The Comparison of Group Life Benefit Schedules

ABSTRACT

A method is suggested for analysing group life insurance benefit schedules with respect to certain criteria. The procedure is illustrated by means of simple numerical examples.

by

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I. INTRODUCTION

Group life insurance often represents an important component of the typical employee benefit package. The purpose of the present paper is to explore some aspects of different benefit schedules. More specifically we consider the situation where the total amount to be spent on the group life plan has already been fixed. Benefit schedules can be constructed to meet various criteria and a number of these are examined. A method is suggested whereby an arbitrary group life benefit schedule can be evaluated in terms of these criteria. It is suggested that such considerations could be helpful in the design of benefit schedules. It is hoped that the approach developed here may have some practical value[1].

There are a number of factors that must be considered in settling the level and nature of group life benefits. As a preliminary the objective of the plan must be articulated. Perhaps the objective might be to set up a plan that will best meet employee needs for a given premium outlay. This raises the further issues of what exactly is meant by best and how employee needs are to be determined. Since there may be death benefits payable from other benefit plans such as employer pension plans and various government plans these should be considered when determining the group life benefits. Differences within the group can give rise to a number of potential problems. Married individuals may perceive the need for more life cover than single individuals. The cost of one year term insurance is considerably cheaper for females and this cost difference is often not reflected in a group life plan in the sense that both sexes may pay the same contribution rates and obtain the same level of benefits. To the extent that the plan is viewed as providing survivor benefits it may be appropriate to have the benefits consist of income

1. The present author found this type of approach helpful in analysing the group life arrangements for the Faculty of a large Canadian University.

benefits payable either for a definite period or throughout the lifetime of the survivor. In this case the question of inflation protection has to be considered. The tax treatment of premiums and benefits has to be examined as has the issue of a contributory plan versus a non-contributory plan.

A common group life benefit schedule consists of a death benefit which is a constant multiple of salary (say 1 or 2) for each plan member, alternatively the benefit may be a constant flat amount in respect of each member. Sometimes the plan provides for higher amounts of coverage for the younger members of the plan so that the benefit pattern corresponds approximately to those available under a survivor's income benefit plan. Since group life premium rates are relatively much more expensive at the older ages a plan which provides a constant multiple of salary for each member implies that the benefits in respect of older members are much more costly than those of the younger ones. This cross-subsidy may not always be relished by the younger members of the plan. Very often complaints of younger members are answered by pointing out that as they become older they in turn will be subsidised. However the younger members often experience the highest turnover rates so that only a proportion of them will remain on to obtain this subsidy. Clearly a plan that would overcome this objection would be one where the benefit level corresponded to that purchased by each member's contributions in a given year. Assuming that contributions are a fixed percentage of salary this will in general produce a schedule where the younger members are covered for higher multiples of salary than the older members. This no-subsidy plan is similar to a money-purchase pension plan in the sense that each individual receives the benefit of his own contributions. Thus the no-subsidy plan and the survivor's income plan both produce decreasing scales of benefit levels and it may be of interest to compare them.

Section II develops a model which shows how the group life benefit schedules can be calculated according to different criteria. These schedules will include those discussed in the previous paragraph. In addition Section II will also examine schedules which have some attractive theoretical properties. Section III gives a number of numerical examples which illustrate the procedures developed in Section II. The final Section provides a summary and discusses some of the issues that have not been dealt with in the paper and comments on the difficulties involved in applying an expected utility approach to this problem.

II. ANALYSIS OF DIFFERENT GROUP LIFE BENEFIT SCHEDULES.

In this Section the following assumptions are made. The group life plan consists of n members and benefits are financed by a constant percentage (100k) of each member's salary. The total contributions are denoted by C. The members are labelled in age order from 1 to n and the salary of the ith member is S_i. The expression for C is thus

$$C = \Sigma_{i=1}^{n} kS_i \qquad (1)$$

It is assumed that the one-year mortality rate for the ith member is q_i. If two members have the same age the corresponding q's are equal since all members are assumed to be male. It is also assumed that the cost of providing a one-year group-life insurance benefit of 1 unit for member i is q_i.[2] This means that premium loadings and interest adjustments are ignored. Hence the cost of providing a benefit of m times salary in respect of each member is

$$m \quad \Sigma_{i=1}^{n} \quad q_i \quad S_i \qquad (2)$$

If m is given (1) and (2) can be equated to find k. Alternatively if k is fixed then the benefit level m can be obtained from the same equation.

