Stochastic Pricing for Embedded Options in Life Insurance and Annuity Products

Milliman, Inc

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I. Introduction

The genesis of this project was a request from the Society of Actuaries for research regarding stochastic pricing of embedded options.

In addressing this topic, our approach focuses on two popular insurance products that have different sensitivities to market conditions. The two products are a variable annuity (VA) with a guaranteed lifetime withdrawal benefit (GLWB) and a universal life product with a secondary guarantee (ULSG). Detailed summaries of the specific products are provided in Sections II and III and associated appendices of this report.

Our research focuses on the challenges associated with market-consistent valuation as called for by FAS 133. The examination includes the following:

- 1. Closed-Form Solutions Can answers be reached using formulas rather than stochastic analysis?
- 2. Scenario Generation
 - a. What models are appropriate for replicating market prices of derivatives?
 - b. What sources are there for calibrating models?
 - c. How many scenarios is enough?
- 3. Liability Assumptions What dynamic lapse, dynamic utilization and other assumptions are appropriate?
- 4. How can results be validated and understood?

While our research will touch on topics such as C3 Phase II and III, VACARVM, and principle-based reserves, our primary focus will be on a "fair value" assessment of embedded options. The ancillary topics will be addressed in a qualitative rather than quantitative manner.

Our research goes beyond simply calculating values toward understanding results both on a point-in-time basis and period-to-period change. It is our opinion that this is where much of the research is needed. There are numerous groups that calculate the value of embedded options, but few or none are focusing on understanding and communicating the results. A key area of our research report explains period-to-period changes in the value. This is a critical area for the pricing of embedded options because the value itself is often not as important as the volatility of the value and understanding or even hedging the volatility.

The final phase of our research uses the results of the prior phases to discuss financial projections and product pricing. This phase will also discuss the challenges of stochastic-on-stochastic models and suggests ways to address these challenges.

As the products offered by insurance companies become more and more complex, often incorporating sophisticated guarantee mechanisms, a method of accounting for the value of these embedded options becomes more important.

II. Universal Life with Secondary Guarantees: Specifications and Assumptions

The universal life with secondary guarantee (ULSG) product specifications and assumptions were chosen to be representative of the features that are available in the current marketplace. The product was priced to arrive at an after-tax, after-capital internal rate of return of 9.0 percent. It is our opinion that this is representative of the pricing target for ULSG products. Detailed product pricing results are provided.

The secondary guarantee for this product is provided by utilizing a shadow account. The shadow account parameters vary cell-by-cell to achieve a competitive premium while maintaining a reasonable profit.

The base UL chassis is typical of what is seen in this market. It will produce cash values, but they are certainly not the focus of the product. Appendix A contains detailed product specifications and assumptions.

Table II.2 is a summary of the source of profits for the block of cells under the specifications and assumptions detailed in the appendix. The margins are defined in Table II.1.:

	TABLE II.1 – SUMMARY OF MARGIN COMPONENTS						
<u>Margin</u>	Margin Components						
Interest Margin	Interest earned on account value and product cash flow less interest credited to the account value						
Mortality Margin	Account value released on death plus COI charges less death benefits						
Surrender Margin	Account value released on surrender less surrenders paid						
Expense Margin	Policy loads less maintenance expenses less acquisition expenses less commissions						
AV/Reserve Margin	Change in account value less change in reserve						

Table II.2: ULSG - Source of Profits

Face Amount:

\$250,000

t	INTEREST MARGIN	MORTALITY MARGIN	SURRENDER MARGIN	EXPENSE MARGIN	AV/RESERVE MARGIN	Pre-Tax Profit	DAC Tax	Other FIT	After-Tax Profit	After-Tax Int on Req Capital	Increase in Required Capital	Stock Holder Divs
1	(299)	569	-	(6,236)	1,241	(4,726)	115	(1,654)	(3,187)	-	641	(3,829)
2 3	109 191	799 878	271 436	278 254	(475)	982 1,025	93 74	397 423	492 529	26 28	41 41	476 516
4	272	931	436	234	(733) (1,638)	278	56	425 212	529 9	28 29	67	(29)
5	398	977	420	218	(2,430)	(418)	41	(79)	(380)	32	92	(440)
6	567	1,025	326	205	(2,462)	(339)	28	(59)	(307)	36	92	(363)
7	736	1,078	227	195	(2,500)	(264)	16	(39)	(242)	40	95 99	(297) (257)
8 9	906 1,081	1,129 1,174	155 93	187 182	(2,598) (2,784)	(221) (255)	6 (4)	(25) (36)	(202) (214)	43 47	106	(237)
10	1,268	1,238	88	176	(2,883)	(113)	(13)	1	(101)	52	101	(150)
11	1,458	1,277	83	223	(3,364)	(323)	(16)	(82)	(225)	56	109	(277)
12 13	1,668 1,869	1,305 1,304	70 59	216 209	(3,318)	(59) 187	(14) (12)	(4) 72	(41) 127	60 64	98 88	(79) 103
13	2,060	1,304	48	209	(3,253) (3,180)	407	(12)	142	275	68	77	266
15	2,240	1,222	38	193	(3,023)	671	(11)	220	462	71	64	469
16	2,405	1,104	30	186	(2,801)	924	(11)	300	634	74	52	655
17	2,553	947	22	178	(2,580)	1,121	(11)	367	764 933	76 77	42 30	798
18 19	2,682 2,780	614 66	15 9	166 140	(2,093) (1,435)	1,384 1,559	(11) (11)	461 524	1,046	79		981 1,106
20	2,846	(261)	4	114	(985)	1,718	(11)	579	1,150	79	8	1,222
21	2,885	(692)	-	(11)	(145)	2,037	(12)	692	1,357	80	(9)	1,446
22	2,883	(964)	-	(47)	37	1,908	(12)	646 672	1,274	79 79	(12)	1,365
23 24	2,876 2,856	(1,016) (1,124)	-	(46) (45)	164 350	1,978 2,037	(12) (12)	672 692	1,317 1,357	79	(21) (31)	1,418 1,466
25	2,830	(1,224)	-	(43)	573	2,070	(12)	707	1,375	70	(40)	1,400
26	2,777	(1,283)	-	(41)	693	2,145	(12)	714	1,443	75	(53)	1,571
27	2,716	(1,353)	-	(39)	799	2,124	(12)	708	1,427	73	(58)	1,558
28 29	2,650 2,569	(1,434) (1,812)	-	(37) (51)	930 1,239	2,110 1,946	(12) (12)	706 652	1,416 1,305	71 68	(63) (64)	1,550 1,437
30	2,309	(1,795)	-	(47)	1,277	1,940	(12)	643	1,284	66	(70)	1,437
31	2,384	(2,053)	-	(46)	1,590	1,876	(11)	631	1,256	63	(75)	1,394
32	2,272	(2,169)	-	(53)	1,631	1,680	(11)	564	1,127	60	(73)	1,260
33 34	2,162 2,049	(2,144) (2,139)	-	(50) (46)	1,667 1,701	1,635 1,565	(11) (11)	549 527	1,097 1,048	57 54	(77) (77)	1,231 1,179
35	1,937	(1,963)	-	(40)	1,576	1,509	(11)	506	1,048	50	(77)	1,179
36	1,827	(2,033)	-	(39)	1,679	1,434	(10)	484	959	47	(78)	1,085
37	1,715	(1,968)	-	(36)	1,649	1,360	(9)	458	912	44	(77)	1,033
38 39	1,602	(1,968)	-	(38)	1,605	1,200	(9)	404 381	805	41	(73)	919
39 40	1,495 1,390	(1,871) (1,868)	-	(35) (34)	1,544 1,545	1,134 1,034	(9) (8)	348	761 693	38 35	(72) (70)	871 799
41	1,286	(1,838)	-	(31)	1,556	973	(8)	327	653	32	(69)	755
42	1,181	(1,832)	-	(30)	1,552	872	(7)	292	586	29	(66)	681
43	1,081	(1,787)	-	(27)	1,547	813	(7)	272	548	27	(63)	638
44 45	981 888	(1,765) (1,692)	-	(28) (28)	1,491 1,406	678 574	(6) (6)	225 189	459 390	24 22	(57) (52)	540 464
46	802	(1,624)	-	(25)	1,365	518	(5)	172	351	20	(49)	419
47	722	(1,484)	-	(23)	1,243	458	(5)	153	310	18	(45)	373
48	647	(1,386)	-	(20)	1,167	408	(4)	136	276	16	(42)	334
49 50	578 514	(1,232) (1,192)	-	(18) (16)	1,040 1,033	368 339	(4) (3)	121 113	250 229	14 13	(39) (37)	303 279
51	452	(1,093)	-	(10)	955	300	(3)	101	202	11	(34)	248
52	396	(992)	-	(13)	875	265	(3)	88	180	10	(31)	221
53	344	(906)	-	(11)	805	232	(2)	77	158	9	(29)	195
54 55	297 256	(788) (699)	-	(10) (8)	698 621	198 170	(2) (2)	65 56	135 115	7 6	(25) (22)	167 144
56	220	(621)	-	(7)	565	157	(2)	52	106	6	(20)	131
57	187	(529)	-	(6)	476	127	(1)	41	87	5	(17)	109
58	159	(491)	-	(5)	444	107	(1)	36	73	4	(15)	92 77
59 60	133 111	(425) (365)	-	(5) (4)	385 330	89 73	(1) (1)	30 24	60 50	3	(14) (12)	64
61	92	(317)	-	(3)	288	60	(1)	19	41	2	(12)	54
62	75	(279)	-	(3)	256	50	(1)	17	34	2	(9)	45
63	61	(223)	-	(2)	205	41	(1)	13	28	2	(8)	37
64 65	49 39	(202) (154)	-	(2) (1)	184 147	29 30	(1) (0)	10 10	20 21	1	(6) (5)	28 27
66	30	(137)	-	(1)	143	35	(0)	12	24	1	(5)	29
67	23	(96)	-	(1)	93	18	(0)	6	13	1	(3)	17
68 69	17 13	(84)	-	(1)	81	13	(0)	4	9 7	0	(3)	12 9
69 70	13	(64) (49)	-	(1) (0)	62 48	10 7	(0) (0)	3 2	5	0	(2) (2)	9 7
71	7	(40)	-	(0)	38	5	(0)	2	3	0	(1)	5
72	5	(29)	-	(0)	27	3	(0)	1	2	0	(1)	3
73 74	3	(20)	-	(0)	20	3	(0)	1	2	0	(1)	3
74	2 1	(15) (10)	-	(0) (0)	15 10	2 2	(0) (0)	1	2 1	0	(1) (0)	2 1
76	1	(10)	-	(0)	18	12	(0)	4	8	0	(1)	8
Pres	ent Value of I	Margins, disco	unted at 8.0%									
	15,174	5,645	1,860	(4,005)	(16,093)	2,582	273	1,207	1,101	569	1,121	550

III. Variable Annuity Product: Specifications and Assumptions

The variable annuity (VA) product specifications and assumptions used in this report were chosen because they are representative of the features available in the current marketplace. The mortality and expense (M&E) charge was solved for to arrive at an after-tax, after-capital return on assets (ROA) of around 20 basis points. The ROA was calculated by discounting the statutory after-tax, after-capital earnings at 8 percent and dividing by the present value of the average annual account value. A breakdown of the sources of profit is provided.

The guaranteed minimum death benefit (GMDB) chosen for this model is a simple return of premium design. This was chosen so that the focus of the analysis remains on the guaranteed living benefit. The cost of the GMDB was assumed to be in the 10 basis-point range and has been added to the M&E charge at cost.

The guaranteed lifetime withdrawal benefit (GLWB) specifications were chosen because they are representative of the types of benefits seen in the marketplace. The charge for the GLWB, 65 basis points of account value, was set to be typical in the marketplace, with coverage of the hedge cost and a modest additional margin for capital and reserves plus volatility in the hedge cost.

Appendix B contains the detailed product specifications and assumptions.

The following Table III.1 is a sources of profit report that outlines the effects that each element of the profits for this product has over the total profitability of the product. The revenue area is composed of investment income on reserves and cash, M&E charges, the withdrawal margin (surrender charges), and revenue sharing. The expense area is composed of death benefits (excess of the amount of death benefits above the account value released at time of death), acquisition expenses, maintenance expenses and

commission. The timing difference of the release in reserves is accounted for by the difference in the change of account value by the change in reserves. The pre-tax profit is obtained by subtracting from the total revenue the total expenses and then adding the reserve allowance. Taxes are subtracted to obtain the after-tax profit and required capital is taken into account to obtain the adjusted after-tax statutory profits. Each of the elements is discounted back to time zero and divided by the average account value in force to obtain a sources of profit report that measures the return on account value (ROA).

	Embedded Options-					
	SOURCES OF	F PR	OFITS			
Projection Module Summary						Page 1
Description: Sources of Profit (bps)				SOA Res	earch Project	
Projection Mode: Annual					9/17/2007	10:25:14 AM
Projection Date: 12/2006		Μ	G-ALF	FA 6.4.2	238 / 931RDO03-59	_VA_09-17-
						07
Projection Cycles: 30 (Annual)		 	~ .			1
ROA in bps of AV				al Strai		
Investment Income	-14.28		PV Yr	1 Prof	it/ PV Premium	-1.82%
M&E Charges	99.36					
Withdrawal Margin	9.05		PV of			847.88
Revenue Sharing	36.13			emium		50,000
Total Revenue	130.26	$\left \right $	PV of	Profit/	PV Premium	1.70%
DB Benefits	0.00		GIRR			18.30%
Acq Exp	20.38					
Main Exp	12.44		PV of	Profit/	PV of Avg AV	0.21%
Net Commission	80.96					
Total Expense	-113.78					
Change In AV	695.40					
Change In Reserves	-671.29					
Total Chg	24.12					
Total Profit	40.60	$\left \right $				
Taxes	-15.29	$\left \right $				
Total After-Tax Profit	25.31	$\left \right $				
	23.31	$\left \right $				
Interest on TS	2.73	\square				
Change in TS	-6.71					
Capital	-3.99					
Total Adjusted Profit	21.33	$\left \right $				

Γ

IV. Valuing Embedded Options

A life insurance contract or an annuity can contain many embedded options. Some of those embedded options, such as minimum interest rates, are fairly explicit while others, such as guaranteed face amount increase options and the mortality aspects of guaranteed annuitization options, are less obvious. Historically, many of these options were included in the contract without explicitly being priced. Many of the options were thought to be conservatively designed and would rarely, if ever, come into play. However, the recent low interest rate period has certainly proven that theory wrong.

