Nonstandard Solutions To Problems In Life Contingency Examinations

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INTRODUCTION

Many instructors preparing examination candidates introduce methods which quickly eliminate certain incorrect answers and in many cases yield the correct answer without actually solving the problem. Unfortunately, these nonstandard solutions can be developed so extensively that a student well trained in these methods has a distinct advantage over a candidate who attempts only textbook solutions. This paper is an attempt to eliminate this advantage by making these techniques available to all students. It is ultimately hoped that the examination questions will be framed in such a manner that these nonstandard solutions fail. On the other hand, these nonstandard solutions do provide a valuable check for a practicing actuary.

An excellent introduction to some nonstandard solutions is given in [1], section 21; this is recommended reading for all candidates.

The gist of all nonstandard solutions is that a general relation between mathematical variables must hold in any specific case. Hence, the method of nonstandard solutions simply involves finding specific (usually simple) cases which identify the incorrect answers.

POPULATION PROBLEMS

The following stationary population is very useful in a variety of situations. Each instant, exactly one person is born; everyone lives until the exact age w and then dies. This population can be imagined as objects on a conveyer belt. All objects are placed on the beginning of the belt, one per notch, and ride until the end of the belt.

We now consider two applications of this "special" population.

S76M10[1] Let Group A represent those persons in a stationary population, now between ages 5 and 25, who will die between ages 40 and 80.

Let Group B represent those persons in the same population, now between ages 25 and 35, who will die between ages 40 and 80.

Find the average age of death of Group B minus the average age of death of Group A.

a) o b) 5 c) 10 d) 15 e) 20

Nonstandard Solution: Since the result must hold for any stationary population, it must hold for our special population. Let w = 80. Clearly, the average age at death for each Group is 80. Hence, the difference is 0. Answers b) through e) are in general impossible. The correct answer must be a).

F75M1 Which of the following is exactly equal to

$$\frac{\mathring{e}_{x+n}}{\mathring{e}_x - \mathring{e}_{x:\overline{n|}}} - \frac{e_{x+n}}{e_x - e_{x:\overline{n|}}}$$

a) o b) 1/2 c) 1 d) $_np_x$ e) $_nq_x$

Nonstandard Solution: Although this is not a stationary population problem per se, it is a general result and must hold for any law of mortality. If our special population is applied to this problem with $x+n<w$, we have $\mathring{e}_x = e_x$ and $\mathring{e}_{x+\overline{n|}} = e_{x+\overline{n|}}$. Thus, for our special population, the answer is zero for all x and n where $x+n<w$. Since $_np_x = 1$, answers b) through d) are eliminated. To finish the problem, note that all the functions in the statement of the problem are continuous in n. Therefore, $\lim\limits_{n\to x-w}\left[\frac{\mathring{e}_{x+n}}{\mathring{e}_x - \mathring{e}_{x:\overline{n|}}} - \frac{e_{x+n}}{e_x - e_{x+\overline{n|}}}\right] = 0$; but $_{w-x}q_z = 1$. Therefore, e) is impossible and a) must be the correct answer.

Problems Using Special Laws of Mortality

F75M25 The death benefit to (x), payable at the end of the year of his death, is an amount equal to the accumulated value of 1 payable at the end of each year

1 All examples are taken from old S.O.A. examinations. S76M10 means Spring 1976 Morning, problem 10.

that the policy was in force, including the year of death. Find the net single premium.

a) a_x b) υa_x c) $\upsilon\ddot{a}_x$ d) $1+\upsilon\ddot{a}_x$ e) $\ddot{a}_x$

<u>Nonstandard Solution</u>: Let everyone die during the first year; i.e., ${}_tp_x = o$ for $t = 1, 2, 3\ldots$ The death benefit is 1 payable at the end of the first year; the premium must be υ. Since $a_x = o$ and $\ddot{a}_x = 1$, the correct answer must be c).

S76M8 A whole life policy to (x) has a death benefit of \$1 plus return of all net premiums without interest. Annual premiums are payable for life. Find the annual premium.

a) $\frac{A_x}{(I\ddot{a})_x}(1+i)$ b) $\frac{A_x}{(I\ddot{a})_x}(1+\frac{1}{i})$ c) $\frac{A_x}{(I\ddot{a})_x}(1+\frac{1}{d})$ d) $\frac{A_x}{(I\ddot{a})_x}(\frac{1}{1+i})$ e) $\frac{A_x}{(I\ddot{a})_x}(\frac{1}{1+i})$

<u>Nonstandard Solution</u>: Let $p_x = o$; then $A_x = \upsilon$, $(I\ddot{a})_x = 1$ and the equation for the premium reduces to: $P.1 = \upsilon + P.\upsilon$ or $P = \frac{1}{i}$. The answers reduce to:

a) 1 b) $\frac{1}{i}$ c) $\upsilon + \frac{1}{i}$ d) υ^2 e) i

Clearly, b) is the choice.

S76A16 Express as a single symbol:

$$\frac{1}{d} \cdot A_{\overline{x:\overline{n|}}} + a_{\overline{x:\overline{n|}}} - \upsilon^n \cdot {}_nq_x$$

a) $\frac{1}{d}$ b) $\frac{1}{i}$ c) o d) i e) d

<u>Nonstandard Solution</u>: Let everyone live forever. The first and third terms vanish; the second term becomes a perpetuity which has value $\frac{1}{i}$. b) is the correct answer.

