A CLAIM RESERVE SYSTEM

by

William A. Bailey and Bruce E. Nickerson

ABSTRACT

A claim reserve system is described. While the system does produce the traditional type of single-point estimates of claim reserves, it also provides the user with frequency distributions of claim reserves. In developing these univariate frequency distributions of claim reserves, bivariate frequency distributions of individual claim dollars (or numbers) before and after a "split date" are convoluted for sums in order to produce *a priori* bivariate frequency distributions of aggregate claim dollars (or numbers) before and after the valuation date. Recent claim experience is then used, together with these *a priori* distributions, to produce posterior frequency distributions of claim reserves.

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GENERAL

All claim reserve systems endeavor to derive estimates of the unpaid portion of claims which have been incurred over a given period of time, making use of information on numbers of claims and amounts paid on claims as of the valuation date. In the absence of other stable and reliable predictive data (which is rarely available), the procedure relies on the information contained in the body of past claims experience, making the assumption that the claim payment patterns are comparable, period to period. Different methods of estimating claim reserves from a given claim history file depend on the strategy used to extract information from the file and the strategy used to apply the resulting information in deriving final estimates.

To some extent, the strategies employed must depend on the nature of the benefit and on the information available in the basic file. The Claim Reserve System of this paper requires that the user's basic data file include claim number, the calendar month of incurral and the amount and calendar month of each payment made on an opened claim. The benefit may entail a single payment (e.g., accidental death) or a series of payments (e.g., major medical). Since the system extracts and organizes data on the basis of monthly time intervals, it may not be sufficiently sensitive for benefits characterized by rapid reporting and payment (e.g., short term weekly disability income). For the same reason, the System may be unwieldy when the basic data includes a small number of claims with a long run-off (e.g., "permanent" claims under long-term disability income). In this latter case, it may be desirable to "close" the claim at an appropriate duration with a "payment" equal to an individually calculated claim reserve (such as the net single premium for a disabled life annuity).

Attention should be paid to what is being estimated. The system described in this manual presumes that the data file contains dollars of claim payments and payment dates. Accordingly, the estimates produced are for amounts not yet paid (whether or not accrued) on claims incurred. Some of the estimates are further divided between claims in course of payment (i.e., at least one payment has been made) and claims not yet in course of payment (reported and unreported combined).

The system can be used to produce other estimates if the content of the data file is changed. For example, if a "payment" of zero dollars (0.00) were recorded as being made when the claim is first reported, then reserve estimates for claims not yet in course of payment would become estimates for unreported claims. Similarly, if the date of accrual is substituted for the date of payment, then the estimated claim reserve will be limited to amounts not yet due on reported claims. If the date used is the later of the accrual date or the date reported, then the estimated claim reserve will also include the liability (accrued and unaccrued) for incurred but unreported claims.

Although various such manipulations are possible, consideration should also be given to alternative ways of obtaining desired subdivisions of the aggregate claim reserve. It may be satisfactory, for example, to use the estimate of the total number of claims not yet in course of payment and a count of the number of reported claims not yet in course of payment as basis for estimating the unreported portion of the reserve for claims not yet in course of payment.

COMPLETION FACTOR METHODS

From the claim data, an estimate is made of the "Completion Factor" for each duration since incurral. (The symbol is CF_t, where t = calendar months since month of incurral.) The assumption is that, for an incurral month "t" months before the valuation date,

$$CF_t \times \text{(total claims incurred)} = \text{(total claims paid to date)}.$$

Hence, the estimate of total claims incurred for the month is equal to the total claims paid to the valuation date divided by the Completion Factor. The claim reserve attributable to the incurral month is then the excess of estimated total claims over the amount paid to date.

Under this method, the estimate of total claims for an incurral month is directly proportional to the accumulated data. If applied directly for recent incurral months, where the accumulated data is small and not reliable as an indicator, minor fluctuations in the data could significantly affect the estimated reserve. The system therefore permits the user to specify a confidence level (expressed as a minimum value of CF_t) below which the accumulated data for the incurral month will not be given full credibility. When this option is selected, the estimate of total claims is based on a seasonal extrapolation of total claims based on prior incurral months with credible experience. The system also permits the user to specify the number of credible incurral years on which the extrapolation is to be based, with a default option of all years.