As competitive pressures in the insurance industry have continued to increase, including stronger guarantees (more favorable to the policyholder) in product offerings have been one way for companies to distinguish their products from those of their competitors. The two products that are the focus of this research project have certainly experienced this company practice.

The variable annuity market is largely dominated by various types of guarantees. Guaranteed minimum death benefits appeared on variable annuities approximately 15 years ago. Living benefits have been offered on variable annuities for just over 10 years. Today, guaranteed minimum income benefits (GMIB), guaranteed minimum withdrawal benefits (GMWB), guaranteed lifetime withdrawal benefits (GLWB), and guaranteed minimum accumulation benefits (GMAB) are significant drivers of variable annuity sales. In fact, a guaranteed living benefit (GLB) is offered as a rider on at least 90 percent of variable annuity sales and the GLB is elected in over 70 percent of sales¹.

The simplest form of the GLB is the GMAB. The benefit closely resembles a put option. The typical structure might be a return of premium guarantee after a 10-year waiting period. If the account value is less than the GMAB value (the premium in this example),

¹ 2007 Milliman Guaranteed Living Benefit Survey

the account value will be "topped-off" to equal the GMAB. Therefore, the benefit is simply equal to the following:

Max (0, GMAB - AV) of the 10th anniversary

This benefit is identical to a 10-year put option except for the following:

- 1. The GMAB is dependent on the survival of the annuitant and the persistency of the base contract, while a put option is a free-standing contract.
- The account value of the VA has fees that reduce the performance. Typical fees might include 120 bps M&E, 100 bps fund management fees, and 50 bps for the GMAB versus an S&P 500 index.
- 3. The charge for the GMAB is typically assessed against the account value or benefit base versus an upfront premium for the put option.

Because of the relative simplicity of the GMAB, the value of the embedded option provided by the GMAB can be approximated by a Black-Scholes formula. The inputs into the formula and the formula itself are as follows.

Risk-free rate	= 5.0%					
Total fees	= 270 bps					
Assumed volatility	= 17.0%					
Duration	= 10 years					
Strike price	= \$1,000					
Premium	= \$1,000					
10^{th} Anniversary Persistency = 50%						

Let

 S_t = Current price of the fund m = Margin deducted from the policyholder fund $Sm_t = S_t^*(1-m)^{T-t}$ = Starting Account Value adjusted for regular charges deducted from it K = Exercise price or Strike price r = Risk-free rate T = maturity t = current duration

BSP = Black-Scholes European Put Option = $Ke^{-r(T-t)}\Phi(-d_2) - Sm_t\Phi(-d_1)$ where:

$$d_1 = \frac{\log\left(\frac{Sm_t}{K}\right) + (T-t)(r + \frac{\sigma^2}{2})}{\sqrt{T-t} \sigma}$$

 $d_2 = d_1 - \sigma \sqrt{T - t}$

GMAB = BSP * Persistency

Table IV.1 below contains an example of GMAB embedded option prices using the Black-Scholes formula and the above inputs.

Table IV.1 - Example of GMAB EmbeddedOption Prices as a Percent of GMAB Strike Price								
Time	At-the- money	Out-of-the- money 20%	In-the- money 20%					
0	4.01%	2.52%	6.39%					
5	4.16%	2.12%	7.82%					

So, a closed-form equation like the Black-Scholes formula can be used to value the embedded option for a GMAB. However, a more complicated benefit like a GLWB requires a stochastic method. A GLWB has a stream of benefits, not just a single payment. A GLWB also provides flexible benefits, where the customer may decide when partial withdrawals start and stop.

A GLWB guarantees a stream of partial withdrawals for the annuitant's life, regardless of the account value. For instance, given a \$100,000 premium, a 5 percent for life GLWB would guarantee partial withdrawals of \$5,000 per year for the life of the annuitant. In a poor investment return scenario, the account value may eventually go to zero. When this happens, the insurance company will typically issue a single premium immediate annuity

for the remaining payments for the life of the annuitant. The present value of the SPIA at the moment the account value goes to zero is the embedded option that will be captured for the GLWB.

A detailed example of this calculation is provided in Section VI.

Largely in response to the low interest rate environment and the reserve pressure associated with long duration term insurance products, the life insurance industry began to offer more and more generous no-lapse guarantees on universal life contracts. These no-lapse guarantees are often called secondary guarantees. The product category in general is often referred to as universal life with secondary guarantees (ULSG). They guarantee that as long as a specific criterion is met, the contract will not lapse, even if the cash value goes to zero. Without the no-lapse guarantee, the traditional universal life contract would lapse once the cash value reaches zero. This could happen during a period of low interest rates, where the credited interest and premiums paid are not sufficient to cover the contract charges (e.g., cost-of-insurance, other loads, etc.).

No-lapse guarantees are generally structured in one of two ways, although the distinction is blurred. Originally, these guarantees were based on the cumulative premiums paid, exceeding a specified annual amount times the number of years that had passed. This type of criteria is a "specified premium" design. As this design evolved, interest credits (e.g., 5 percent) were applied to excess premiums that were paid and the specified annual amount may no longer be a constant amount (typically increasing if not level).

The other no-lapse guarantee structure involves the use of a second account value-type calculation, often called a shadow account. The shadow account typically functions much like the account value, except that all of the parameters are guaranteed, including the interest rate. The fact that all values are guaranteed allows the customer to see exactly how long the contract will remain in force. It also allows the customer and agent to solve

for the premiums necessary to keep the contract in force until maturity (lifetime guarantee).

For the purposes of this research report, we chose a shadow account design, because this is generally the method used for more competitive products. Actually, the structure, shadow account versus specified premium, to determine the guarantee really does not matter for our purposes. The length of the guarantee provided by the specific premium paid is the only item needed for our research. We chose to use a lifetime guarantee because this is the most frequent guarantee period used in the industry for these products.

The length of the guarantee is the only item that matters in this exercise, because the research project measures the amount of death benefits paid, which would not have been paid without the no-lapse guarantee. This is defined as death benefits paid minus premiums paid once the cash value has gone to zero. If no additional premiums need to be paid to keep the no-lapse guarantee in effect, this collapses to just the death benefits paid once the cash value has gone to zero.

A detailed example of this calculation is provided in Section VII.

V. Generation of Scenarios

When the embedded option is determined on a stochastic basis, a key determinant of the value of the embedded option is the stochastic scenarios used. There are many choices that can be used to calibrate and generate scenarios. Table V.1 below is a sample of the categories that scenarios can fall under:

,	TABLE V.1: SAMPLE CATEGORIES OF SCENARIOS							
Category	Interest Rates	Equity Returns						
Historic (realistic, probabilistic)	Interest rate scenarios start at the current interest rate curve and reflect historic volatility. There may be a reversion to a mean component. There may be restrictions or requirements for a number of inverted yield-curve occurrences. C3 Phase I or II scenarios would fit this category.	Equity returns are based on historic returns and volatility. Returns might be based on excess over a risk-free rate or might be based purely on average historic returns. Consideration should be made for "fatter" tails than are generally seen with lognormal distributions. Historic correlations should be reflected. C3 Phase II scenarios would fit this category.						
Long-Term Risk Neutral	Similar to market consistent but using a smoothed current interest rate curve based on a period of a few years and potentially a "normal" shape.	Similar to market consistent, but with a longer term view of the mean rate and volatility. May use a historic value for both, possibly with some additional conservatism.						
Market Consistent	Generally arbitrage free interest rate scenarios where the forward rates are driven by the current yield curve. Interest rate volatility assumptions are being gathered from market instruments such as Swaptions and other interest rate derivatives. The scenarios should be able to replicate derivatives found in the marketplace.	Categorized as risk neutral. Mean returns are based on the current swap curve. Volatility assumptions are based on a term structure of volatility, which needs to be gathered from instruments in the marketplace such as long-dated calls and puts. Beyond the period where instruments are available (generally after five to 10 years), volatilities may remain at the last observed value or may grade back to a historic level. Scenarios should be able to replicate derivative prices found in the market.						

Each of the scenarios will result in materially different results for the embedded value calculation. The distinction between the long-term risk neutral and the market consistent scenarios is the frequency of adjusting the parameters. The long-term risk neutral scenario set described above relies on a longer term view of market parameters. Conceptually, this type of a scenario set can be appropriate for a pricing exercise where it may not be possible to adjust the price on a frequent basis, thereby facilitating a price that will be appropriate in a variety of market conditions. Market consistent scenarios are appropriate for calculating current hedge costs, for calculating derivative prices, for market value calculations like FAS 133 and for any situation where the value needs to be consistent with other instruments available in the market.

For historic scenarios, we used the scenarios generated for C3 Phase II. Scenarios can be downloaded from the American Academy of Actuaries (AAA) Web site. Table V.2 below shows the indices used in our model, and the C3 Phase II dataset used for each of the indices.

TABLE V.2: INDICES USED IN MODEL					
Model Index	C3 Phase II Dataset				
S&P 500	US				
Russell 2000	SMALL				
NASDAQ	AGGR				
SB BIG	LTCORP				
EAFE	INTL				
Money Market	MONEY				

For long-term risk neutral scenarios, a lognormal scenario generator was used. The assumptions used to populate the generator were chosen to include a modest degree of

conservatism over historically accurate parameters. The mean return was chosen as a conservative estimate of the forward rates that would be generated by a typical swap curve. The volatility rates are 10-year average realized market volatilities increased by 100 to 200 basis points for conservatism. Table V.3 below shows the assumptions:

TABLE V.3 - LONG-TERM RISK NEUTRAL SCENARIO PARAMETERS							
Model Index	Mean Return	Volatility					
S&P 500	5.0%	19.0%					
Russell 2000	5.0%	21.5%					
NASDAQ	5.0%	28.0%					
SB BIG	5.0%	5.0%					
EAFE	5.0%	19.5%					
Money Market	5.0%	1.0%					
1-yr Treasury	4.9%	6.1%					
7-yr Treasury	5.0%	4.3%					
20-yr Treasury	5.1%	3.4%					

Also, for the long-term risk neutral scenarios, the following correlation matrix, Table V.4, applies. The correlation matrix is based on a 10-year history.

Tab	Table V.4 - Long-Term Risk Neutral Scenarios – Correlation Matrix								
	S&P500	Russell 2000	NASDAQ	SBBIG	EAFE	Money Market			
S&P 500	1.000	.806	.833	095	.687	.005			
Russell 2000	.806	1.000	.851	135	.631	025			
NASDAQ	.833	.851	1.000	130	.588	019			
SBBIG	095	135	130	1.000	136	.035			
EAFE	.687	.631	.588	136	1.000	034			
Money Market	.005	025	019	.035	034	1.000			

For the market consistent scenarios, separate sets were used for the ULSG product and the VA product. The ULSG product used a set of arbitrage free interest rates based on the 9/30/2007 yield curve. These scenarios were generated using a Hull-White scenario generator. Appendix C contains a summary of the Hull-White scenario generator.

For the VA product, a lognormal model was used with the parameters summarized in Charts V.1 and V.2 and Table V.5. The values in Charts V.1 and V.2 can be found in Appendix D.

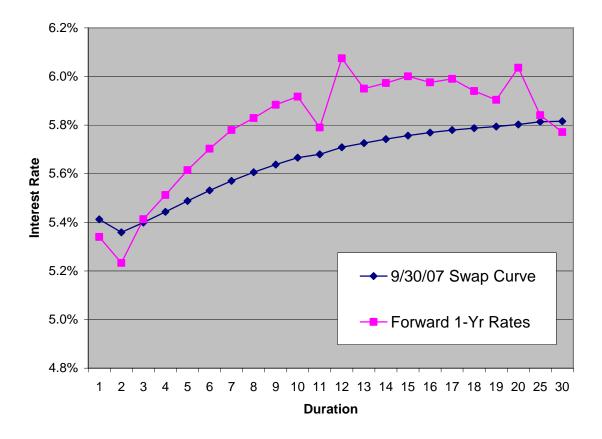
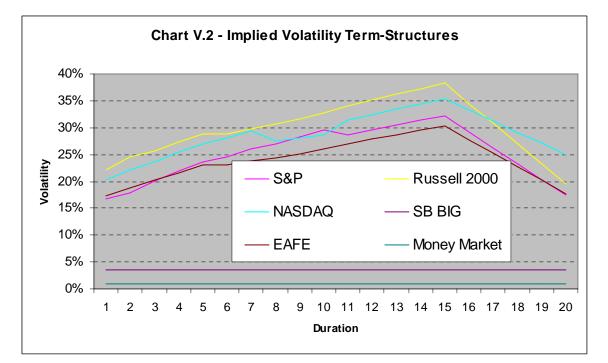


Chart V.1 - Swap Curve and Resulting Forward Rates



The volatility surface was determined using implied volatility data for the first 15 years and then grading to the historic average over the next five years. Implied volatility data was derived from market prices on available put options as of 9/30/07. Implied volatility term structures were then used to create the forward one-year volatilities.

