alpha{}_A\}\right]$$

All linear programs have a dual version

In this case dual variables are scenario weights λ^A that maximize the liability present value

Dual Problem: $V(LCF,\boldsymbol{Z}) = \max_{\boldsymbol{\lambda}} \sum_A L_A\, \lambda^A$

Dual Constraints

$$\sum_A \lambda^A = 1, \qquad \sum_A H^\alpha{}_A \lambda^A = Z^\alpha,$$

$$0 \le \lambda^A \le \frac{1}{N(1-a)}.$$

Optimization process extracts a calibrated subset of scenarios

More on Dual Approach

- If the dual is feasible then the primal is bounded and the two optimal values agree
- $\frac{\partial V}{\partial Z^{\alpha}} = b_{\alpha}$ candidate for replicating portfolio
- $\frac{\partial V}{\partial b^{\alpha}} = Z^{\alpha} - \sum_{A} H^{\alpha}{}_{A} \lambda^{A}$ (Tasche's Theorem)
  - This result can be used to develop an interior method for solving the linear program
- $\frac{\partial V}{\partial L_{A}} = \lambda^{A}$ - useful if you forgot a contract

Static Control Summary

- Convex optimization: can be solved if *CTE* level is high enough
- Model produces a calibrated scenario subset $\mathcal{S}^* \subset \mathcal{S}$
- Liability value $V(LCF, Z)$ is average PV over $\mathcal{S}^*$
- Optimal hedge portfolio weights are also "greeks"

$$\frac{\partial V(LCF,Z)}{\partial Z^{\alpha}} = b^{*}{}_{\alpha}$$

- Calibrated scenario set reflects whatever liquidity premium we put into the universe of hedge instruments
- Mechanics are manageable
- Key issue is how we model *risk adjusted* asset cash flows
- Asset cash flow should reflect best estimate defaults and appropriate cost of capital

4

Methods that depend only on liability characteristics

John Manistre

A Bottom Up Approach

- Basic Idea: Asset prices reflect liquidity premiums for a variety of reasons
- Main issue: holding an illiquid asset means the value may change abruptly when you try to sell it
- Owner should then hold capital for a potential shock to liquidity spreads
- Liquidity spread itself should pay for cost of holding liquidity risk capital (circular)
- If assets back an illiquid liability then there is less risk that the asset will need to be sold at a loss
- Liquidity risk capital can be reduced and the spread used to subsidize liability pricing/valuation
- For more detail see John M's 2015 ERM Symposium paper on this topic, still a work in progress

5

John and Dariush Debate