The only difference among the Completion Factor methods lies in the procedure for estimating values of CF_t. Each of the methods views a Completion Factor as a product of Completion Ratios. In symbols, $CF_t = \prod_{i=t}^{\omega} CR_i$ and $CR_t = CF_t/CF_{t+1}$. The Completion Ratio for duration t is thus the ratio of claims paid from incurral through duration t (symbolized C_t) to claims paid from incurral through duration t+1 (C_{t+1}).

In the following descriptions of the Completion Factor methods, a subscript i (e.g., $CR_{t,i}$) identifies the value as based on data from incurral month i.

Completion Factor Method 1

An aggregate Completion Ratio is calculated for each duration t, using the total claims paid through duration t and through duration t+1 for all incurral months at least t+1 months before the valuation:

$$CR_t = \Sigma\, C_{t,i} \div \Sigma\, C_{t+1,i}$$

The Completion Factors are then derived recursively:

$$CF_t = CF_{t+1} \times CR_t$$

Completion Factor Method 2

A Completion Ratio is calculated for each combination of duration t and incurral month i for which data is available:

$$CR_{t,i} = C_{t,i} \div C_{t+1,i}$$

Each of these Completion Ratios is assumed to be an equally probable random observation, and the aggregate Completion Ratio for duration t is assumed to be the average of the observed values:

$CR_t = \frac{1}{n} \sum_i CR_{t,i}$, where n is the number of observations.

Finally, the Completion Factors are derived recursively:

$$CR_t = CF_{t+1} \times CR_t$$

Completion Factor Method 3

A Completion Factor is calculated (or estimated) for each combination of duration t and incurred month i. In the case of incurral months for which complete cumulative claims data are available, this calculation is simply:

$$CF_{t,i} = C_{t,i} \div C_{\omega+1,i}$$

For more recent incurral months, however, it is necessary to just extrapolate beyond the experience period to obtain $C_{\omega+1,i}$ (see below).

Each set of Completion Factors for an incurral month is assumed to be an equally probable random observation. (The use of Completion Factors, rather than Ratios, means that independence is not assumed for successive durations within an incurral month.) Noting that the aggregate Completion Factors will be used as divisors in making the final estimates, the reciprocal of the aggregate Completion Factor for duration t is assumed to be the average of the reciprocals of the duration t Completion Factors for the individual incurral months:

$$1/CF_t = \frac{1}{n} \sum_i (1/CF_{t,i})$$

For the incomplete incurral months, Completion Ratios are calculated, as described under Completion Factor Method 2, at each duration t for which data is available at duration t+1 (i.e. where $C_{t+1,i}$ is available); namely,

$$CR_{t,i} = C_{t,i} \div C_{t+1,i}$$

The extrapolation for incurral months with incomplete experience is done recursively as follows, where we are assuming that complete experience (values of $C_{\omega+1,i}$) is available for incurral months 1 through m, so that incurral month m+1 will have calculated Completion Ratios through duration ω-1, incurral month m+2 will have calculated Completion Ratios through duration ω-2, etc.

The formulas given below may be easier to understand if reference is made to the following two tables.

Table of $C_{t,i}$

t/i =	1	2	...	m	m+1	m+2	...	n
0	$C_{0,1}$	$C_{0,2}$	...	$C_{0,m}$	$C_{0,m+1}$	$C_{0,m+2}$	...	$C_{0,n}$
1	$C_{1,1}$	$C_{1,2}$	...	$C_{1,m}$	$C_{1,m+1}$	$C_{1,m+2}$	...	$C_{1,n}$
2	$C_{2,1}$	$C_{2,2}$	...	$C_{2,m}$	$C_{2,m+1}$	$C_{2,m+2}$	...	$C_{2,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$		$\vdots$
$\omega-2$	$C_{\omega-2,1}$	$C_{\omega-2,2}$	...	$C_{\omega-2,m}$	$C_{\omega-2,m+1}$	$C_{\omega-2,m+2}$	...	$C_{\omega-2,n}$
$\omega-1$	$C_{\omega-1,1}$	$C_{\omega-1,2}$	...	$C_{\omega-1,m}$	$C_{\omega-1,m+1}$	$C_{\omega-1,m+2}$	...	
ω	$C_{\omega,1}$	$C_{\omega,2}$	...	$C_{\omega,m}$	$C_{\omega,m+1}$			
$\omega+1$	$C_{\omega+1,1}$	$C_{\omega+1,2}$	...	$C_{\omega+1,m}$				