	Table V.5 – Correlation Matrix									
	S&P500	Russell 2000	NASDAQ	SBBIG	EAFE	Money Market				
S&P 500	1.000	.806	.833	095	.687	.005				
Russell 2000	.806	1.000	.851	135	.631	025				
NASDAQ	.833	.851	1.000	130	.588	019				
SBBIG	095	135	130	1.000	136	.035				
EAFE	.687	.631	.588	136	1.000	034				
Money Market	.005	025	019	.035	034	1.000				

A key for confirming that the market consistent scenarios are in fact market consistent is to test whether the price of options available in the marketplace can be reproduced. This was done for the ULSG scenarios by capturing a 9/30/07 swaption quote. The implied volatility embedded in the observable price of an at-the-money swaption with a seven-year tenor and one-year option term on 9/30/07 was 18.6 percent. Our scenarios resulted in a comparable value of 18.4 percent. The implied volatility embedded in the observable price of an at-the-money swaption term on 9/30/07 was 17.2 percent. Our scenarios resulted in a comparable value of 17.8 percent.

For the VA scenarios, a long dated at-the-money put option is used to confirm the validity of the scenarios. On 9/30/2007, the market observable price for a 10-year, at-the-money, S&P 500 put was 7.26 percent. The scenarios were run through an Excel model

to calculate the cost of the 10-year put over the 1,000 scenarios. A sample for a single scenario is shown below in Table V.6.

Table V.6 – Sample Calculation for Single Scenario			
S&P Index at time 0 (Index ₀)	1,000.00		
S&P Index at time 10 (Index ₁₀)	986.25		
Strike Price (at-the-money)	1,000.00		
Value of Put Option (Max[Strike - Index ₁₀ ,0])	13.75		
Present Value Factor (Continuous Risk Free Rate)	0.56992		
Present Value Of Put Option	7.84		

The average cost of the put option over the 1,000 scenarios was 7.10 percent.

VI. Valuing Annuity Product Embedded Options

Recall that the embedded option being valued for the GLWB is a stream of partial withdrawals for the annuitant's life, regardless of the account value. If/when the account value goes to zero, a single premium immediate annuity is issued to cover the remaining payments for the life of the annuitant. The present value of the SPIA at the moment the account value goes to zero is the embedded option that will be captured for the GLWB.

Table VI.1 below is an example of this calculation for a specific scenario:

	Table VI.1 – VA Embedded Options DetailVA Embedded OptionsProj.002GLWB Hedge Cost								
Liabili	ity Audit.	Scenario 250		[002* 065*	•	U			
	Mode: An		—	•				SOA Resear	rch Project
LOB a	& Charac	teristics: Var	Ann XM N	AN 1S 2Y 3A	41			11/19/2007 9:	5
Plan:	GLWB-V	A			MG-ALFA 6.4	4.279 / TEMP.	P05 / 931R		
Issue /	Age: 65 E	Deferral Perio	d 10 Year						
							GLWB		
					Partial	Cumulative	Excess		
		Beginning	GLWB	Maximum	Withdrawal	GLWB	Over	Beginning	
		Month	Benefit	Annual	Year-To-	Withdrawal	Acct	of Month	GLWB
Year	Month	Acct Val	Base	GLWB	Date	Amount	Val	Persistency	Credit
1	1	50,000	50,204	2,510	0	0	0	1	
5	12	29,972	63,814	3,829	0	0	0	0.858894	
10	12	50,936	81,445	4,887	0	0	0	0.358988	
15	12	34,368	81,445	4,887	4,887	24,433	0	0.182364	
20	12	14,379	81,445	4,887	4,887	48,867	0	0.095504	
21	12	7,035	81,445	4,887	4,887	53,754	0	0.088276	
22	12	2,231	81,445	4,887	4,887	58,640	0	0.081226	
23	4	757	81,445	4,887	1,629	60,269	0	0.078356	
23	5	366	81,445	4,887	2,036	60,676	42	0.077275	3.21
23	6	0	81,445	4,887	30,241	88,881	28,204	0.076806	2,166.27
	PV of C	LWB Credit	@ 5% =	672.69					

The example shows that the account value (AV) runs out 23 years and five months into the scenario. In fact, the partial withdrawal in 23rd year and fifth month is \$366 of account value and \$42 of GLWB value. At that point, the guaranteed benefit stream of \$4,887 per year is discounted back to the date the account value went to zero with 5 percent interest

and the assumed mortality table. This equals \$28,204 on an undecremented basis and \$2,166.27 on a decremented basis. Discounting the \$2,166.27, plus the \$3.21, which is the decremented \$42 payment top-off, back to time zero at a 5-percent interest rate gives \$672.69.

The above calculation relies on the portion of policies in force as of the date the AV goes to zero as part of the calculation. This portion is determined by the mortality assumptions and the lapse assumption. The lapse assumption used in the example has a dynamic component that reflects the amount the GLWB is in-the-money. The details of the formula used are presented in Appendix B.

A typical dynamic lapse formula used in pricing VA guarantees decreases lapses as the value of the guarantee, the GMWB in this case as measured by the present value of the guaranteed income stream, increases relative to the account value. The theory is that customers will be more likely to persist to realize the value of the guarantee than they would be without the guarantee. The impact of this assumption can be substantial. For instance, for a 65 year old who waits 10 years to start taking withdrawals and assuming 100 percent aggressive growth asset allocation under market consistent scenarios, the PV of GLWB credit is \$1,800. The same PV of GLWB without the dynamic lapse multiplier is \$1,037. That is a 42-percent decrease in this liability, due to the dynamic lapse assumption.

Recall that a number of different scenario sets and the times when each was appropriate were discussed. The difference between the market consistent, long-term risk neutral, and historic scenarios is often quite large, as shown by the Tables VI.2, VI.3 and VI.4. All three of these tables assume 100 percent of the VA account value is allocated to the aggressive growth asset allocation.

TABLE VI.2 – PV OF GLWB WITH AGGRESSIVE GROWTH ASSET ALLOCATION					
	MARKET CONSIS	TENT SCENARIO	S		
	Deferral Period				
Issue Age	0 5 10				
55	\$2,897 \$2,259				
65	\$1,511 \$2,717 \$1,800				
75	\$1,126 \$1,607 \$816				
85	85 \$512				

Deferral period in Table VI.2, and subsequent tables, is the number of years before the first withdrawal is taken. Recall that our GLWB product accumulates premiums at 5 percent until the earlier of the 10th anniversary or the first withdrawal. The 10-year deferral will maximize the value of the benefit base. However, deferring withdrawals does not necessarily maximize the PV of the GLWB. For instance, note that the PV of GLWB is \$2,717 for issue age 65, five-year deferral and \$1,800 for issue age 65, 10-year deferral. Recall that the payout percent is based on the attained age and would be 6 percent for both the five-year and 10-year deferrals. Even though the benefit base is higher for the 10-year deferral, starting withdrawals younger, 70 for the five-year deferral versus 75 for the 10-year deferral, has a larger impact on the PV of GLWB.

In Table VI.3 below, the long-term risk neutral scenario set is used instead of the market consistent scenarios.

TABLE VI.3 – PV OF GLWB WITH AGGRESSIVE GROWTH ASSET ALLOCATION					
LO	NG-TERM RISK N	EUTRAL SCENAR	RIOS		
	Deferral Period				
Issue Age	0 5 10				
55		\$2,776	\$2,113		
65	\$1,437 \$2,609 \$1,649				
75	\$1,096	\$1,518	\$716		
85	\$503				

The values in Table VI.3 are slightly lower than the values in Table VI.2. This is due to the relationship between the long-term risk neutral parameters and the market consistent parameters as of 9/30/07.

In Table VI.4 below, the historic scenario set is used instead of the market consistent scenarios.

TABLE VI.4 – PV OF GLWB WITH AGGRESSIVE GROWTH ASSET ALLOCATION					
	HISTORIC	SCENARIOS			
	Deferral Period				
Issue Age	0 5 10				
55	\$490 \$297				
65	\$236 \$567 \$280				
75	\$230 \$366 \$126				
85	\$113				

Not surprisingly, the values in Table VI.4 are considerably lower than the values in Table VI.2 or Table VI.3. This is due to the higher mean return associated with equity subaccounts within the VA.

Another interesting comparison is the impact of the three assumed asset allocations. Focusing back on Table VI.2 for a moment, if we move from the aggressive growth asset allocation (approximately 75 percent equity / 25 percent bond) to the moderate aggressive growth asset allocation (approximately 65 percent equity, 35 percent bond), as shown in Table VI.5 below, we see a marked reduction in the PV of GLWB.

TABLE VI.5 – PV OF GLWB WITH MODERATE AGGRESSIVE GROWTH ASSET ALLOCATION					
	MARKET CONSISTENT SCENARIOS				
	Deferral Period				
Issue Age	0 5 10				
55		\$2,257	\$1,751		
65	\$1,125 \$2,162 \$1,400				
75	\$861	\$1,266	\$626		
85	\$388				

As the asset allocation is now changed to the moderate growth asset allocation (approximately 50 percent equity, 50 percent bond) another significant drop in the PV of GLWB is seen as shown in Table VI.6 below.

TABLE VI.6 – PV OF GLWB WITH MODERATE GROWTH ASSET ALLOCATION					
	MARKET CONSISTENT SCENARIOS				
	Deferral Period				
Issue Age	0 5 10				
55		\$1,396	\$1,068		
65	65 \$633 \$1,391 \$873				
75 \$512 \$810 \$381					
85	\$234				

Focusing on a specific cell for a moment, Issue Age 65 with 10 year deferral, the values are \$1,800 for the aggressive growth asset allocation, \$1,400 for the moderate aggressive growth asset allocation and \$873 for moderate growth asset allocation, a reduction of 51.5 percent from the highest to the lowest. Recall that in the market consistent scenarios, we assume that all subaccounts have the same mean growth rate dictated by the swap curve. However, as the allocation is moved more toward the bond and money market subaccounts, the overall volatility of the underlying account value is reduced. This is why the PV of GLWB is cut in half, going from the aggressive growth asset allocation to the moderate growth asset allocation.

Now, in looking at the issue age 65 and 10-year deferral period cell using the historic scenarios, the values are \$280 for the aggressive growth asset allocation, \$248 (value can be found in Appendix E) for the moderate aggressive growth asset allocation and \$213 for the moderate growth asset allocation, a reduction of 23.9 percent from the highest to the lowest. This is a significantly lower reduction going from the aggressive growth asset allocation to the moderate growth asset allocation. In the historic scenarios, a significant reduction in overall volatility of the underlying account value would again be expected. However, in the historic scenarios, there is also a corresponding reduction in the mean

return expectation as the asset allocation is moved from aggressive growth to moderate growth, thus the lower percent reduction in PV of GLWB.

The tables above presented the average PV of the GLWB benefits across 1,000 trials. This prompts the question of how many scenarios are enough. The answer to this question is not always an easy one. It depends on what types of measures are being examined and the underlying distribution. For instance, the focus here is on the average of all the scenarios. It will generally take fewer scenarios to arrive at a high degree of confidence in the result for an average rather than a 95th or 99th percentile. Percentiles are highly dependent on only the tail of the distribution, rather than dependency spread across the whole distribution as an average.

One brute force way to see if the number of scenarios is enough is to simply rerun the model with a new random number seed one or preferably two more times. If the results are all within an acceptably narrow range, then a level of confidence can be achieved that a sufficient number of scenarios is being run. A comparison can also be made between each of the scenario sets results and the combined results of scenario sets.

A word of caution, however; if the underlying distribution is extremely long-tailed, such as catastrophe modeling, then the number of scenarios, even focusing on the average, may need to be much larger. If the distribution is almost entirely focused in a narrow range with a few very large values relative to the narrow range, then many more scenarios will likely be required to arrive at an acceptable level of confidence.

The bibliography at the end of this report contains a list of reports and presentations that focus on the question of how many scenarios is enough.

VII. Valuing Life Insurance Embedded Options

Recall that the embedded option being valued for the ULSG product has cash flows defined as death benefits paid minus premiums paid once the cash value has gone to zero. If no additional premiums need to be paid to keep the no-lapse guarantee in effect, this collapses to just the death benefits paid once the cash value has gone to zero. The length of the guarantee is the only item that matters in valuing the embedded option since the goal of the research project is to measure the amount of death benefits paid which would not have been paid without the no-lapse guarantee.

Another way to characterize the embedded option is that it is equivalent to the company being short a series of put options (policyholder is long a series of put options). The policyholder has the option of continuing to pay premiums to keep the no-lapse guarantee in force. As the death benefit, which will eventually be paid, becomes more valuable to the policyholder through the passage of time, the policyholder may continue to select against the insurer and continue to pay premiums to keep alive the promise of payment of the death benefit.

Table VII.1 below	is an example of	of this calculation	for a st	pecific scenario:

Table VII.1 – Example Calculation for ULSG Embedded Option ULSG - Embedded Option Analysis - Scenario 1009: AAJ[002000001045MA]							
Cash Mode: Annual							
LOB & Characteristi	cs: FlexPrem XM MA	-		11:06:30			
Plan: 000001		MG-ALFA 6.4.238	/ TEMP.P58 / 0931F	RDO01-59			
Issue Age: 45							
When Issued: Cash C	Cycle 1						
	End of Year			End of Year	Embedded Option		
Year	Account Value	Premiums	Death Benefits	Persistency	Cash Flows		
1	1,433.39	2,218.75	69.00	0.99972	0.00		
10	13,074.47	1,355.36	228.44	0.60995	0.00		
20	25,616.97	1,274.68	678.73	0.57179	0.00		
30	42,607.43	964.29	1,428.12	0.42890	0.00		
40	28,900.55	629.86	2,682.36	0.27315	0.00		
43	5,093.17	541.03	3,128.99	0.23133	0.00		
44	0.00	513.05	3,272.17	0.21814	2,759.13		
50	0.00	330.61	3,381.85	0.13548	3,051.24		
60	0.00	91.56	1,677.96	0.03455	1,586.41		
70	0.00	8.33	268.31	0.00268	259.98		
76	0.00	0.74	83.86	0.00000	83.12		
	PV of Embedded Option Cash Flows(all years) @ 5% = 4,182.13						

Table VII.1 shows the account value (AV) running out in year 44. At that point and beyond, the policyholder continues to pay premiums into the contract and the embedded option cash flows are discounted back to time zero at 5-percent interest.