Since member i's contribution is kS_i his insurance coverage in the no-subsidy case is $\dfrac{kS_i}{q_i}$

2. We also assume that the cost of 1 unit of group life insurance (on a one-year term basis) is independent of the benefit schedules. In other words the premium rates do not depend on the particular benefit schedule adopted.

This corresponds to a multiple of salary of $\frac{k}{q_i}$. Interestingly enough this does not necessarily imply that the coverage expressed as a multiple of salary decreases with age. The reason is that there is a dip in the q's in many mortality tables in the 20-30 age range corresponding to accidental deaths from motor accidents and other violent causes.

In order to calculate the benefit schedule available for a total contribution of C which has the features of a survivor income benefit one must make assumptions with regard to the duration and nature of the income benefits. For the moment assume that the income benefit in respect of member i is a constant proportion of S_i. Let this constant proportion be h and assume that the present value of the income benefit payable on i's death is $hS_i\omega_i$. The proportion h can then be obtained from the equation

$$h = \frac{C}{\Sigma_{i=1}^{n} \; q_i S_i \omega_i} \tag{3}$$

An arbitrary benefit schedule which provides a multiple a_i of member i's salary on death will cost

$$\Sigma_{i=1}^{n} \; a_i q_i S_i \tag{4}$$

It has been assumed that the premium rates are fixed independent of the benefit schedule. An insurer might well be interested in the variance of the cost (to it) under the different benefit schedules . The variance of the claims paid by the insurer for the schedule which provides member i with a_i times salary is

$$\Sigma_{i=1}^{n} \; a_i^2 \; S_i^2 \; q_i(1-q_i) \tag{5}$$

In these circumstances it could be argued that the insurer would find the benefit schedule which minimised total claim variance appealing and it is of interest therefore to obtain this schedule.

Since the total cost is fixed the minimum variance schedule can be obtained using the method of Lagrange multipliers. Let

$$V = \Sigma_{i=1}^{n} a_i^2 S_i^2 q_i(1-q_i) + \lambda(C-\Sigma_{i=1}^{n} a_i q_i S_i)$$

$$\frac{\partial V}{\partial a_i} = 0 \Rightarrow 2a_i S_i^2 q_i(1-q_i) = \lambda q_i S_i$$

$$\text{or} \quad a_i = \frac{\lambda}{2S_i(1-q_i)}$$

$$\frac{\partial V}{\partial \lambda} = 0 \Rightarrow C = \Sigma_{i=1}^{n} a_i q_i S_i$$

By substituting the expression for a_i in the equation for C an equation for λ in terms of known quantities is obtained. This can be used to solve for the a_i to give

$$a_i = \frac{\dfrac{C}{S_i(1-q_i)}}{\Sigma_{i=1}^{n} \dfrac{q_i}{1-q_i}} \tag{6}$$

It may be of interest to analyse an arbitrary benefit schedule $(a_1, a_2, \cdots, a_n)$ in terms of its departure from some reference schedule labelled by the series $(a_1^*, a_2^*, \ldots a_n^*)$. In the reference schedule a_i^* denotes the multiple of salary for which individual i is covered. One way to measure the difference would be to plot a graph of the sums assured versus age for both schedules on the same axes. To characterise the differences algebraically one could construct an index composed of the weighted average of the absolute differences in death benefits at each age. It is worth pointing out that the choice of the weighting factors is subjective. For some purposes it may be meaningful to weight the differences in death benefits at each age by the corresponding premium rate per unit sum insured. On this basis the difference between the two benefit schedules is measured by

$$D(a, a^*) = \Sigma_{i=1}^{n} q_i s_i \; |a_i - a_i^*| \tag{7}$$

Recalling that

$$\Sigma_{i=1}^{n} q_i a_i S_i = \Sigma_{i=1}^{n} q_i \cdot a_i^* S_i = C$$

and assuming that the a's are non-negative it follows that

$$0 \leqslant D(a, a^*) \leqslant 2C \qquad (8)$$

(Of course the q's and the S's are assumed to be strictly positive.) This suggests the definition of a departure index DI(a, a*);

$$DI(a, a^*) = \frac{D(a, a^*)}{2C} \qquad (9)$$

On this basis one could analyse a set of group life benefit schedules in terms of their departures from a given reference schedule. As has been pointed out different criteria will give rise to different reference schedules.