Table of $CF_{t,i}$

t/i =	1	2	...	m	m+1	m+2	...	n
0	$CF_{0,1}$	$CF_{0,2}$	...	$CF_{0,m}$	$\hat{CF}_{0,m+1}$	$\hat{CF}_{0,m+2}$	...	$\hat{CF}_{0,n}$
1	$CF_{1,1}$	$CF_{1,2}$	...	$CF_{1,m}$	$\hat{CF}_{1,m+1}$	$\hat{CF}_{1,m+2}$	...	$\hat{CF}_{1,n}$
2	$CF_{2,1}$	$CF_{2,2}$	...	$CF_{2,m}$	$\hat{CF}_{2,m+1}$	$\hat{CF}_{2,m+2}$	...	$\hat{CF}_{2,n}$
⋮	⋮	⋮		⋮	⋮	⋮		⋮
ω−2	$CF_{\omega-2,1}$	$CF_{\omega-2,2}$	...	$CF_{\omega-2,m}$	$\hat{CF}_{\omega-2,m+1}$	$\hat{CF}_{\omega-2,m+2}$	...	$\hat{CF}_{\omega-2,n}$
ω−1	$CF_{\omega-1,1}$	$CF_{\omega-1,2}$	...	$CF_{\omega-1,m}$	$\hat{CF}_{\omega-1,m+1}$	$\hat{CF}_{\omega-1,m+2}$	...	$\hat{CF}_{\omega-1,n}$
ω	$CF_{\omega,1}$	$CF_{\omega,2}$	...	$CF_{\omega,m}$	$\hat{CF}_{\omega,m+1}$	$\hat{CF}_{\omega,m+2}$	...	$\hat{CF}_{\omega,n}$

$\hat{CF}_{t,i}$ denotes estimated values of $CF_{t,i}$ where the values of $CF_{t,i}$ are unknown because claims for the i^{th} incurral month are incomplete.

We calculate $\hat{CF}_{\omega,m+1} = \dfrac{1}{\frac{1}{m}\sum_{t=1}^{m}\frac{1}{CF_{\omega,t}}}$.

Then $\hat{CF}_{t,m+1}$ for $t-1 = \omega-1,\ldots,0$ is calculated recursively by $\hat{CF}_{t-1,m+1} = \hat{CF}_{t,m+1} \cdot CR_{t-1,m+1}$ that is, the other values of $\hat{CF}$ higher up in the $m+1^{st}$ column can be filled in, once $\hat{CF}_{\omega,m+1}$ is available.

In general, we have $\hat{CF}_{t,s} = \hat{CF}_{t,m+1+\omega-t}$ for $s > m+1+\omega-t$; i.e., once $\hat{CF}_{t,m+1+\omega-t}$ has been calculated, each other $\hat{CF}$ (to the right) in that row is assumed to be equal to $\hat{CF}_{t,m+1+\omega-t}$.

Considering column $m+2$ we calculate

$$\hat{CF}_{\omega-1,m+2} = \frac{1}{\left[\frac{1}{m+1}\left(\sum_{t=1}^{m} \frac{1}{CF_{\omega-1,t}} + \sum_{t=m+1}^{m+1} \frac{1}{\hat{CF}_{\omega-1,t}}\right)\right]}$$

Then $\hat{CF}_{t-1,m+2} = \hat{CF}_{t,m+2} \cdot CR_{t-1,m+2}$ for $t-1 = \omega-2,\ldots,0$;

that is, the other values of CF (above) in the $m+2$nd column can be filled in, once $\hat{CF}_{\omega-1,m+2}$ has been estimated.