No specific dynamic lapse formula was used to determine excess lapses due to competitor behavior. The lapse assumption has a dynamic component such that, when the AV falls to 10 percent of the original face amount, lapses grade from 100 percent of the current lapse assumption to 0 percent in a linear fashion as the AV reaches zero. This grading to zero of lapses is a more reasonable assumption than if lapses all of a sudden step down to zero when the AV goes to zero. For any interest rate scenario, the AV credited rate is a portfolio earned rate less a policy spread, but it is floored at the

guaranteed credited interest rate. The AV credited rate is uncapped and can increase and decrease as the portfolio earned rate moves, thus potentially mitigating excess lapses due to competitor behavior with regard to credited rate practices.

Recall that a number of different scenario sets and the times when each was appropriate were discussed. The difference between the market consistent, long-term risk neutral and historic scenarios can often be quite large. Table VII.2 below summarizes these differences for a few examples.

Table VII.2 - Average	Table VII.2 - Average PV of Embedded Option Cash Flows for Three				
Different S	Scenario Sets ov	ver 1,000 Scena	arios		
	45, Male 65, Male Total Weighted				
	Preferred	Preferred	<u>Liability</u>		
Scenario Set	<u>NS</u>	<u>NS</u>	Portfolio		
Historic	5,779	15,976	11,447		
Long-Term Risk Neutral	4,855	14,084	10,405		
Market Consistent	2,522	10,415	7,134		

One of the characteristics of market consistent scenarios is that the shape of the yield curve determines the forward rates that will serve as the average return in arbitrage-free stochastic scenarios. A typical upward sloping yield curve will result in forward rates significantly higher than current rates. This characteristic is what makes the embedded values in the above table lower for the market consistent scenarios versus the long-term risk neutral or historic scenarios.

In the current accounting regimes, the embedded option we are valuing in this report is handled in different ways. Statutory reserves would be determined based on Actuarial Guideline AXXX. This guideline, like most current statutory calculations, does not attempt to reflect current market conditions. The valuation interest rate is locked-in, based on the issue date and the calculation is formulaic rather than stochastic. Of course,

cash flow testing is always an aggregate requirement that does incorporate some stochastic features and current market conditions. Principle-based reserve methods will be discussed in generalities later in this report.

On the GAAP side, AICPA SOP 03-1 addresses the case where additional reserves may be required for UL-type contracts if the amounts assessed for insurance benefits are assessed in a fashion that is expected to develop profits followed by losses. This situation can be common in UL designs. A liability is set up at issue that recognizes a portion of the assessments that offsets benefits to be provided in the future.

ULSG contracts are subject to SOP 03-01, stated in paragraph 3. The methodology of SOP 03-1 results in the development of a "benefit ratio," which is defined, at issue, as the ratio of the present value of the excess benefits to the present value of policy assessments. Policy assessments usually include policy loads, surrender charges, COIs, and investment spread, but excess benefits may not have a clear definition. Once the benefit ratio is determined, the additional reserve is a retrospective accumulation with interest of policy assessments collected, multiplied by the benefit ratio minus any excess benefits paid during the accounting period.

In a recent Milliman survey, respondents discussed the implementation challenges of SOP 03-1. There was not an overriding consensus on what the assessment was after the account value went to zero, answers included zero or the stipulated premium. Most respondents agreed the benefit in this situation was the death benefit. The definition of excess benefits was more problematic and answers included the death benefits paid after the account value goes to zero, the death benefits paid, less the reserve released, after the account value goes to zero, or some function of the income statement losses. Scenarios generated to project assessments and excess benefits were a mix of historic and market consistent scenarios. The discount rate used was consistent with the rate used to amortize DAC. The AAA practice note on SOP 03-01 provides some interpretation of the

requirements of SOP 03-01 to help companies perform these evaluations, but it does not provide clear answers to the issues stated above.

The evaluation of the additional reserve for this project is grounded in evaluating the embedded option in a market consistent scenario framework. Present values of death benefits paid less the stipulated premiums, after the primary account value goes to zero, are averaged over many stochastic paths where the embedded option cash flows are discounted using the one-year rates along each stochastic path. This process would be performed at each future valuation date and the results would be added to the base reserve at each valuation date.

VIII. Attribution Analysis

The calculated value for the embedded option is important, but on its own provides little insight into the nature of the liability. To really understand the value, to assess the reasonability and to determine what conditions will cause the embedded value to change, an attribution analysis can be helpful.

An attribution analysis, as defined here, will look at the change in the embedded option value over a period of time and break the change into components. It is often the case that the entire change cannot be explained as part of the attribution analysis. The point of the attribution analysis is to show how the major drivers of the value flowed through the change in the embedded option value.

VA with GLWB - Sources of Volatility that Combine to Determine Volatility of the Embedded Option Values

• Interest Rates: Interest rates impact the embedded option value in a few ways. First, most variable annuities have a variety of fixed income investment options as well as a fixed account. An increase in interest rates may have a short-term effect on the value of a fixed income subaccount, but also may have a long-term impact on higher subaccount returns.

The second way interest rates impact the embedded option value is in discounting future benefits. Since most GLWB benefits will be far out in the future, the discount rate can have a significant impact on the present value.

The third way significantly impacts the market consistent embedded value calculation. Since the current interest rate environment is used to calculate forward rates that are the

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mean growth rate in future periods, relatively small changes in interest rates and/or the shape of the curve can have a major impact on the projected future benefits and the embedded value calculation.

• Equity Returns: Directional movements in the market will have an impact on the embedded value calculation. Positive equity returns will reduce the value of the GLWB liability. In the early contract durations, the impact of equity returns is small relative to the impact of interest rate changes. As the contract ages and as the GLWB potentially is more and more in-the-money, the impact of equity returns becomes more significant.

• Implied Volatility: In a market consistent embedded value calculation, implied volatilities are used as the volatility assumption in the stochastic scenarios. Increases in implied volatility, particularly in the early years, can have a significant impact on the liability.

• Policyholder behavior: The various policyholder behavior assumptions required to perform the stochastic modeling all have an impact on determining the embedded values. The lapse assumption typically includes a dynamic lapse component that reduces the lapses as the GLWB becomes more in-the-money. The combination of the base lapse assumption and this dynamic component determine the projected number of contracts available to use the GLWB. Other assumptions such as mortality and withdrawal rates also impact the embedded value calculation. To the extent these assumptions do not match experience, the attribution analysis can demonstrate the portion of the change in the embedded value caused by the difference between assumption and experience.

<u>ULSG - Sources of Volatility that Combine to Determine Volatility of the Embedded</u> <u>Option Values</u>

• Interest Rates: As interest rates fluctuate, so do the cash flows and the valuations of existing and new assets that determine overall portfolio yields when credited interest rates are driven by a portfolio earned rate approach. As the yield curve changes from period to period, at a particular valuation date a new set of risk-neutral scenarios are developed, which may cause the embedded option cash flows to change. Also, discount rates (one-year rates are used) are likely to change and are dependent on the new level and shape of the yield curve at the current valuation date. An example is provided in this section explaining this situation and implications in more detail.

• Policyholder Behavior: Policyholder behavior through disintermediation, additional premium payments, or stopping premium payments, can add to the difficulty of valuing the embedded option. As the uncertainty of policyholder behavior increases, assumption development to value the embedded option becomes more difficult and the need for sensitivity testing increases in order to evaluate embedded optionality appropriately.

Embedded Option Attribution Example

GLWB Example

For the GLWB product the attribution analysis is completed by relying on the sensitivity of the embedded option based on the various measures of the Greeks.

In the following example, the value of the GLWB embedded option was determined at time 1 and time 2 of scenario 3,178 of the C3 Phase II pre-packaged scenarios from the AAA. At each valuation point the yield curve is converted to forward rates. These rates along with the implied volatility surface and correlation matrix are utilized to develop

market consistent scenarios. In this example, to project forward and obtain the present value of the liabilities, 250 market consistent scenarios were developed. The value of the liability at time 1 was \$2,989, and the value of the liability at time 2 was \$2,510. Table VIII.1 shows the index value of the various funds and the average forward rate at both of these times.

Table VIII.1: Index Values of Funds At Various Times								
	Time 1	Time 2	Change					
S&P	1,091	1,069	-2.0%					
Russell	1,520	1,485	-2.3%					
NASDAQ	840	727	-13.5%					
SBIG	1,049	1,058	0.9%					
EAFE	1,010	1,046	3.5%					
Money Market	1,034	1,074	3.9%					
Total Allocation	1,123	1,112	-0.9%					
Average Fwd Rate	4.70%	5.16%	0.46%					

At time 1 various measures of the Greeks were also developed to measure the sensitivity of the value of the liability to changes in the economic environment. To calculate the Greeks it was necessary to repeat the process of valuating the liability changing one input of the scenario generator at a time. In this example, the process was repeated 16 more times to obtain the delta, rho and vega values of the liability.

The delta measure is the sensitivity of the value of the liability with respect to changes in price of the underlying funds. To calculate this measure, each of the six funds was shocked up and down by 1 percent independently. For each fund, the delta is equal to the average difference between the two values. In this example, the value of the liability adding 1 percent to the S&P fund is 2,979.43. The value of the liability reducing the S&P fund by 1 percent is 2,997.60. The value of the delta for this fund is then $\frac{(2,979.43-2,997.60)}{2} = -9.09$.

Since this embedded option can be viewed as a put option, it is expected that the value of the delta will be negative. It is also expected as this option gets more in-the-money that the value of the delta will be more negative, hence more sensitive to small changes in the underlying funds. Chart VIII.1 shows the liability values at time 1 and the deltas for each of the underlying funds as well as the delta for the overall allocation.



Chart VIII.1: Embedded Option Deltas and Option Values

The rho measure is the sensitivity of the value of the liability with respect to moves in interest rates. To calculate this measure a parallel shift in the 1 year forward curve of 0.1 percent up and down was assumed and the liability was revalued. The value of the embedded option with a 0.1-percent increase in interest rates is 2,854.98. The value of the embedded option with a 0.1-percent decrease in interest rates is 3,128.48. The value of rho is then calculated in the same way as the values of the deltas were by averaging the change of the two values. In this example the value of rho is $\frac{(2,854.98-3,128.48)}{2} = -136.75$.

Rho is larger for embedded options that are in-the-money, and is also larger as the time to exercise is farther away. As the time to exercise gets shorter the value of rho gets smaller.

These two effects are the consequence of longer discount periods and larger amounts of the liability as the embedded option is more in-the-money.

In real life, the yield curve does not move in parallel shifts as the assumption made to measure rho in this example. Therefore, in practice, it is possible to shock different points on the yield curve to obtain a more accurate measure of the change in the liability due to changes in interest rate.

The vega measure is the sensitivity of the value of the liability with respect to moves in the implied volatility. To calculate this measure a parallel shift to the volatility surface of 1 percent up and down was assumed and the liability was revalued. The value of the embedded option with a 1-percent increase in implied volatility is 3,165.99, while the value of the embedded option with a 1-percent decrease in implied volatility is 2,815.94. Using the same logic to value the change in liability used for the other measures, the value of vega is $\frac{(3,165.99 - 2,815.94)}{2} = 175.03$.

The Greeks can be used to approximate the value of the liability given the changes in the underlying assumption, and therefore to attribute the change in the embedded value from period to period. As an example of how to calculate the change in embedded option value from one period to the next using the Greeks, consider rho. In this example, the value of rho is -136.75 for every 0.1-percent change in the interest rates. The interest rate changed from period 1 to period 2 by 0.46 percent, so the approximated change in option value from period 1 to period 2 due to the changes in interest rates is $\frac{0.46\%}{0.1\%}$ *-136.75 = -631.07 (value differs due to rounding). Similarly the rest of the adjustments can be calculated.

Table VIII.2 – Change in Value of Embedded Option								
Option Value at time 1	Amount 2,988.69	Greek Measure	Change in Assumption					
S&P Adjustment Russell Adjustment NASDAQ Adjustment SBBIG Adjustment EAFE Adjustment	18.37 16.02 47.57 -3.38 -26.13	-9.09 -7.05 -3.52 -3.90 -7.40	-2.0% -2.3% -13.5% 0.9% 3.5%					
MM Adjustment Sum of individual fund adjustment	<u>-23.17</u> 29.28	-5.99	3.9%					
Delta (Equity) Adjustment Rho (Interest) Adjustment	34.59 -631.67	-36.95 -136.75	-0.9% 0.46%					
Vega (Volatility) Adjustment Discount (Theta) Adjustment	0.00	175.03 1 yr fwd rate =	0.0% 4.1%					
Total Adjustment Attributed Option Value	-473.90 2,514.79							
Actual Option Value Percent of Change Explained by Attribution Analysis	2,509.85 99.0%							

Table VIII.2 shows the change in value of the embedded option by attribute.