III. NUMERICAL EXAMPLES

In the present Section the procedures developed in the last Section are illustrated in terms of simple numerical examples. The basic data for the examples is given in Table 1. It is assumed that the group consists of eleven members ranging in age from 25 to 60. Salaries increase until age 50 and decline thereafter with total salary roll equal to $200,000. In addition Table 1 gives the values of the q's for each age concerned.

Table 1 Details of plan membership

Age x	Number of members aged x	Salary of member aged x $	Value of q for member aged x
25	2	13000	.0010
30	2	15000	.0009
35	2	17000	.0012
40	1	20000	.001785
45	1	23000	.0035
50	1	25000	.0060
55	1	22000	.0100
60	1	20000	.0160

Let us consider first a conventional benefit schedule which provides each member with a death benefit of twice salary. Using formula (2) it is found that total contributions C for this plan will amount to \$1800 so that the benefits can be financed by a contirbution rate of .9% of member's salaries. In the sequel it is assumed that the various benefit schedules considered all cost \$1800 and that all such schedules are costed on the basis of the same premium rates for the individual ages. Using the results of Section II we can find the benefit schedule which corresponds to the no-subsidy case. These are displayed in Table 2 as is a benefit schedule corresponding to the coverage produced under an income benefit. In the income benefit case it has been assumed that each member is covered for a term certain annual income equal to a fixed proportion of his salary. The term of the payment stream is equal to 65 minus the individual's age. Thus for example the member aged 40 is covered for an amount equal to

$$(20000) \times h \times a_{\overline{25|}}$$

where $$a_{\overline{25|}} = \sum_{j=1}^{25} \frac{1}{(1+r)^j}$$

and r is the appropriate discount rate. For this set of calculations r was set equal to 3% per annum. Thus in the notation of Section II

$$\omega_i = a_{\overline{65-i|}} \tag{10}$$

Alternative schedules could be calculated under different assumptions with regard to the nature of the income benefit (for example it might correspond to a widow's pension). Having selected ω_i according to equation (10) formula (3) can be used to find h. For the present example h was found to be equal to .2015. The benefits corresponding to this value of h are also given in Table 2. With the aid of formula (6) the benefits corresponding to the minimum variance schedule can be calculated and these are also displayed in Table 2.

Table 2 Benefit patterns under different group life benefit schedules.

	TYPE OF SCHEDULE*							
Age	2 × Salary $		No Subsidy $		Income Benefit $		Minimum Variance $	
25	26000	(2)	117000	(9)	60549	(4.7)	41039	(3.2)
30	30000	(2)	150000	(10)	64945	(4.3)	41035	(2.7)
35	34000	(2)	127500	(7.5)	67142	(3.9)	41047	(2.4)
40	40000	(2)	100840	(5.0)	70175	(3.5)	41071	(2.1)
45	46000	(2)	59143	(2.6)	68950	(3.0)	41142	(1.8)
50	50000	(2)	37500	(1.5)	60138	(2.4)	41246	(1.6)
55	44000	(2)	19800	(.9)	37815	(1.7)	41412	(1.9)
60	40000	(2)	11250	(.6)	18456	(.9)	41665	(2.1)

*Figures in brackets show death benefits expressed as multiples of salary. The Income Benefit has been expressed as a lump sum.

From an inspection of Table 2 it is noted that the no-subsidy schedule gives the highest amounts of death benefits at the younger ages and the lowest amounts at the older ages relative to the other schedules. Note also that at age 30 the maximum life cover-expressed as a multiple of salary is attained. This is because the q corresponding to age 30 is smallest in the group. For the income benefit schedule the sums insured increase until age 40 and decline thereafter. It may be more revealing to examine this schedule in terms of multiples of salary. On this basis the size of the multiple of salary decreases steadily from age 25 through to 60. Thus the income benefit schedule has many of the characteristics of the no-subsidy scheduly. The interesting feature of the minimum variance schedule is that all the sums insured are nearly equal. From equation (6) the ratio of the amounts of death benefit for members i and j is

$$\frac{a_i S_i}{a_j S_j} = \frac{1-q_j}{1-q_i} \tag{11}$$

and since the q's are quite small this last ratio is nearly unity. Group life schedules where all the members are covered for the same sum insured are often found in practice and it is interesting to note that such schedules correspond closely to minimum variance schedules.