Considering column $m+3$ we calculate

$$\hat{CF}_{\omega-2,m+3} = \frac{1}{\left[\frac{1}{m+2}\left(\sum_{t=1}^{m} \frac{1}{CF_{\omega-2,t}} + \sum_{t=m+1}^{m+2} \frac{1}{\hat{CF}_{\omega-2,t}}\right)\right]}$$

Then, $\hat{CF}_{t-2,m+3} = \hat{CF}_{t-1,m+3} \cdot CR_{t-2,m+3}$ for $t-1 = \omega-3,\ldots,0$;

that is the other values of $\hat{CF}$ (above) in the $m+3$nd column can be filled in, once $\hat{CF}_{\omega-2,m+3}$ has been calculated.

We continue in this fashion, until $\hat{CF}_{t,i}$ has been calculated so as to fill in the Table of $CF_{t,i}$ for $i = 1,2,\ldots,m$ and $\hat{CF}_{t,i}$ for $i = m+1, m+2,\ldots,n$, with $t = 0,1,\ldots,\omega$.

The above completion factor methods can be refined by determining $IPCF_t$ and $NIPCF_t$, where the assumption is that, for an incurral month "t" months before the valuation date,

$IPCF_t$ x (total claims incurred on claims In-Payment as of the valuation date)

= total claims paid prior to the valuation date.

and

$NIPCF_t$ x (total claims incurred on claims Not-In-Payment as of the valuation date)

= total claims paid prior to the valuation date.

(this is semi-artificial)

Thus,

$$\text{Total incurral claims} = \frac{\text{(total claims paid prior to the valuation date)}}{CF_t}$$

$$= \frac{\left(\begin{array}{l}\text{total claim paid prior}\\ \text{to the valuation date}\end{array}\right)}{IPCF_t} + \frac{\left(\begin{array}{l}\text{total claim paid prior}\\ \text{to the valuation date}\end{array}\right)}{NIPCF_t}$$

and

$$CF_t = \frac{1}{\left(\frac{1}{IPCF_t} + \frac{1}{NIPCF_t}\right)}$$

NONCREDIBLE MONTHS

Some recent months may be handled differently because the value of $C_{t,i}$ may be too small to be considered to be a reliable predictor. For these months, two options are available:

(a) a least squares estimate based on estimated ultimate aggregate dollars of claims for each credible incurral month; and

(b) a seasonal adjustment which estimates the ultimate aggregate dollars of claims for a particular current month of incurral by calculating the average of the ratios of (i) to (ii), where

(i) = the currently estimated ultimate of dollars of claims in the same month of a prior fiscal year, and

(ii) = the aggregate currently estimated dollars of claims for the preceding "credible" months in that prior fiscal year,

and applying that average to the aggregate of the currently estimated dollars of claims for the credible months in the current fiscal year.

To see option (a) in more detail, temporarily designate the current estimates of ultimate claims for credible incurral months by x_i ($i = i_0; i_1, \ldots, i_n$) where each of the i's represents a lag month and i_n is the maximum lag month for which data is available. The linear least squares estimate uses values $a = \hat{a}$ and $b = \hat{b}$ which minimize the value of

$$\sum_{i=i_0}^{i_n} (a \cdot x_i + b - \overline{x})^2$$

where $\overline{x} = \frac{1}{i_n} \cdot \sum_{i=i_0}^{i_n} x_i$ is the average (mean) of the individual observations.

Having a_0 and b_0, we can calculate $a_0 \cdot x_i + b_0$ for $i = i_{-1}, i_{-2}, \ldots$; that is, estimated ultimate claims for the noncredible lag months $i_{-1}, i_{-2}, \ldots$.