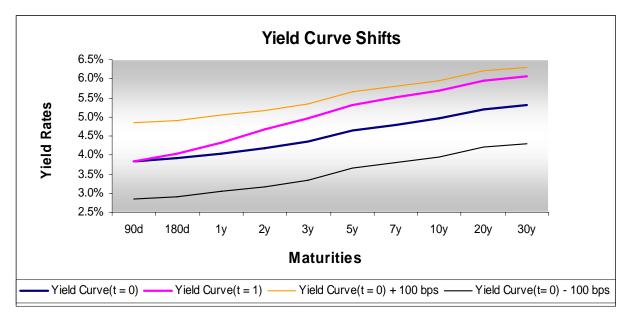
The Greek measures are good at predicting small changes in the underlying attributes. When changes in these attributes are large, for example the changes in the NASDAQ fund of -13.5 percent, other measures are necessary to improve the approximation of the option value. This can be seen in our example with the discontinuity of the sum of the changes due to individual funds of 29.28 versus the calculation of the changes to the option value on the overall allocation of equity of 34.59. Calculating gamma (the change

in delta) would improve the adjustment to the option due to the individual changes in the funds, and the two figures presented prior would be closer to each other.

Another measure worth mentioning at this time is theta. Theta measures the sensitivity of the embedded option as time passes. In our example, the value of theta is calculated by the amount of interest needed to accumulate the option value from period 1 to period 2.

ULSG Example

Chart VIII.2 presents the yield curves that are used in the embedded option valuation and attribution examples that follow. The yield curve at time t = 1 steepened by 75 bps at the 10-year maturity with smaller increases on the shorter end of the yield curve. Parallel yield curve shifts, to the yield curve at time t = 0, of +/-100 bps are also shown.



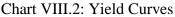
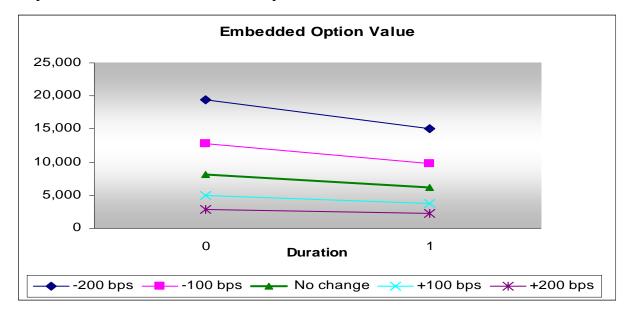


Chart VIII.3 below exhibits the valuation of the embedded option for all new business cells at time t = 0 and time t = 1 where the yield curves (using the above yield curves at time t = 0 and time = 1) are shocked by +/-100 bps and +/- 200 bps to assess the volatility of the embedded option values given that a steepening of the yield curve has occurred



Graph VIII.3: Valuation of Embedded Option of New Business

At each duration and yield curve shift a new set of 500 risk-neutral scenarios was developed, which causes embedded option cash flows to change. In the graph above at time t = 1, the yield curve steepened and projected forward rates are higher than they were at time t = 0. These forward rates seed the risk-neutral interest rate generator, in general projecting higher interest rates, which imply that the embedded option value should usually be lower than it was at time t = 0. This decreased embedded option value stems from higher credited rates driving higher account values making the embedded option more out-of-the-money (account value is positive longer pushing embedded option cash flows further out) and higher discount rates are used to value those option cash flows .

Table VIII.3 below is an example using option risk measures to attribute the change in embedded option value from period to period.

Table VIII.3 – Change in Embedded Option Valuation Using Option Risk Measures

Embedded Option Values and Option Risk Measures

· · · · · · · · · · · · · · · · · · ·			
<u>Time 0</u>	Time 1	<u>Change</u>	
12,837			
8,115	5,997	(2,118)	
4,954			
-48.57			
1,924.15			
Option Value	Option Value		
Change	Change using		
using Rho	Convexity		
-2,956	439		
	12,837 8,115 4,954 -48.57 1,924.15 Option Value Change <u>using Rho</u>	12,837 8,115 5,997 4,954 -48.57 1,924.15 Option Value Option Value Change Lusing Rho Convexity	12,837

Partial Durations

	Rate	Partial	Embedded	
Yield Curve Maturity	<u>Change (bps)</u>	<u>Durations</u>	Value Change	
1 year	28	-0.12	-3	
10 year	75	26.83	1,633	
30 year	75	-74.71	-4,547	
			-2,917	

Embedded Option Value Attribution

Option $\lambda(a)ua(t=0)$	8,115			
Option Value(t=0)	0,110			
Option Value change due to:				
+ Rho(t=0) Adjustment	-2,956			
+ Convexity(t=0) Adjustment	439			
+ Discounting Adjustment	392			
= Estimated Option Value(t=1)	5,990	% of Actual(t=1) =	99.9%	
+ Lapse Adjustment	-47			
= Estimated Option Value(t=1)	5,943	% of Actual(t=1) =	99.1%	

In the attribution example above, a valuation at time t = 0 of the embedded option produces a value of \$8,115. To approximate the embedded option value change from time t = 0 to time t = 1, a couple of option risk measures were determined. The yield curve at time t = 0 was shifted up and down by 100 basis points to calculate a rho of -48.57 (sign convention consistent with the option value change when rates rise), essentially an effective duration calculation, which generates a 48.57-percent or a \$2,956 decrease in option value for the 75 basis-point (bp) increase in rates at the 10-year maturity.

Embedded option value changes exhibit a convexity relationship as can be seen by the asymmetric relationship of the calculated embedded option values after the +/- 100 bps parallel shifts of the yield curve. As stated previously, as rates increase, account values increase and the embedded option becomes more out-of-the-money as the account value will be positive longer. Similarly, as rates decrease, account values decrease and the embedded option becomes more in-the-money as the account value depletes faster. The amount of convexity in the option value depends on how fast the growth or the depletion occurs when rates change, relative to the no shift case, how the policy has aged, and where the floor is on the credited interest rate. The effective convexity was approximated as $(12,837 + 4,954 - 2 \times 8,115) / [8,115 \times (.5 \times .02)^{2}] = 1,924.15$ (actual unrounded). The approximation for effective convexity used equals $(P^- + P^- - 2 \times P^0) / [P^0 \times (.5 \times (y^+ - 2 \times P^0))]$ $(y^{-})^{2}$, where "+" indicates a +100 bps parallel shift and "-" indicates a -100 bps parallel shift in the yield curve, P is the option value, and y is the interest rate. For this example, the embedded option exhibits positive convexity and the effective convexity generates a .5 x 1,924.15 x 8,115 x (.0075) $^2 =$ \$439 increase in embedded option value at time t = 0 assuming a 75 basis point (bp) increase at the 10-year maturity.

Rho and effective convexity together predict an embedded option value of \$5,598 at time t = 1 or approximately 93.3 percent of the calculated option value of \$5,997 at time t = 1.

Since the embedded option has a finite number of cash flows, given no change from time t = 0 to time t = 1 except for a small change in account values, the embedded option value would grow by embedded option value (t = 0) x discount rate (year 1) = \$392. Rho, effective convexity, and the discounting adjustment predict an embedded option value of \$5,990 at time t = 1 or approximately 99.9 percent of the calculated option value of \$5,997 at time t = 1.

Suppose that over the year from time t = 0 to time t = 1, lapse experience was 25 percent higher than expected and that the embedded option value at time t = 1 does incorporate this assumption. Assuming an overall weighted lapse rate of 8.9 percent, the lapse adjustment of .089 x .25 x (-2,956 + 439 + 392) = \$ -47 should be added to the embedded option value estimate of \$5,990 at time t = 1 giving an estimate of \$5,943 or approximately 99.1 percent of the calculated option value of \$5,997 at time t = 1.

Rho and effective convexity option risk measures are generally used to approximate option values for small instantaneous movements in rates. This example shows that these measures can be used as a way to predict a substantial amount of the change in embedded option value from period to period to a reasonable degree of accuracy.

The actual yield curve change from time t = 0 to time t = 1 was not a parallel shift up +75 bps. A yield curve steepening occurred on the long end of +75 bps with smaller yield changes for the shorter maturities. The 10-year rate served as the rate used in the attribution since the 10-year rate was the primary driver of portfolio earned rates, which drive credited interest.

Partial durations were calculated to assess the sensitivity of the embedded option to nonparallel shifts in the yield curve. Partial durations were determined at three key maturities. The change in the embedded option value using only the partial durations was -\$2,917 versus -\$2,956 (using Rho and the 10-year rate change to approximate the change in embedded option value which assumes a parallel shift in the yield curve). For the example above, the embedded option was evaluated at issue and at one year out and the sensitivity of the embedded option value is primarily driven by the long part of the yield curve as can be seen by the 30-year partial duration of -74.71. This relationship makes sense since the embedded option has very long-dated cash flows for a newly issued policy and the late duration credited rates really determine whether the option is in- the- money or not. Early duration credited rates are less important as the COI charges dominate due to high net-amounts-at-risk.

The 10-year partial duration at issue is positive. Credited interest rates are a function of a moving average of 10-year rates, but early duration COI charges predominantly dominate and the late duration credited rates are more important to the sensitivity of the embedded option values.

Chart VIII.4 below exhibits the progression of the 10- and 30-year partial durations.

Progression of Partial Durations 30 20 Partial Durations 10 0 -10 13 15 1 3 5 9 11 17 25 27 29 -20 -30 -40 -50 -60 -70 -80 Duration 30 yr Partial Duration 10 yr Partial Duration -

Chart VIII.4: Progression of the 10 and 30-year Partial Durations

As the policy ages, the point at which the sensitivity of the embedded option becomes inthe-money moves closer to the current valuation date. For example, in the early durations of the block, the long part of the curve (maybe 25-30 years out) is the most important driver of embedded option value changes. If the block has aged 20 years or so, the longer part of the curve becomes a less dominate driver of the change in embedded option value. The convergence of the 10 and 30-year partial durations in Chart VIII.4 exhibits this characteristic.

IX. Longitudinal Analysis

The prior section demonstrated the importance of understanding the items that influence the change in the embedded option value. This section will examine how the sensitivity of the embedded option varies over time and based on market conditions. It is important to understand these items so that fluctuation in the embedded option value can be anticipated and explained.

GLWB Example

The GLWB example below uses a specific stochastic scenario to observe the sensitivity of the embedded option to the various drivers over time and over various market conditions.

Chart IX.1 below is a graph of the scenario. In this scenario, interest rates rise substantially over the first 9 years and then fall dramatically over the next two years. Equity performance is generally favorable in this scenario other than the flat period between years 10 and 20.

Chart IX.1: Interest Rate Scenario

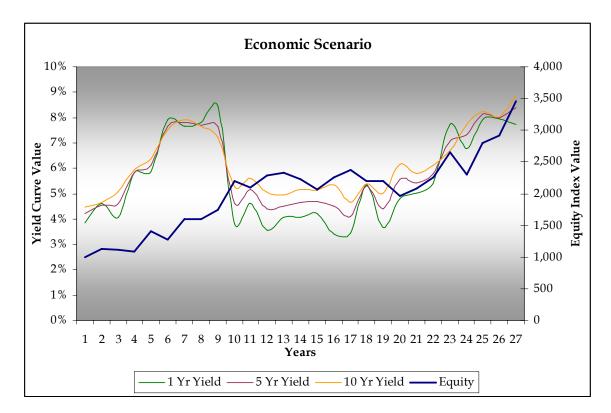
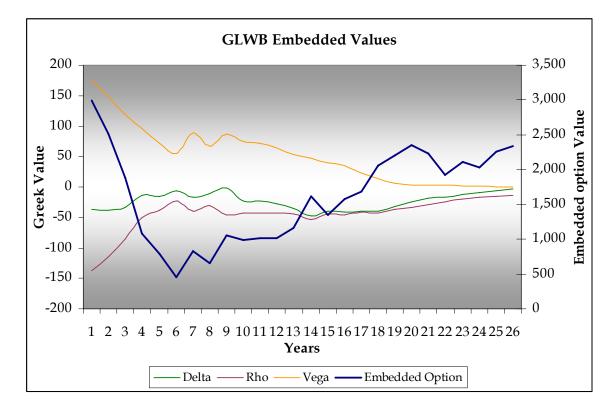


Chart IX.2 below shows the relevant Greeks for the GLWB embedded option through this scenario. The graph also shows the embedded option value. The embedded option value falls quickly over the first six years largely due to the rapid increase in interest rates. The embedded option then climbs overtime.

A pattern that is typical to this type of analysis is apparent in the following graph. The sensitivity to interest rate (rho) and implied volatility (vega) starts high and then grades toward zero as the date for paying (or not paying) any GLWB payments approaches. This makes sense because as the ultimate benefit amount is approached, the average rate of fund growth (determined by interest rates) and the fund growth volatility are less important.

Chart IX.2: GLWB Embedded Values



ULSG Examples

Chart IX.3 below shows the evolution of embedded option values for the block of cells for three deterministic scenarios, each scenario starts with the 9/30/07 swap curve. The flat scenario has future rates held at the initial yield curve rates, the increasing scenario has rates increase by 15 bps per year for each yield curve maturity, and the decreasing scenario has rates decrease by 15 bps per year for each yield curve maturity. The 15 bps increases and decreases occur over each duration of the 30-year scenarios. A set of 300 risk-neutral scenarios were generated at each valuation date to value the embedded option.

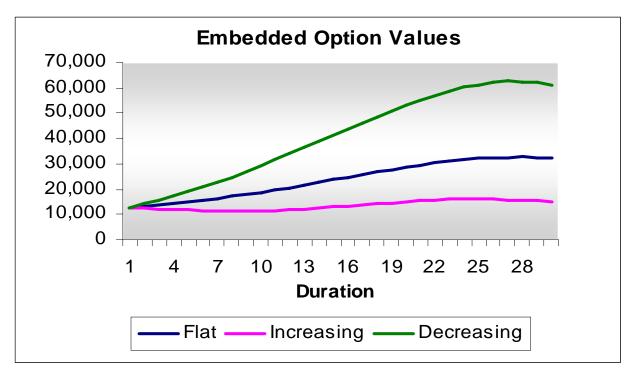


Chart IX.3: Evolution of Embedded Option Values for Three Scenarios

As can be seen the first embedded option value is the same for each of the three scenarios, since each scenario has the same initial yield curve. Embedded option values slowly increase for the flat scenario. For the decreasing scenario, embedded option values rise faster than the flat scenario due to decreasing credited and discounting rates. For the increasing scenario the opposite occurs, embedded option values decrease faster than the flat scenario due to rising credited and discounting rates.