F75A15 Simplify: $\bar{a}_{\overline{n|}} - \bar{a}_{x:\overline{n|}} + \int_o^n v^s({}_{n-s}E_x + \delta\bar{a}_{x:\overline{n-s|}})ds$

a) $\bar{a}_{\overline{n|}}$ b) $\bar{a}_{x:\overline{n|}}$ c) $\bar{A}^1_{x:\overline{n|}}$ d) $\bar{A}_{x:\overline{n|}}$ e) $\bar{s}_{\overline{n|}}$

<u>Nonstandard Solution</u>: Let everyone die immediately. The expression in question becomes $\bar{a}_{\overline{n|}}$. Only answer a) has this property.

<u>Problems Using Special Rates of Interest</u>

S76M32 Express in simplest form: $\frac{1}{2}(P_{xx} - P_{\overline{xx}})\ddot{a}_{xx}\ddot{a}_{\overline{xx}}$

a) $\ddot{a}_x$ b) $\ddot{a}_{\overline{xx}}$ c) $a_{\overline{xx}}$ d) a_{xx} e) $\ddot{a}_{x|x}$

<u>Nonstandard Solution</u>: Let $i = \infty$; Then, $P_{xx} = P_{\overline{xx}} = 0$. Since a) and b) equal 1, they are eliminated. Let $i = 0$; then, $A_{xx} = A_{\overline{xx}} = 1$ and the problem becomes $\frac{1}{2}(\ddot{a}_{\overline{xx}} - \ddot{a}_{xx}) = \frac{1}{2}(a_{\overline{xx}} - a_{xx})$. Since it pays when just one is alive, d) is eliminated. Because $\frac{1}{2}(a_{\overline{xx}} - a_{xx}) < a_{\overline{xx}}$, c) is eliminated. The correct answer must be e).

S75M37 If $s(x) = e^{-x}$, find $\bar{A}_{\underset{1}{x}y\overset{2}{z}}$

a) $\frac{3-\delta}{2+\delta}$ b) $\frac{2-\delta}{3+\delta}$ c) $\frac{\delta}{(2+\delta)(3+\delta)}$ d) $\frac{1}{(3+\delta)(2+\delta)}$ e) None

<u>Nonstandard Solution:</u> Let $i = \delta = \infty$. Since $\bar{A}_{\underset{1}{x}y\overset{2}{z}} = 0$, answers a) and b) are eliminated. Let $i = \delta = 0$; $\bar{A}_{\underset{1}{x}y\overset{2}{z}} = {}_{\infty}q_{\underset{1}{x}y\overset{2}{z}} > 0$. This disposes of answer c). An answer of the type e), poses the most difficult challenge to nonstandard methods and, in fact, such methods can never eliminate e). On the other hand, answer d) already agrees in two separate boundary conditions ($i=0$ and ∞) and if $x=y=z$, ${}_{\infty}q_{\underset{1}{x}y\overset{2}{x}} = 1/6$ which further agrees with d) in the case $i = 0$. Answer d) seems an excellent bet.

<u>Problems Involving Special Laws of Mortality and Special Interest Rates</u>

S76M1 Express in simplest form.

$$\frac{(I_{\overline{n}|}\, a)_x}{(I_{\overline{n}|}\, a)_{x+1} + (I_{\overline{n}|}\, A)_{x+1}}$$

a) d b) ${}_n p_x$ c) ${}_n q_x$ d) p_x e) q_x

<u>Nonstandard Solution</u>: If $i = 0$, $(I_{\overline{n}|}\, a)_x \neq 0$ but $d = 0$. Eliminate answer a). Let everyone live forever; $(I_{\overline{n}|}\, A)_{x+1} = 0$ and the expression equals one. This eliminates answers c) and e). Let $p_x = 1$ and $p_{x+1} = 0$. The expression becomes 1, while answer b) equals 0. Hence, d) is the correct choice.

S76M21 If $s(x) = \upsilon^x$, simplify $A_{x_1, x_2, \ldots, x_m}$

a) $\frac{1}{m\delta}$ b) $\frac{m}{m+1}$ c) $\frac{m-1}{m}$ d) $\frac{1-\upsilon^m}{1-\upsilon^{m+1}}$ e) $\frac{\upsilon-\upsilon^{m+1}}{1-\upsilon^{m+1}}$

N. Sltn. If $s(x) = \upsilon = 1$; this implies $i = 0$ and everyone lives forever. Since, $A_{x_1x_x\cdots x_m} = 0$, answers a), b) and c) are incorrect. Let $\upsilon = 0$; this means that $i = \infty$ and everyone dies immediately. Hence, $A_{x_1x_2\cdots x_m} = 0$. This eliminates d) and leaves only e).

Solution by Specific Cases:

S76M28 Simplify: $a_x + a_y + a_z - (a_{\overline{xy}:z} + a_{\overline{xz}:y} + a_{\overline{yz}:x})$

a) $a_{\overline{xyz}}$ b) $a_{\overline{xyz}}^{\;2}$ c) $a_{\overline{xyz}}^{[1]}$ d) $a_{\overline{xyz}} - a_{\overline{xyz}}^{[2]}$ e) $a_{\overline{xyz}}^{[2]}$

Nonstandard Solution: The following table is quickly constructed:

Survivors	value	not possible
x	1	b), e)
xy	0	a)
xyz	0	d)

The correct answer must be c).

The Method of Undetermined Coefficients

See reference [1] section 16.

References

[1] Bambrough Brian, Problem Solving in Life Contingencies