As an example of the seasonal adjustment [option (b)], suppose the current valuation date is 12-31-78, the currently estimated ultimate dollars of claims (temporarily designated by $c_1, c_2, \ldots, c_9$) for January thru September of 1978 are deemed to be credible, and we are estimating the ultimate dollars of claims (temporarily designated by c_{10}, c_{11}, c_{12}) for October thru December, of 1978, and the currently estimated ultimate dollars of claims are temporarily designated by $c_1(-t), c_2(-t), \ldots, c_{12}(-t)$ for the t^{th} year prior to 1978. Then,

$$c_{10} = \sum_{j=1}^{9} c_j \cdot \frac{1}{n} \cdot \sum_{t=1}^{n} \frac{c_{10}(-t)}{\sum_{i=1}^{9} c_i(-t)}$$

$$c_{11} = \sum_{j=1}^{9} c_j \cdot \frac{1}{n} \cdot \sum_{t=1}^{n} \frac{c_{11}(-t)}{\sum_{i=1}^{9} c_i(-t)}$$

$$c_{12} = \sum_{j=1}^{9} c_j \cdot \frac{1}{n} \cdot \sum_{t=1}^{n} \frac{c_{12}(-t)}{\sum_{i=1}^{9} c_i(-t)}$$

where n = a whole number of prior years experience available.

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FREQUENCY DISTRIBUTION METHODS

The Frequency Distribution methods endeavor to estimate the probability that a given level of claim reserves will prove to be inadequate, adequate or excessive. Numerical convolution techniques are used to derive a representative of the frequency distribution of amounts not yet paid on incurred claims. From this distribution, percentiles, means, variances and other statistics are available, and probabilities can be estimated as to the adequacy of a given level of claim reserve.

The focus of these methods is on estimating the frequency distribution of the aggregate claim reserve. If the reserves are adequate in the aggregate, then the determination of the components (incurred but not reported, etc.) can be viewed as an allocation problem. A satisfactory allocation could then be made using any of a number of approximation techniques.

Distributions of certain components of the total claim reserve are also derived as intermediate steps, and are available for analysis. It should be noted, however, that the mean values are the only values obtained from the component distributions which may be added to produce the corresponding value in the distribution of the aggregate claim reserve. For example, the aggregate claim reserve with a 90% chance of being adequate will normally be significantly smaller than the sum of claim reserve components each of which has a 90% likelihood of adequacy.

CLAIM RESERVE SYSTEM - INDIVIDUAL RISK APPROACH

Approach and Theory

The starting point for this frequency distribution method is the recognition that the future payments to be made on each individual claim is a variable with an (unknown) frequency distribution. When the historical claim experience is analyzed on a claim-by-claim basis, however, this experience constitutes an observed frequency distribution of payments on individual claims. Subject to the basic assumption that future claim payment patterns will be comparable to those experienced in the past, the observed distributions may be used as representations or estimates of the theoretical individual claim frequency distributions.

Therefore, given a count of the number of individual claims which have been opened, a reasonable estimate of the number of claims not yet opened, and an appropriate frequency distribution for the amounts of future payments to be made on each category of claim, the frequency distribution of aggregate future payments for all claims can (in theory) be generated by a straightforward application of probability theory. Given the number of possible "amounts" for each single claim (even in a discrete representation of what is theoretically a near-continuous distribution) and a reasonable number of individual claims, however, this theoretical approach rapidly becomes infeasible because of the number of calculations required. Even a "Monte-Carlo" sampling approach to estimating the distribution of aggregate reserves normally entails too many calculations to be considered for routine claim reserve estimation.

The Claim Reserve System in this paper uses numerical convolution techniques to generate discrete representations of the desired aggregate distributions. These techniques assure that the resulting distributions have the theoretically correct mean and variance at all stages, while permitting the number of calculations required to be controlled and limited to a practical level. The final discrete distributions are then converted to continuous distributions from which probability estimates of the adequacy of various levels of reserves can be drawn.

The numerical convolution techniques themselves have general application to a wide variety of probability and statistical problems. A paper entitled

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Numerical Convolutions will appear in the Skandinavian Actuarial Journal in the near future. The discussion below is limited to the application of these techniques in deriving representations of claim reserve distributions.