Chart IX.4 below shows absolute values of rho under the increasing and decreasing scenarios described above. An interesting phenomenon in the early durations is that the value sensitivity of the embedded option is larger in the increasing versus decreasing scenario. Credited interest rates were floored at 3.0 percent, so this will effectively limit the amount of embedded option cash flows in a down scenario and this dampens the volatility of the embedded option values to some degree (i.e. when

rho is calculated at a valuation date, the current yield curve is shocked +/- 100 bps to determine rho).

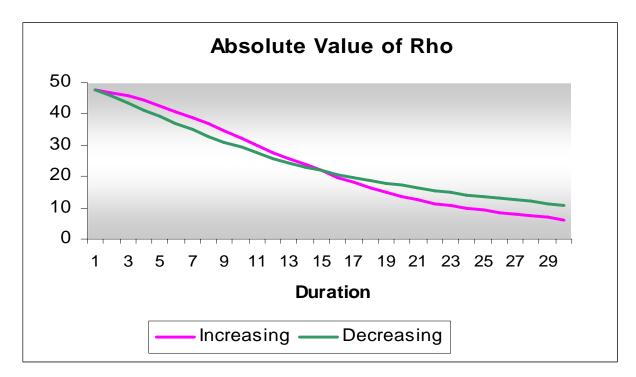


Chart IX.4: Absolute Values of Rho under Increasing and Decreasing Scenarios

Chart IX.5 below shows the evolution of embedded option values for the block of cells for a C3-Phase II scenario. A set of 300 risk-neutral scenarios were generated at each valuation date to value the embedded option.

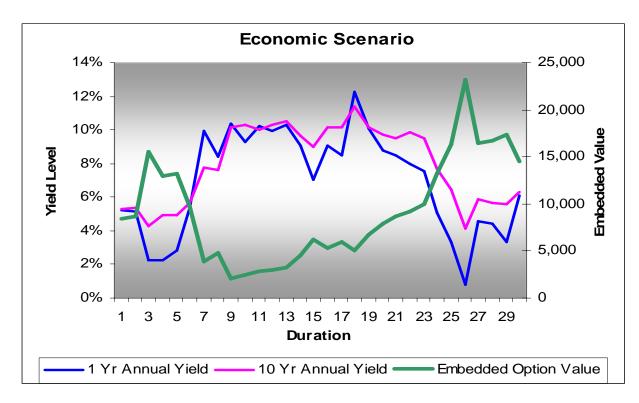


Chart IX.5: Evolution of Embedded Option Values for C3-Phase II Scenario

The embedded option value changes are essentially consistent with the movements of the 10-year rates. In general, as the 10-year rates increase (decrease), embedded option values decrease (increase). For this more realistic interest rate scenario, as compared to the deterministic scenarios above, the determined embedded option values are quite volatile. Incorporation of the embedded option values into reserves for a pricing exercise would necessitate a stochastic framework to properly evaluate the embedded optionality. A balance needs to be struck between the number of risk-neutral scenarios to run at each valuation date as well as the number of pricing scenarios (outer loop scenarios) to use as the exercise is computationally intensive, but well suited for a grid computing platform.

X. Integration into Pricing

One of the questions this research report intended to answer has to do with how the value of the embedded option can be integrated into pricing. Stochastic pricing has been used in insurance product pricing for many years to estimate the range of potential results. Stochastic analysis has been facilitated by improvements in technology, namely computing power. The kinds of stochastic analysis done today could not have been contemplated 20 years ago.

The emergence of stochastic analysis in pricing primarily started with products that had obvious sensitivity to interest rate movements. For instance, it has long been understood that fixed annuity policyholders were more likely to lapse when the crediting rate on their current product is less than prevailing market rates. This is called disintermediation lapse. Since an insurance company generally buys bonds with five- to seven-year maturities to back an annuity with a seven-year surrender charge schedule, a large number of lapses in a market where interest rates have risen significantly would result in capital losses from the forced asset sales to cover the lapse payments. This type of a dynamic is impossible to see in a single-level interest rate environment scenario. Stress test scenarios like the "New York 7" can show these dynamics, but may be too extreme to allow the pricing actuary to assess the risk of this type of behavior.

As variable products became more popular and especially as guarantees on variable products became common place, stochastic analysis became a core component of the product pricing effort. Rather than being a secondary test to evaluate the potential range of values, stochastic analysis became the tool for setting prices. In the early emergence of guarantees on variable products, stochastic analysis was done using historic scenarios and then setting a price so that the cost would be covered in 80 to 90 percent of the scenarios. More recently, the focus has been on calculating the cost to hedge the guarantee. This type of analysis focuses on average claim costs under risk-neutral scenarios.

With the advent of C3 Phases 1 and 2, VACARVM, and principle-based reserves, future reserves and capital requirements are no longer generally known, or at least not easily calculated based on a specific projected path, at issue. This has forced the pricing actuary to consider stochastic-in-stochastic methods for generating the range of cash flows and profitability that will emerge from a product.

As attention shifts to topics like economic capital and enterprise risk management, actuaries are being asked to assess how a new product initiative would look in the context of a company's existing balance sheet and the risks the company currently faces. This becomes an exercise in stochastically modeling a new product in conjunction with the entire company. Obviously the expectations of the pricing actuary and the models used by the pricing actuary continue to advance.

In the context of the pricing implications of this research project, if the focus is placed on the results generated by the market consistent scenarios, the value of the embedded option can be thought of as the price the insurer would have to pay to remove the liability from its balance sheet. If the embedded option value calculated here is added to or integrated with all the other embedded options in the contract and this sum is added to the realized value in the contract, for which the cash value is a good proxy, we have a sum that is representative of the total fair value of the liability.

Suppose that this fair value of the liability was used as the reserve in a typical income statement and the assets backing the reserve were in two forms. The assets backing the cash value could be typical fixed income assets. The assets backing the embedded value would be the appropriate hedge instruments to match the market consistent embedded value. These hedge assets would transfer the risks associated with the embedded option to the capital markets. The insurer would be left with a low risk product. If all embedded options were accounted for, the product could be thought of as no risk.

The income statement generated by this approach would result in projected cash flows. These cash flows could be discounted to calculate a profit margin. If these cash flows were discounted at the risk-free rate under the assumption that all embedded options are accounted for, then a reasonable expectation would be a zero profit margin. Any amount in excess of this would be an excess return in a no-risk situation.

The pricing approach described above is not generally how companies would ultimately determine a price for a product for two reasons. First, it is difficult to measure all embedded options and second, insurers are in the business of taking on risk and getting paid for taking on this risk. However, the pricing method above could be thought of as a secondary threshold, which would confirm that if all embedded options were transferred to the capital markets, the product would still return the risk-free rate. Since insurance companies are in business to earn more than a risk-free rate of return, appropriate risks need to be taken and compensated for so that the overall return will be some reasonable margin above the risk-free rate of return.

An examination of embedded options and their sensitivities can also provide insights into risks that can be mitigated through diversification and ones that cannot. For instance, given a large enough population, the risks associated with an individual's mortality are diversifiable, while over-all trends in population mortality are not. Risks associated with the interest rate environment and equity markets overall can often be hedged reasonably efficiently but are generally not diversifiable. Knowing the risks associated with the embedded option allows the insurance company to assess the cost of hedging, keeping or seeking to diversify these risks.

ULSG Example of Pricing Regime

Embedded option values were incorporated into reserves to assess profitability under 200 C3-Phase II scenarios (outer loop scenarios). Pre-tax profit margins were captured for each scenario where pre-tax profits and premium were discounted at the one-year rates at

each duration. The reserve equals the cash value + embedded option value at each duration. The reserve can be thought of as two independent pieces: (1) Cash value is for the primary guarantee in that the policy will stay in force as long as the cash value is positive, and (2) The embedded option value is for the secondary guarantee when the cash value is depleted and the primary guarantee lapses, but the policy stays in force as long as the policy's secondary guarantee requirements are met. Essentially this exercise is a market consistent evaluation of statutory pre-tax profits where the product design has not been modified.

As discussed above, it is expected that the average of the pre-tax profit margins will be close to zero. The average pre-tax profit margin for this calculation was 48 bps of the present value of premiums. This is significantly lower than a typical pricing profit margin for a ULSG product. Taxes are a confounding factor in this exercise and were ignored.

One of the challenges of creating results like those above is the stochastic-in-stochastic issue. The results above are based on a set of "outer-loop" stochastic scenarios that are historic scenarios. Along each of these stochastic scenarios, an "inner-loop" is taking the economic environment based on the outer-loop scenario to generate a set of market consistent scenarios. These inner-loop scenarios are used to value the embedded option.

One of the major challenges of this type of analysis is run-time. This stochastic-instochastic run has a huge number of computations to perform. While grid computing systems can spread stochastic-in-stochastic runs over many processors, there still is a tremendous amount of data created that has to be summarized in the results. It is essential to have a high degree of confidence in the model before attempting stochastic-instochastic runs, as it is unfeasible to perform detailed calculation checks. However, the types of attribution analysis described in Section VIII of the report can provide a certain level of comfort that the model is working correctly.

XI. Comparison to Similar Calculations

Many of the calculations actuaries perform are migrating from relatively straightforward, although often complex, formulaic calculations to calculations that involve stochastic models. C3 Phase I, C3 Phase II, principles-based reserves (PBR), FAS 133, FAS 157, international financial reporting standards (IFRS) and market consistent embedded value (MCEV) all rely on stochastic analysis.

The first three items on the above list are statutory measures focused on company solvency. All three of these calculations rely on historic scenario sets with an amount of conservatism added. The primary calculation for each of these involves the accumulation of surplus. The accumulated surplus is examined each year within the stochastic projection to confirm sufficiency along that path or to determine the additional amount needed if not sufficient. The final calculation looks at any additional surplus required for each scenario and calculates the average of a certain portion of the worst scenarios (e.g. 90 CTE is the average of the worst 10 percent of scenarios).

The statutory solvency focus of C3 Phase I, C3 Phase II, and PBR is very different from the primary calculations we have been using in this report to examine the embedded option. First, our calculations have centered on the present value of specific cash flows. This focus does not attempt to address sufficiency of the interim amount of surplus. Second, the three statutory measures use historic based scenarios to generate future values. Third, our embedded option calculations involved the average of the individual scenarios rather than concentrating on the tail of the distribution the way the statutory solvency focused calculations do.

The next two measures, FAS 133 and FAS 157, are GAAP measures explicitly for determining the value of embedded derivatives. The methodology of these calculations is to rely on market consistent methods to assess the value of the embedded derivative. This

generally is consistent with the methods used in this report. One exception is that GAAP uses adjustments to ensure certain characteristics. For instance, the starting GAAP reserve has to equal the amount received by the insurance company. For our GLWB contract, the \$50,000 premium would start a \$50,000 account value that would be a part of the GAAP reserve. Any additional reserve for the embedded option generally needs to equal zero at the moment of issue. This is done by computing an imputed charge such that the present value of future benefits minus the present value of future imputed charges equals zero at issue.

For the last two items, IFRS and MCEV, the similarities between these methods and the methods in this report seem to be the strongest. IFRS is yet to be fully defined but it seems to be moving toward a fair value type calculation, where the entire liability is measured on a market consistent basis of some form. This is really a step beyond the valuation of the embedded option described in this report.

MCEV relies on the bifurcation of the liability into an embedded option and a host contract. The host contract is valued using a formulaic method. The embedded option is, as the name implies, valued on a market consistent basis. This is very similar to the methodology described in Section X.

XII. Conclusions

At the beginning of this report we set out to answer a number of questions. The first question was whether closed-form solutions could be used in assessing the value of the embedded option. An example was shown for a relatively simple GMAB on a variable annuity. It was demonstrated that a closed-form solution could be used for this relatively simple guarantee. The point was then made that for more complicated products such as GMWBs and ULSG products, no closed-form solution existed.

One advantage of closed-form solutions is that they do not have stochastic runtime issues. This can be especially beneficial in a stochastic-in-stochastic situation where the innerloop stochastic could potentially be avoided. However, with the ever increasing complexity of the products being sold, it does not appear that expecting a closed-form solution for each type of liability is reasonable.

The second question had to do with generating stochastic scenarios. The paper described three different types of stochastic scenarios: historic, long-term risk neutral and market consistent. The paper discussed and demonstrated how parameters could be set for each of these models and, in the case of the Market Consistent models, how the resulting scenarios could be validated relative to prices observable in the marketplace.

For the two products used in this paper, each of the three scenario sets was used to calculate values. The paper showed that the value of the embedded option is highly sensitive to the scenario set used. The paper also discussed the situations where each of the values might be appropriate.

The third question was in regard to liability assumptions. In describing the two products used in this report, the assumptions that were used in calculating the embedded value were described in detail. The rationale behind the assumptions was also discussed. However, the paper also mentioned that often there is little or no historic data that can be used to set these assumptions and this is where judgment needs to come into play. Judgment, combined with conservatism and an appreciation for the sensitivity of the results due to a change in the assumption, is often the best an actuary can do.

The fourth and likely the most significant question was regarding validating and understanding the results. The paper made the point that the embedded option value itself had value, but only limited value to a company without an understanding of the items that influence the value and an understanding of the period-to-period change in the value. A significant portion of the paper was spent on an attribution analysis demonstration. The attribution analysis was used to explain period-to-period changes in the embedded option value by breaking the change into pieces that captured the sensitivity of the embedded option value to changes in market levels, interest rates, volatility assumptions and policyholder behavior.

