Article From:

The Actuary

SKEPSIS AVAUNT

by Ralph Garfield

One of my students recently asked me what happens to Px, the whole life premium, if the interest rate changes. The answer, of course, is that as the interest rate rises the value of Px decreases. The common sense reason is that the more interest earned by the insurance company on the investment of those premiums, the more it can pass on to the policyholders by way of a premium reduction. The student, however, wanted a mathematical proof. The following is one such proof:

Consider

$$\frac{d}{di}(Px) = \frac{d}{di}\frac{A_x}{\ddot{a}_x}$$

$$= \frac{(\ddot{a}_x)(-v(IA)_x) - (A_x)(-v(Ia)_x)}{(\ddot{a}_x)^2}$$

$$= \frac{-v}{(\ddot{a}_x)^2}\left[\ddot{a}_x(IA)_x - A_x(Ia)_x\right]$$

Examine

$$\ddot{a}_x(IA)_x - A_x(Ia)_x$$

$$= \ddot{a}_x\left[\ddot{a}_x - d(I\ddot{a})_x\right] - (1-d\ddot{a}_x)(I\ddot{a})_x + (A_x)(\ddot{a}_x)$$

$$= (\ddot{a}_x)^2 - (I\ddot{a})_x + (A_x)(\ddot{a}_x)$$

This is positive, proof is as follows:

$$(I\ddot{a})_x = \ddot{a}_x + {}_1E_x\,\ddot{a}_{x+1} + 2{}E_x\,\ddot{a}_{x+2} + 3{}E_x\,\ddot{a}_{x+3}+\cdots$$

$$< \ddot{a}_x + {}_1E_x\,\ddot{a}_x + 2{}E_x\,\ddot{a}_x + 3{}E_x\,\ddot{a}_x+\cdots$$

$$< \ddot{a}_x\left[1 + {}_1E_x + 2{}E_x + 3{}E_x + \cdots\right] = (\ddot{a}_x)^2, \text{ and}$$

because $(A_x)(\ddot{a}_x)$ is positive, it is clear that

$$(\ddot{a}_x)(IA)_x - A_x(Ia)_x$$

is positive.

Thus: $\frac{d}{di}(P_x) < 0$ i.e. as i increases Px decreases

$$Q.E.D.$$

Ed. Note: Can readers produce any simpler proof, or a convincing way to general reason the result?

OUT OF ORDER?

The May, 1984 examination results include the name Virgilio C. Ty as a successful candidate for Parts 1 and 2, *and* as both a new associate and a new fellow. Frank Di Paolo, who brought this interesting item to our attention, congratulates Mr. Ty on what appears to be a new record.

The records show that Mr. Ty never previously attempted either of Parts 1 or 2, but they throw no light on why he took the actuarial exams in such an unusual order.

Would it be "out of order" to ask our readers for other interesting cases of this same phenomenon; or even to suggest, if Mr. Ty is so inclined, that he might fill us in? □

ERRATA LIST FOR NEW ACTUARIAL MATH TEXTBOOK AVAILABLE

A 5-page Errata and Addenda for "Actuarial Mathematics", Chapters 3-10, 14, 15, dated May 1, 1984, will be sent to those requesting it by phone or letter to Society headquarters in Illinois.

DEATHS

Welburn J. Adams, F.S.A. 1936
Kenneth J. Hedley, A.S.A. 1951
Simone Matteodo, Jr., F.S.A. 1957
Ronald R. Puffer, F.S.A. 1977

JOHN P. TILLINGHAST PRIZE

In memory of John P. Tillinghast, F.S.A., the Atlanta firm of which he was a co-founder has presented the Society with a capital sum, the yield on which is to be used at the Society's discretion in the form of annual awards to writers of papers, presumably for the Transactions.

UNRIDDLING

Paul E. Angelo sent us this from Group Experience Studies, 10GB-104-79, p. 22:
 The average benefits per claim would be converted into an average benefit per dollar of daily benefit provided for each amount of daily benefit provided and an average obtained for all daily benefits provided by applying the percentage distribution of business by amount of daily benefit provided obtained from the study of claims costs.

Kent M. Bergene offered one from Life & Health Insurance Law, Merger, p. 30:
 In that case, there is no contract because the meeting of the minds is made ineffective by the nonexistence of the subject matter (a horse offered for sale) with respect to which the purported contract was made.
Mr. Bergene submitted his own 12-word solution; others are invited.

Christopher Doyle found a candidate in the Multiemployer Pension Plan Amendments Act of 1980, for which of course no blame attaches to any study note writer. It now appears in the amended ERISA, Sec. 4244 (e)(5); Mr. Doyle isn't sure what it means.
 The first year in which the plan is in reorganization is the first of a period of 1 or more consecutive plan years in which the plan has been in reorganization not taking into account any plan years the plan was in reorganization prior to any period of 3 or more consecutive plan years in which the plan was not in reorganization.

Ed. Note: Our columns are open for both (a) solutions to the above examples, and (b) other tough sentences in our syllabus that interested or anguished readers care to send to our new masthead address. □