Distributions for Individual Claims

The basic assumption used in this frequency distribution method is that the pattern* of claim payment for any given calendar month of claim payment will be the same as the pattern* for the same calendar month in the prior calendar year. This association of experience for corresponding calendar months is intended to avoid the masking of seasonal variations, which can be acute in some types of coverages (e.g., comprehensive medical with a calendar year benefit period) and can be significant in many other types. Thus, the pattern of future claim payments for claims incurred in October of a given year is assumed to follow the pattern of the previous October, rather than a pattern derived by blending the experience of an entire prior year.

There may be a liability for future payments on incurred claims in course of payment and there may be a liability for incurred claims not yet in course of payment. For each incomplete incurral month, the experience of the same calendar month in the preceding calendar year is analyzed, claim-by-claim, to produce a bivariate frequency distribution of amounts (x) paid before and amounts (y) paid after the corresponding valuation date in the preceding year. This distribution is then divided into a distribution (A_1) consisting of all claims with $x > 0$ and a second distribution (A_2) consisting of all claims with $x = 0$. Each pair, x and y, in A_1 is an observed pattern for a claim in course of payment and each value in A_2 is an observation for a claim not yet in course of payment.

Example: The valuation date is 12/31/77. February 1977 is an incomplete incurral month for which a claim reserve must be established.

* in a sense to be made more precise in the following pages.

The basic assumption is that future payments (after 12/31/77) on 2/77 incurrals will be similar to payments after 12/31/76 on 2/76 incurrals. A_1 is a bivariate distribution, derived from the February 1976 experience, of amounts paid before and after 12/31/76 on claims in course of payment as of 12/31/76. A_2 is a corresponding distribution of amounts paid on claims where the first payment was after 12/31/76.

In creating the basic distributions, A_1 and A_2, each observed claim is given equal weight. In other words, no prior assumption is made as to the relative likelihood of any particular observation. Each claim actually occurred, and is treated as being as credible an estimator as any other observation. While a particular observation may deviate substantially from the range in which most of the observations lie, and could even be deemed to "distort" the frequency distribution of reserves attributable to a particular incurral month, the user should recall that the primary objective of the system is to estimate the frequency distribution of the aggregate claim reserve for all open incurral months combined. Failure to appropriately recognize that "exceptional" claims do arise from time to time can affect the mean and, particularly, the variance of the final aggregate distributions.

Some of the A_1 and A_2 distributions would be based on experience which is incomplete. For a description of how the system estimates the remaining future payments on these claims, see the Appendix.

Frequency Distributions of Claim Reserves for In-Payment Claims

There is the usual difficulty, inherent in all claim reserve systems, of the interpretation of the recent very incomplete months of incurral in that relatively minor variances in the total-claims-paid-to-date figure are blown up out of proportion; that is, the emerging experience is taken as being proportional to the final ultimate experience, and the factors of proportionality in this area are large. So, special methods are applied in these cases (the so-called "noncredible" or "starred" months of incurral).

For each open (incomplete) month two reserve distributions are derived, one for In-Payment claims and another for Not-In-Payment claims. The resulting distributions (referred to as C_1 and C_2, respectively) are convoluted together for sums to produce the final reserve distributions. The reserve distributions for In-Payment claims are constructed differently from the reserve distributions for Not-In-Payment claims. An option is provided in the case of the Not-In-Payment claims to handle the very incomplete incurral months (i.e. those for which the CF is less than ε) differently from the earlier more complete incurral months.

Given the A_1 distribution, which we trend by comparing average claim sizes in successive 12-month-apart incurral months to produce a "trended" A_1 distribution, designated T_1, and given the fact that n In-Payment claims have occurred we can compute an a priori bivariate distribution of aggregate claims (x) paid prior to, and aggregate claims (y) paid after, a split date; namely, $B_1 = T_1^{\oplus n}$. Given further that \$d of claims have been paid as of the valuation date, we can, from each pair of a priori claim pairs (x,y), derive a new estimate of outstanding claims by computing $d \cdot \left(\frac{y}{x}\right)$; see footnote*. Associating the probability of each pair (x,y) in B_1 with the corresponding resulting $d \cdot \left(\frac{y}{x}\right)$, we have the final distribution of In-Payment claim reserves, denoted by C_1.