Discussing ways to validate and understand the embedded option value is certainly where we see this research report adding the most value. The authors of this report believe this to be a critical issue as the insurance industry and the actuarial profession continue to progress toward stochastic measures of reserves in either a principle-based framework or with regard to International Financial Reporting Standards (IFRS). Without an ability to understand and validate how complex embedded options will behave in these stochastic environments, it would be difficult for an actuary to be comfortable with the stochastic generated embedded option value results. Also, if results cannot be understood and validated, how can actuaries hope to communicate the results to upper-management, industry analysts, rating agencies and regulators?

The authors of this report hope that you have found information and insights that can be applied to your particular situation. Please feel free to contact us if you have questions and/or comments.

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<u>Appendix A – Universal Life with Secondary Guarantee</u> <u>Specifications and Assumptions</u>

Base Universal Life Product Specifications

Account Value Loads	7.0% of premium, all years\$1.20 per unit, years 1-20\$7.50 per month, all years
Cost of Insurance (COI) Charges	Typical reverse select & ultimate schedule
Guaranteed COIs	2001 CSO, Ultimate, M/F, NS/SM, ANB mortality table
Maturity Age	121
Minimum Guaranteed Interest Rate	3.0%
Surrender Charges	Maximum allowable year 1 based on 2001 CSO, Ultimate, M/F, NS/SM, ANB mortality, 3.00% interest (including reduction for any "initial acquisition expense charge"). Amortization period = 19 years, zero surrender charge for 20th policy year
Surrender Charges Commissions and Field Expenses	CSO, Ultimate, M/F, NS/SM, ANB mortality, 3.00% interest (including reduction for any "initial acquisition expense charge"). Amortization period = 19 years, zero surrender charge for 20th
	CSO, Ultimate, M/F, NS/SM, ANB mortality, 3.00% interest (including reduction for any "initial acquisition expense charge"). Amortization period = 19 years, zero surrender charge for 20th policy year
	CSO, Ultimate, M/F, NS/SM, ANB mortality, 3.00% interest (including reduction for any "initial acquisition expense charge"). Amortization period = 19 years, zero surrender charge for 20th policy year Year 1: 125% to target, 4% excess

Shadow Account Specifications

Loads		15% of Premium
Cost of Insurance Charges		Level percentage of the 2001 CSO Select & Ultimate, M/F, SM/NS, ANB mortality rates
		See Table App A.1 Below
Credited Interest		See Table App A.2 Below
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	<u>Table App A.1</u> Shadow Account Cost of Insurance Rates (times 1,000)													
		Issue A		** 11000	1111 UUSI	Issue A		1115 (1111	Issue Age 65					
	Super		Standard				Super		Stan	dard	Super	· Pref	Stan	dard
Yr	M	F	Μ	F	M	F	М	F	M	F	Μ	F		
1	3.33	5.28	3.37	2.71	4.42	2.84	4.39	7.54	12.40	3.61	6.42	11.21		
2	3.97	6.68	4.85	3.12	5.90	4.22	5.29	9.72	16.58	4.20	8.75	15.54		
3	4.42	7.62	6.44	3.54	6.40	4.63	5.95	11.34	20.35	4.82	9.75	15.85		
4	4.77	8.06	8.14	3.91	6.62	5.28	6.51	12.32	23.58	5.38	10.40	16.67		
5	5.16	8.32	9.90	4.27	6.80	6.18	7.13	13.10	26.07	5.95	11.09	17.89		
6	5.51	8.68	11.69	4.59	6.92	7.34	7.73	14.20	27.69	6.50	11.78	19.31		
7	5.90	9.11	13.45	4.83	6.95	8.76	8.43	15.62	28.32	6.96	12.49	20.76		
8	6.18	9.38	15.22	5.03	6.84	10.47	9.02	17.06	28.04	7.40	13.14	22.05		
9	6.31	9.18	17.06	5.15	6.55	12.48	9.45	18.02	26.94	7.78	13.73	22.99		
10	6.26	8.70	20.19	5.18	6.02	14.80	9.67	18.99	26.57	8.06	14.20	23.37		
11	6.24	9.88	23.81	5.09	6.90	17.47	10.01	18.85	25.07	8.25	14.47	22.99		
12	6.20	11.31	28.04	4.93	7.90	20.48	10.43	18.51	29.53	8.38	14.48	21.56		
13	5.77	12.72	33.01	4.61	9.00	23.83	10.37	17.35	34.76	8.36	14.15	25.10		
14	5.05	14.15	38.96	4.14	10.25	27.62	9.91	15.44	41.02	8.21	13.42	29.08		
15	5.53	15.74	46.01	4.62	11.63	31.79	9.32	17.17	48.45	7.85	12.19	33.47		
16	6.20	17.70	52.30	5.14	13.17	35.22	8.70	19.31	55.07	7.28	13.80	37.08		
17	6.98	19.92	58.66	5.71	14.86	38.74	7.83	21.73	61.76	6.47	15.57	40.79		
18	7.84	22.28	65.33	6.31	16.73	42.68	8.80	24.30	68.79	7.15	17.53	44.94		
19	8.75	24.96	73.26	6.95	18.48	46.92	9.82	27.23	77.14	7.87	19.36	49.40		
20	9.71	28.09	82.98	7.62	20.34	51.64	10.90	30.65	87.37	8.64	21.30	54.37		
21	10.76	31.34	92.89	8.26	22.31	56.86	12.08	34.19	97.81	9.36	23.37	59.87		
22	11.85	35.02	104.00	8.96	24.49	62.55	13.30	38.21	109.50	10.16	25.65	65.86		
23	12.94	39.28	116.24	9.73	26.85	68.82	14.53	42.85	122.39	11.02	28.12	72.47		
24	14.42	44.28	129.53	10.60	29.48	75.69	16.18	48.30	136.39	12.01	30.88	79.70		
25	16.07	49.63	142.41	11.54	32.36	83.33	18.04	54.14	149.95	13.07	33.90	87.75		
26	18.68	55.99	157.30	12.62	35.64	102.55	20.97	61.08	165.63	14.30	37.34	107.98		
27	20.51	62.57	169.94	13.82	39.98	106.90	23.02	68.26	178.94	15.66	41.88	112.56		
28	22.91	69.42	183.12	15.16	44.86	115.46	25.72	75.73	192.81	17.18	47.00	121.58		
29	25.44	76.80	197.01	16.61	49.74	128.16	28.56	83.78	207.44	18.83	52.10	134.95		
30	28.11	84.98	211.69	18.21	55.12	144.19	31.55	92.70	222.90	20.64	57.75	151.83		
31	31.02	94.11	227.11	19.98	61.19	163.98	34.83	102.66	239.14	22.64	64.10	172.66		
32	34.20	104.23	241.22	21.92	66.69	182.78	38.39	113.71	253.99	24.85	69.86	192.45		
33	37.89	115.29	256.27	24.06	74.97	202.12	42.53	125.77	269.83	27.27	78.54	212.82		
34	42.20	127.13	272.32	26.42	83.62	205.44	47.37	138.69	286.74	29.95	87.60	216.31		
35	47.17	139.63	289.46	28.97	92.85	216.53	52.96	152.33	304.78	32.84	97.27	227.99		
36	52.60	152.67	307.79	31.82	101.35	234.14	59.05	166.55	324.08	36.07	106.17	246.54		
37	58.78	164.94	322.33	35.69	105.65	252.92	65.98	179.94	339.39	40.45	110.68	266.31		
38	65.21	177.73	337.82	40.06	114.11	273.86	73.20	193.89	355.71	45.40	119.54	288.36		
39	72.14	191.22	354.31	44.41	126.66	296.68	80.99	208.60	373.07	50.33	132.69	312.39		
40	79.83	205.47	371.86	49.22	142.50	321.81	89.61	224.15	391.54	55.78	149.28	338.85		
41	88.40	220.43	390.26	54.63	162.05	348.98	99.24	240.47	410.92	61.91	169.77	367.45		
42	97.91	234.13	409.83	59.54	180.63	376.82	109.92	255.41	431.52	67.48	189.23	396.77		
43	108.30	248.73	430.63	66.94	199.74	405.35	121.57	271.34	453.42	75.86	209.26	426.81		

	Table App A.1 Shadow Account Cost of Insurance Rates (times 1,000)											
	Issue Age 45 Issue Age 55 Issue Age 65											
	Super	r Pref	Stan	dard	Super	Super Pref		dard	Super	r Pref	Stan	dard
Yr	M	F	Μ	F	M	F	Μ	F	M	F	Μ	F
44	119.43	264.31	452.74	74.66	203.02	434.04	134.07	288.34	476.70	84.62	212.69	457.02
45	131.17	280.95	476.22	82.90	213.98	463.93	147.25	306.49	501.43	93.95	224.17	488.49
46	143.42	298.73	501.15	90.49	231.39	494.50	161.00	325.89	527.68	102.55	242.40	520.68
47	154.95	312.85	527.63	94.33	249.94	523.87	173.94	341.29	555.56	106.90	261.84	551.61
48	166.96	327.89	555.73	101.88	270.64	552.36	187.42	357.70	585.15	115.46	283.53	581.61
49	179.63	343.89	585.57	113.09	293.19	578.31	201.65	375.16	616.57	128.16	307.16	608.92
50	193.01	360.92	617.23	127.23	318.02	614.87	216.67	393.73	649.90	144.19	333.17	647.43

	Table App A.2 Shaday Assaurt Cradited Interest Dates											
	Shadow Account Credited Interest Rates Issue Age 45 Issue Age 55 Issue Age 65											
	Super	r Pref		dard	Super	r Pref		dard	Super Pref Standa			dard
Yr	M	F	Μ	F	M	F	Μ	F	M	F	Μ	F
1-7	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
8	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.8%	5.8%	5.9%	5.9%
9	5.0%	5.0%	5.0%	5.0%	5.7%	5.9%	5.9%	5.9%	6.5%	6.7%	6.8%	6.9%
10	5.0%	5.0%	5.0%	5.0%	6.4%	6.8%	6.8%	6.8%	7.3%	7.5%	7.7%	7.8%
11	5.0%	5.0%	5.0%	5.0%	7.1%	7.7%	7.6%	7.7%	8.0%	8.3%	8.6%	8.7%
12	5.0%	5.0%	5.0%	5.0%	7.8%	8.6%	8.5%	8.6%	8.8%	9.1%	9.6%	9.7%
13	5.0%	5.0%	5.0%	5.0%	8.5%	9.5%	9.4%	9.6%	9.5%	10.0%	10.5%	10.6%
14	5.0%	5.0%	5.0%	5.0%	9.2%	10.4%	10.3%	10.5%	10.3%	10.8%	11.4%	11.5%
15	5.9%	5.9%	5.9%	5.8%	9.9%	11.3%	11.2%	11.4%	11.0%	11.6%	12.3%	12.4%
16	6.7%	6.8%	6.8%	6.7%	10.5%	12.2%	12.0%	12.3%	11.8%	12.5%	13.2%	13.4%
17	7.6%	7.7%	7.7%	7.5%	11.2%	13.1%	12.9%	13.2%	12.5%	13.3%	14.1%	14.3%
18	8.4%	8.6%	8.6%	8.4%	11.9%	14.0%	13.8%	14.1%	13.3%	14.1%	15.0%	15.2%
19	9.3%	9.5%	9.5%	9.2%	12.6%	14.9%	14.7%	15.0%	13.3%	14.1%	15.0%	15.2%
20	10.1%	10.4%	10.3%	10.1%	12.6%	14.9%	14.7%	15.0%	13.3%	14.1%	15.0%	15.2%
21	11.0%	11.3%	11.2%	10.9%	12.6%	14.9%	14.7%	15.0%	13.3%	14.1%	15.0%	15.2%
22	11.8%	12.3%	12.1%	11.7%	12.6%	14.9%	14.7%	15.0%	13.3%	14.1%	15.0%	15.2%
23	12.7%	13.2%	13.0%	12.6%	12.6%	14.9%	14.7%	15.0%	13.3%	14.1%	15.0%	15.2%
24	13.5%	14.1%	13.9%	13.4%	12.6%	14.9%	14.7%	15.0%	13.3%	14.1%	15.0%	15.2%
25+	14.4%	15.0%	14.8%	14.3%	12.6%	14.9%	14.7%	15.0%	13.3%	14.1%	15.0%	15.2%

Distribution Assumptions

Gender	60% Male / 40% Female
Death Benefit	100% option A
Issue Age	45: 25%
	55: 40%
	65: 35%
Risk Classes	Super Preferred Non-Tobacco: 60%
	Standard Non-Tobacco: 40%

Assumptions

Face amount	\$250,000
Net Investment Earnings Rate	6.25%, Deterministic
	Portfolio Earned Rate (PER), Stochastic
Credited Interest Rate	6.25% - 2.25% = 4.0%, Deterministic
	Max(PER - 2.25%, Guaranteed Rate), Stochastic
Acquisition Expenses	Per Policy
	Issue age 45: \$300
	Issue age 55: \$400
	Issue age 65: \$450
	Percent of Premium
	10.0% to Target
	10.070 to Taiget
	Per Unit
	Issue age 45: \$0.70
	Issue age 55: \$0.90
	Issue age 65: \$1.10

Maintenance Expenses	\$60 per policy increasing by 3% per year1% of premium2.5% of premium for premium tax
Lapse Rate by Policy Year	45: 10, 8, 7, 6, 5, 4, 3, 2, 2% 55: 9, 8, 7, 6, 5, 4, 3, 2, 2%, 65: 8, 8, 7, 6, 5, 3.5, 2.5, 1.5, 1.5%, If account value = \$0, and secondary
	If account value = 0 , and secondary guarantee is keeping policy in force, lapses = 0% .
Partial Withdrawal Rate	None
Policy Loans	None

The mortality rates are a percentage of the 2001VBT Table (ANB). These percentages vary by issue age and class.

