



SOCIETY OF ACTUARIES

Article From:

The Actuary

December 1983 – Volume No. 17, Issue No. 10

MORE ON JOINT AND SURVIVOR FACTORS

by James Ramenda

In the April issue, Ralph Garfield developed a method for determining the impact of a change in interest rates on the factor which converts a life annuity to a 100% joint and survivor annuity. This factor, defined as $f(100)$, is multiplied by the benefit under the life annuity form to derive the new benefit. Thus, for a participant aged x with a beneficiary aged y , the factor is:

$$f(100) = \frac{\ddot{a}_x}{\ddot{a}_{xy}}$$

Mr. Garfield observed that by assuming y to be immortal, one could deduce that the factor is positively correlated with interest rate changes. I would like to suggest an alternative method of solving this problem which can also be generalized to other optional forms of annuities. This alternative approach utilizes the concept of duration, familiar to actuaries from the literature on immunization theory. The duration of a series of payments, C_1, C_2, \dots, C_n , is defined by:

$$D = \frac{\sum_{t=1}^n t v^t p(t) C_t}{\sum_{t=1}^n v^t p(t) C_t}$$

where $p(t)$ is the probability that the t -th payment will be realized.

Students of finance have long used this concept to estimate the percentage change in the price

$$\left(P = \sum_{t=1}^n v^t p(t) C_t \right)$$

of a bond which results from a given change in the interest rate (Δi) in the approximation:

$$\frac{\Delta P}{P} \approx -D(\Delta i)$$

This result can be derived using calculus by noting that

$$\frac{\Delta P}{\Delta i} \approx \frac{dP}{di}$$

and $1+i$ is dropped in the denominator.

This formula implies that the longer the duration of a series of payments, the more its price will move in the opposite direction of interest rates. With this in mind let us consider the following cases:

(1) Conversion from a life annuity to x to a joint and survivor annuity with beneficiary y . The factor in question is

$\ddot{a}_x / \ddot{a}_{xy}$, the denominator of which has a longer duration than that of the numerator. Therefore, the denominator will fall more, relatively, than the numerator will with an increase in interest rates. This, of course, causes the factor to increase. Obviously, the opposite is true for a decrease in interest rates.

(2) Early retirement. The factor that converts a deferred life annuity benefit with payments commencing at age x to an early retirement benefit commencing immediately (assuming retirement five years early) is

$$5 | \ddot{a}_{x-5} / \ddot{a}_{x-5}$$

In this case the numerator has the longer duration so that a result directly opposite of that in (1) above, is reached. Thus, an increase in interest rates will decrease early retirement benefits.

(3) Deferred Retirement. The factor in this case is the reciprocal of the factor in the early retirement case if one now assumes normal retirement at $x-5$ and a five year deferral period. Thus, its behavior will be inversely related to that of the early retirement factor for a given change in interest rates. That is, it will increase with an increase in interest rates.

The method outlined above is fairly safe as long as reasonable judgments can be made as to the relative durations of the annuity forms in question. Furthermore, the approximation can give some indication as to the size of the change in benefits that a change in interest rate will produce if one actually has the values of the necessary durations. These durations, as one can easily see, will typically be functions of increasing annuities. □

Research

(Continued from page 6)

actuarial ideas. My American colleagues will be interested in that.

Another ambition I must achieve before I really retire is to establish a formal unit for medico-actuarial studies. It is time that actuaries understood what medicine does; and that doctors understood what actuaries do. Doctors are now heavily engaged in cohort studies involving survival analysis; they need guidance—indeed Edward A. Lew and I have just run a workshop on the subject for the American Cancer Society in New York. Actuaries want to know more about the insurability of impaired lives—they need to know what “impaired” means. I hope our Assurance Medical Society will sponsor such a unit with a doctor and actuary working together, and that this will be soon. □

INTERNATIONAL CONGRESS NEWS

1. Word from Canada’s National Correspondent, Michael Rosenfelder, is that the nearly 30 who have registered provisionally for Sydney next October have filled that country’s quota. But others interested are welcome to be placed on the waiting list, to replace those who change their minds; just notify Mr. Rosenfelder at his Yearbook address.

2. Provisional registrants in both Canada and the U.S.A. should be aware that the deadline for final registration is January 31, 1984. You need, and should already have, instructions in Congress Announcement No. 3.

3. Those going to Sydney may like to consider a General Insurance Seminar planned at Macquarie University in that city for October 18-19. Its theme—Bridging the gap between the theory and practice of general insurance. For particulars, ask Dr. B. Zehnwith, School of Economic and Financial Studies, Macquarie University, North Ryde, N.S.W. 2113.

E.J.M.

INTERNATIONAL SUMMER SCHOOL

The Association of Swiss Actuaries has announced its third Summer School, the topic this year being Statistical Forecasting Techniques. Lecturers will be our own Prof. James C. Hickman and his colleague at University of Wisconsin, Prof. Robert A. Miller. Registration forms from Rene Held, Swiss Reinsurance Company, Box 172, 8022 Zurich.

ANOTHER INVESTMENT GAME LAUNCHED

Chicago Actuarial Club has followed the lead of Canadian Institute of Actuaries (September issue) by sponsoring an investment game among its members. Organizers are John A. Dewan and Donald S. Boger. The game will run through calendar year 1984.