Article From:

The Actuary

QUANTIFICATION OF A FUNDAMENTAL ERROR INVOLVED IN "ACTUARIAL" ANALYSES MADE BY NON-ACTUARIES

by Robert J. Myers

From time immemorial, actuarial students have learned early in their education that valuing a life annuity by using the amount of an annuity-certain for a period equal to the expectation of life is erroneous, producing an overstatement. This is rigorously proved in a quantitative manner in C. W. Jordan, "Life Contingencies" (second edition, Society of Actuaries, 1968) — and also for actuaries of Ancient Mariner vintage, in their actuarial bible, E. F. Spurgeon, "Life Contingencies" (third edition, Cambridge University Press, 1932).

Non-actuaries find it appealing to take shortcuts when they attempt actuarial calculations. One of these is to assume that everybody now aged x will survive for a number of years equal to their expectation of life and then all drop dead. Another is the erroneous procedure mentioned previously with regard to calculating annuity values. Nowhere, however, has this actuary seen any quantitative analysis of the error therein involved. Without such facts being demonstrated, non-actuaries might complain that actuaries were being critical of their methodology only on theoretical grounds, and that no significant error was present in their shortcut — and simple — procedure.

We can all readily observe that, if the interest rate is zero, then the annuity-certain method gives precisely the correct answer. Let us consider the situation under the U.S. Total Persons Life Tables for 1969-71 at various interest rates. As the following table shows, for a given age the relative excess, or error, increases slowly to a peak and then decreases — until eventually, for an infinite interest rate, the annuity-certain method produces the correct result (namely, zero, except for an annuity with first payment on the valuation date).

The relative error, as would be expected, becomes larger as the expectation of life for the valuation age increases. For those at the retirement age of 65 (which is often the area where non-actuaries tread in making these calculations), the relative error can be as much as 10% at moderate rates of interest.

The maximum value of the error is reached at about 3% interest for females age 20, at 5½% for females age 40, at 4½% for males age 20, and at 6% for males age 40. The author did not attempt to find the relatively high interest rate where this peak occurs for age 65, but apparently for females it is about 10% (but well less than 20%, for which a calculation showed only a 5.3% excess). The peak for males age 65 is probably close to 15%.

One other subsidiary calculation made was the error for a 1% interest rate for the category with the largest expectation of life shown by these tables — namely, females age 1. The excess in this case was only 1.6%.

I now leave it to my younger colleagues, who are closer to their mathematical training and computer expertise than I, to extend this analysis in theoretical directions as compared with merely taking cases.

RELATIVE EXCESSES OF ANNUITY-CERTAIN FOR EXPECTATION OF LIFE OVER ACTUAL ANNUITY VALUE, U.S. TOTAL PERSONS LIFE TABLES FOR 1969-71

Interest Rate	Females Age 20	Females Age 40	Females Age 65	Males Age 20	Males Age 40	Males Age 65
1%	2.1%	2.9%	4.7%	2.7%	3.6%	6.0%
2	2.7	4.0	6.2	3.7	5.0	9.0
3	2.9	4.7	7.4	4.1	6.0	9.4
4	2.8	5.0	8.5	4.2	6.6	10.8
5	2.7	5.1	9.4	4.2	6.9	11.9
6	2.4	5.1	9.9	3.9	7.0	12.9
7	2.0	5.0	10.5	3.6	6.9	13.7
8	1.8	4.7	10.8	3.2	6.8	14.4

Editor's Note: The author now informs us that he is "not the only one to re-invent the wheel". Papers by William Lumsden and Murray Projector explore similar matters in PCAPP.   ☐

COMPETITION RESULTS

Esther Portnoy and Robert D. Hohertz are our co-champions! They solved all ten Actucrossword puzzles correctly for the last fiscal year (Sept. '83 through June '84). Christopher Doyle was next with nine correct solutions, while Sheryl Cuba, Bernard Packer and Joseph S. Raich made the list eight times. These solvers may nominate some non-member to receive a free copy of The ACTUARY through June, 1986 by sending the name and address of their nominee to your C.E. with their next 100% solution.

87 members (or their wives, children, parents, associates or friends) sent in 269 solutions. The number varied from only 12 in March (only 4 were correct—see below) to 46 in November. For the year, 4 out of 5 solutions were correct — as against 5 out of 6 for the previous year. A remarkable feature of the year's results was the fact that almost one-third of the solvers (28 of the 87) submitted only one correct solution and then were never heard from again. Does this mean they had something to prove, proved it, and then went back to work?

The most interesting problem for our solvers came from Clue 28-Down in the March puzzle which read: "Flower love in this cape (4)". With "d" as the second letter and "e" as the fourth, the correct answer was "idle". Seven of the eight solvers who didn't get 100% missed only this clue as follows: Three had "edge" with two comments — "?!!?" and "Just a guess, I haven't the foggiest notion what the word should be". Two changed the second letter to "r" and made the love "true" and "free". Another used "rose" with a comment to the effect that the puzzle seemed to be in error, while the seventh left it incomplete and commented that neither adge, edge, nor idle seemed to fit.

Our wily constructor, Graham Deas, gives the rationale as simply "love-in-idleness" which is defined by Webster as the wild pansy, and "ness" in turn is a cape. While this seems to have proved difficult, it looks fair to him and he's still rather pleased with it. This seems to suggest that we should give the rationale behind some of the more difficult clues when we give each month's solution. Something along this line has been suggested by a number of solvers and your C.E. will do so in the future to the extent that space and time will permit.