Rates below are linearly interpolated between years.

Male, Super-Preferred

Years	45	55	65
1	46%	48%	50%
16	45%	46%	48%
26+	40%	45%	45%
Female, S	uper-Prefer	red	
Years	45	55	65
1	42%	45%	48%
16	40%	42%	45%
26+	38%	40%	42%
Male, Star	ndard		
Years	45	55	65
1	90%	95%	95%
16	85%	90%	90%
26+	80%	85%	85%
<u>Female, S</u>	tandard		
Years	45	55	65
1	85%	90%	92%
16	80%	90%	90%
26+	80%	90%	90%
35%			
7.70%			

Federal Tax Rate

DAC Tax Rate

7.70%

10-year amortization period assumed

Mortality

Statutory Reserve	Reserves for the secondary guarantees will follow the methodologies described in Actuarial Guideline AXXX
	Valuation Mortality Tables: UL CRVM Reserves: 2001 CSO, Select & Ultimate, M/F, NS/SM, ANB, mortality table
	Basic Reserves for secondary guarantees: 2001 CSO, Select & Ultimate, M/F, NS/SM, ANB, mortality table
	Deficiency Reserves for secondary guarantees (not applicable to tax reserves): 2001 CSO, Select & Ultimate, M/F, NS/SM, ANB, mortality table with X- Factors as defined per Regulation XXX
	Tax Reserves: 2001 CSO, Ultimate, M/F, NS/SM, ANB, mortality table
	Calculate Statutory and Tax Reserves to attained age 121
	Valuation Interest Rates : Statutory: 4.00% Tax: 4.00%
Base Case Capital and Surplus	3.5% of reserve0.15% of NAAR4.50% of premium
Reinsurance	None

<u>Appendix B – Variable Annuity with Guaranteed Lifetime Withdrawal</u> <u>Benefit Specifications and Assumptions</u>

Base Variable Annuity Product Specifications

M&E Charge	110 basis points
Surrender Charge	8, 7, 6, 5, 4, 3, 2, 0% of account value Typical of a B-share product
Free Amount	10% of account value
Return of Premium Death Benefit	Death benefit equals the greater of premiums paid and the account value. Partial withdrawals reduce the death benefit on a pro-rata basis.
Commission	7.0% of premium with a 100% chargeback for lapses in the first 6 months and a 50% chargeback for lapses in months $7 - 12$.
	No trail commission

<u>GLWB Product Specifications</u>

Benefit Base	Maximum of 5% compound Roll-Up and annual step-up for a minimum of 10 years or first withdrawal and maximum anniversary value thereafter
Charge as a percent of AV	0.65%
Maximum Benefit Percent	5% "for life" at ages 60-69
	6% "for life" at ages 70-79
	7% "for life" at ages 80-85
	Attained age at first withdrawal sets maximum benefit percent.

Ratchet Once Withdrawals Have Started	If the account value is above the Benefit Base on any anniversary after withdrawals have started, the Benefit Base will be reset to the account value and a new Maximum Benefit Amount will be calculated.
	The Maximum Benefit Percent will not be changed.
Excess Withdrawals	If a withdrawal in any year exceeds the Maximum Benefit Amount, the Maximum Benefit Amount going forward will be reduced in proportion to the amount of the excess withdrawal relative to the account value.
Asset Allocation Restrictions	The entire account value must be allocated to one of the asset allocations listed in the following table.

	GLWB Asset Allocations Available						
		S&P 500	Russell 2000	NASDAQ	SB BIG (Bond)	EAFE	Money Market
	Aggressive Growth	25%	20%	10%	10%	20%	15%
Fund Allocation	Moderate Aggressive Growth	25%	20%	5%	20%	15%	15%
	Moderate Growth	22%	15%	3%	30%	10%	20%

Distribution Assumptions

Gender	50% Male / 50% Female
Tax Status	50% Qualified / 50% Non-qualified
Election of GLWB	100%

Age and GLWB Deferral Period defined as the number of years after issue before first withdrawal taken

Issue Age	0-yr Deferral	5-yr Deferral	10-yr Deferral
55	0%	5%	20%
65	5%	5%	30%
75	10%	10%	10%
85	5%	0%	0%

Assumptions

Initial Premium	\$50,000
	No subsequent premiums
Investment Management Fee	100 basis points
Revenue Sharing	40 basis points
Acquisition Expenses	1.5% of premium plus \$125 per policy
Maintenance Expenses	\$75 per policy increasing by 3% per year
Lapse Rate by Policy Year	1.5%, 2.5%, 3.0%, 3.0%, 4.0%, 5.0%, 8.0%, 40.0%, 10.0, 10.0%,
Dynamic Lapse Multiple	When the GLWB is more in-the-money, the Dynamic Lapse Multiple will be less than 100% and thus when applied to the Lapse Rate, will reduce Lapses.
	Dynamic Lapse Multiple = Max [10%, 100% - 0.75* (GLWB ITM% - 110%)] If GLWB ITM% > 110% Dynamic Lapse Multiple = 100% otherwise
	Where: GLWB ITM% = [PV of GLWB / AV]
Partial Withdrawal Rate	Determined by GLWB Deferral Assumption

Mortality	80% of Annuity 2000 Basic
Federal Income Tax Rate	35%
DAC Tax rate	1.75% for non-qualified10-year amortization period assumed
Base Case Statutory Reserve	Cash Surrender Value
Base Case Capital and Surplus	100 basis points of statutory reserve

Appendix C – Hull-White Model

The Hull-White model considers the stochastic process followed by the instantaneous (short) interest rate. It assumes that the process is both normal and mean-reverting. Specifically,

$$dr(t,t) = [\theta(t) - ar(t)]dt + \sigma dz \tag{1}$$

where r(t,t) is the short rate, σ is the short rate volatility, a is the mean reversion strength, and $\theta(t)$ is a deterministic drift function. No-arbitrage conditions provide a recipe, not only for determining $\theta(t)$, but also for describing the implied evolution of the entire yield curve. A salient feature of the yield curve dynamics implied by the Hull White model is that the volatility of the instantaneous forward rate T years forward is

$$v(t,t+T) = \sigma \exp[-aT] \tag{2}$$

where v(t,t+T) is the time-*t* volatility of the interest rate which is *T* years forward. Naturally, as *T* goes to zero, v(t,t+T) reduces to the short rate volatility σ .

In practice, Monte Carlo implementation requires discretization of the Hull-White process. One common approach is to use the so-called BGM formulation. In this approach, the focus is on discrete rather than instantaneous forward rates, and the volatility term structure described in equation (2) is replaced by a step function. Specifically, equations (1) and (2) are replaced by the equations

$$r'(t,t+T) = r'(t-1,t+T) + \theta'(t,t+T) + v'(t,t+T)\phi$$
(3)

$$v'(t,t+T) = \sigma' \exp[-aT] \tag{4}$$

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The primes indicate that the variables have been discretized and also normalized (so that the length of the discrete time step is 1). Here, r'(t,t+T) is the time-t interest rate for the forward period between t+T and t+T+1. σ' is the volatility of r'(t,t), ϕ is a random normal number, and the volatility term structure v'(t,t+T) describes the volatility of the forward rate r'(t,t+T). The deterministic function $\theta'(t,t+T)$ is again determined by no-arbitrage conditions.

Equations (3) and (4) have the same physical content as equations (1) and (2), but provide a recipe for practical implementation with discrete interest rates and time steps.

9/30/07 S	wap Curve
USSW1	5.412%
USSW2	5.359%
USSW3	5.399%
USSW4	5.443%
USSW5	5.488%
USSW6	5.531%
USSW7	5.571%
USSW8	5.606%
USSW9	5.638%
USSW10	5.666%
USSW11	5.680%
USSW12	5.709%
USSW13	5.726%
USSW14	5.742%
USSW15	5.757%
USSW16	5.769%
USSW17	5.780%
USSW18	5.788%
USSW19	5.794%
USSW20	5.803%
USSW25	5.814%
USSW30	5.816%

Forward 1-yr Rates			
Year 1	5.340%		
2	5.233%		
3	5.413%		
4	5.512%		
5	5.615%		
6	5.703%		
7	5.780%		
8	5.829%		
9	5.884%		
10	5.917%		
11	5.790%		
12	6.075%		
13	5.950%		
14	5.973%		
15	6.001%		
16	5.976%		
17	5.990%		
18	5.940%		
19	5.904%		
20	6.036%		
25	5.841%		
30	5.771%		

Appendix D – Market Consistent Scenarios Parameters for VA with GLWB

	Volatility Surface as of 9/30/07					
	Forward 1-Year Volatilities					
Year	S&P	Russell 2000	NASDAQ	SB BIG	EAFE	Money Market
1	16.8%	22.2%	20.2%	3.5%	17.3%	1.0%
2	17.8%	24.5%	22.2%	3.5%	18.7%	1.0%
3	20.1%	25.7%	23.7%	3.5%	20.2%	1.0%
4	21.9%	27.3%	25.4%	3.5%	21.6%	1.0%
5	23.7%	28.8%	27.0%	3.5%	23.0%	1.0%
6	24.6%	28.8%	28.0%	3.5%	23.0%	1.0%
7	26.1%	29.7%	29.4%	3.5%	23.9%	1.0%
8	26.9%	30.7%	27.6%	3.5%	24.4%	1.0%
9	28.2%	31.7%	28.1%	3.5%	25.2%	1.0%
10	29.5%	32.7%	28.7%	3.5%	26.0%	1.0%
11	28.7%	34.1%	31.5%	3.5%	27.0%	1.0%
12	29.6%	35.2%	32.4%	3.5%	27.9%	1.0%
13	30.5%	36.2%	33.4%	3.5%	28.7%	1.0%
14	31.4%	37.3%	34.4%	3.5%	29.5%	1.0%
15	32.2%	38.3%	35.3%	3.5%	30.4%	1.0%
16	29.3%	34.5%	33.3%	3.5%	27.8%	1.0%
17	26.3%	30.8%	31.2%	3.5%	25.3%	1.0%
18	23.3%	27.0%	29.1%	3.5%	22.7%	1.0%
19	20.3%	23.3%	27.1%	3.5%	20.2%	1.0%
20	17.4%	19.5%	25.0%	3.5%	17.6%	1.0%

Appendix E – Results from VA with GLWB Models

TABLE VI.2 – PV OF GLWB WITH AGGRESSIVE GROWTH ASSET ALLOCATION MARKET CONSISTENT SCENARIOS				
	Deferral Period			
Issue Age	0 5 10			
55	\$2,897 \$2,259			
65	\$1,511 \$2,717 \$1,800			
75	\$1,126 \$1,607 \$816			
85	85 \$512			

TABLE VI.3 – PV OF GLWB WITH AGGRESSIVE GROWTH ASSET ALLOCATION				
LO	NG-TERM RISK N	EUTRAL SCENAR	RIOS	
	Deferral Period			
Issue Age	0 5 10			
55	\$2,776 \$2,113			
65	\$1,437 \$2,609 \$1,649			
75	\$1,096 \$1,518 \$716			
85	\$503			

TABLE VI.4 – PV OF GLWB WITH AGGRESSIVE GROWTH ASSET ALLOCATION					
	HISTORIC SCENARIOS				
	Deferral Period				
Issue Age	0 5 10				
55	\$490 \$297				
65	\$236 \$567 \$280				
75	\$230 \$366 \$126				
85	85 \$113				

TABLE VI.5 – PV OF GLWB WITH MODERATE AGGRESSIVE GROWTH ASSET ALLOCATION					
	MARKET CONSIS	STENT SCENARIO	S		
	Deferral Period				
Issue Age	0 5 10				
55		\$2,257	\$1,751		
65	\$1,125 \$2,162 \$1,400				
75	\$861 \$1,266 \$626				
85	85 \$388				

TABLE VI.6 – PV OF GLWB WITH MODERATE GROWTH ASSET ALLOCATION				
MARKET CONSISTENT SCENARIOS				
		Deferral Period		
Issue Age	0 5 10			
55	\$1,396 \$1,068			
65	\$633 \$1,391 \$873			
75	\$512 \$810 \$381			
85	\$234			

TABLE VI.7 – PV OF GLWB WITH MODERATE AGGESSIVE GROWTH ASSET ALLOCATION				
LO	LONG-TERM RISK NEUTRAL SCENARIOS			
	Deferral Period			
Issue Age	0 5 10			
55	\$2,185 \$1,661			
65	\$1,067 \$2,093 \$1,304			
75	\$833 \$1,202 \$557			
85	\$380			

TABLE VI.8 – PV OF GLWB WITH MODERATE GROWTH ASSET ALLOCATION				
LONG-TERM RISK NEUTRAL SCENARIOS Deferral Period				
Issue Age	0 5 10			
55	\$1,500 \$1,138			
65	\$661 \$1,495 \$919			
75	\$545 \$849 \$386			
85	\$249			

TABLE VI.9 – PV OF GLWB WITH MODERATE AGGESSIVE GROWTH ASSET ALLOCATION					
	HISTORIC SCENARIOS				
	Deferral Period				
Issue Age	0 5 10				
55	\$415 \$250				
65	\$193 \$528 \$248				
75	\$198 \$327 \$112				
85	85 \$100				

TABLE VI.10 – PV OF GLWB WITH MODERATE GROWTH ASSET ALLOCATION				
	HISTORIC SCENARIOS			
	Deferral Period			
Issue Age	0 5 10			
55	\$334 \$208			
65	\$141 \$455 \$213			
75	\$160 \$286 \$98			
85	85 \$83			