To examine the relationships between these different schedules it is convenient to construct the departure index, given by equation (9) for each pair of schedules. These indices are disolaved in matrix form in Table 3. This matrix has zeros in the leading diagonal and is symmetric since $DI(a,a^*) = DI(a^*,a)$. Using this basis any one of the four schedules can be analysed in terms of its closeness to the remaining three schedules. Thus the schedule which provides a benefit of twice salary most closely resembles the minimum variance schedule and departs furtherest from the no-subsidy schedule with the income benefit schedule lying somewhere between. The no-subsidy schedule is closest to the income benefit schedule. On the other hand the income benefit schedule

Table 3 Matrix of departure indices

	BENEFIT SCHEDULES			
Benefit Schedules	2×Salary (2S)	No-Subsidy (NS)	Income Benefit (IB)	Minimum Variance (MV)
2S	0	.4317	.2259	.0530
NS	.4317	0	.2587	.4029
IB	.2259	.2587	0	.2263
MV	.0530	.4029	.2263	0

is closest to the twice-salary schedule and furtherest from the no-subsidy schedule. In this case the values of the three departure indices are grouped closely together. An arbitrary schedule can be analysed in terms of its departures from these cour schedules. The number of reference schedules can be expanded or contracted. One possible use of the table might be to compare two benefit schedules with reference to some criterion or property. For example the two schedules under consideration might be twice-salary and the income benefit. If it is decided to select whichever of these best reflects the no-subsidy feature then the matrix indicates that the income-benefit schedule should be picked.

The variances of the claims under the various schedules can be calculated using equation (5) and the sums insured given in Table 2. These variances are displayed in Table 4. Note that since the sums insured in Table 2 have been rounded to the nearest dollar this will induce slight inaccuracies in the variance but these are of no consequence. The variance of the total claims under a

Table 4 Variances and standard deviations of claims under various group life benefit schedules

Type of Schedule	Variance of claims $	Standard deviation $
Twice salary	75 238 042	8674
No-subsidy	151 361 548	12303
Income benefit	92 161 189	9600
Minimum variance	73 796 614	8590
Level sum insured of $41393.584 per member	73 799 201	8591

fifth benefit schedule where each member is covered for the same sum insured of $41393.584 has also been calculated. As would be expected from Table 2 the variance under this schedule is only very slightly in excess of the minimum variance. Of the first three schedules in Table 4 the no-subsidy schedule has the greatest variance followed by the income-benefit schedule and then the twice-salary schedule. This of course confirms the impression gained from the fourth row of Table 3 which suggests that the twice-salary schedule lies closest to the minimum variance schedule and that the no-subsidy schedule is furtherest away from the minimum variance schedule.

IV SUMMARY AND CONCLUDING OBSERVATIONS

This paper has suggested procedures which may sometimes be helpful in examining certain aspects of group life insurance benefit schedules. The scope of the paper has been restricted to a comparison of schedules each of which has the same total cost. A method was proposed for analysing an arbitrary schedule in terms of its closeness to a particular reference schedule and the method was illustrated by means of some simple numerical examples.

It may be worthwhile to stress that the approach suggested here deals only with a few aspects of benefit schedule comparisons and design. In a practical setting there remains the important problem of selecting appropriate criteria for the design of the benefit schedule. In theory different criteria can be reflected in the construction of different reference schedules and these can be used to analyse an arbitrary schedule. The approach developed here may serve to pinpoint conflicting objectives but it does not offer any recommendations with regard to their resolution.

On the broader issue of the design of group-life benefit schedules it would seem that in practice the dominant criteria are often administrative simplicity and comparison with similar groups. It might be useful to spend more time and energy surveying the plan membership to see what type of benefit schedules they would prefer. Using this approach their preferences could be reflected at least partially in the overall plan design.

On the theoretical front it seems difficult to develop a model which would cast the group life schedule selection problem in terms of expected utility maximisation. One reason for this is that it is only under strong assumptions about each individual's utility function that one can form a group utility function [1]. If the group life plan is sponsored by the employer the employer's utility function must also be taken into account which adds an additional element of complexity to the problem. Another reason is that from the individual member's perspective the relevant states of the world are life and death. It is not intuitively obvious what constitues reasonable assumptions regarding the shape and nature of an individual's utility function corresponding to the state of death. Arrow [2] has touched on this point in a recent paper on optimal insurance and deductibles from the perspective of a single individual.

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A C K N O W L E D G E M E N T S

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