* Optionally, you can use only those ratios $\frac{y}{x}$ for which $d - j < x < d + j$, where j is a (dollar) parameter you pick; in this case, the probabilities must be "normalized" to sum to unity.

Frequency Distributions of Claim Reserves for Not-In-Payment Claims

Not-In-Payment reserves require a more elaborate approach, since (contrary to what is the case for In-Payment reserves)

(1) the number of Not-In-Payment claims is unknown, and

(2) there have been no payments yet made on these claims (i.e. there is no \$d figure available). Consequently, we derive

(a) from comparisons of claims in successive 12-month-apart incurral months, trend factors which are applied to "trend" A_2 to produce T_2,

and

(b) from actual numbers of claims for each credible month of incurral, a bivariate distribution (N_ℓ) of

(1) number (x) of claims on which some payment was made prior to lag month ℓ, and

(2) number (y) of claims on which the first payment was made after lag month ℓ,

attaching equal probabilities to each of these number pairs (x,y).

Then, for each possible number (y) of Not-In-Payment claims (appearing in the bivariate distribution N_ℓ) we convolute T_2 that number of times, thus deriving a fragment of an absolute dollar distribution of Not-In-Payment claims. (Note that, unlike In-Payment claims, the x values in T_2 are zero and there is no \$d available.) The probabilities in each such fragment are weighted (multiplied) by the probability associated with that possible number of Not-In-Payment claims, and the resulting fragments merged to form the final Not-In-Payment claim dollar distribution.

Frequency Distributions of Claim Reserves

For each incurral month the resulting distributions are convoluted together, to derive distributions of claim reserves combining In-Payment and Not-In-Payment claim reserves for that incurral month.

Finally, the distributions of claim reserves for the various months of incurral are convoluted together, to derive the overall claim reserve distribution.

For more details, see Appendix.

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APPENDIX

In what follows, the "split date" is the date exactly one year prior to the valuation date.

<u>Frequency Distributions of Claim Reserves for In-Payment Claims</u>

Let: β be a given current incurral month

n be the number of In-Payment claims (claims in course of payment as of the valuation date), for current incurral month β

m be number of claims in the historical month (12 months before current month β) which have a first payment before the split date.

A_1 be the In-Payment die $\left[x_i, y_i, p_i\right]$ for the incurral month 12 months before β, where

x_i = total payments before the split date, for the i^{th} In-Payment claim
y_i = total payments after the split date, for the i^{th} In-Payment claim
p_i = probability associated with the number pair (x_i, y_i)
($p_i = \frac{1}{m_1}$ if claim is considered to be complete by the valuation date; for claims still considered to be incomplete as of the valuation date, see Subappendix #1

A_1 is "trended" to produce T_1; see Subappendix #2

d be actual aggregate dollars of claims paid prior to the valuation date, for current incurral month β.

B_1 be the a priori bivariate frequency distribution $\left[x_i, y_i, p_i\right]$ of In-Payment claims dollars for month of incurral β; more specifically, given the distribution T_1 and the number n,

x_i = aggregate dollars of claims which might have been paid prior to the split date, for claims incurred in the month 12 months before β.

y_i = aggregate dollars of claims which might have been paid after the split date, for claims incurred in the month 12 months before β, corresponding to x_i.

p_i = probability of the occurrence of the number pair (x_i, y_i)

Frequency Distributions of Claim Reserves for In-Payment Claims (continued)

C_1 be the frequency distribution of claim reserves for In-Payment claims, for current incurral month β.

Then: $B_1 = T_1^{\oplus n_1} = \underbrace{T_1 \oplus \cdots \oplus T_1}_{T_1 \text{ appears } n_1 \text{ times}}$ (i.e. n_1-1 convolutions)

$C = [d \cdot y \div x, 0, p] B_1$ (See footnotes (a) and (b).)

The following numerical example illustrates these operations:

Let: $T_1 = \begin{bmatrix} 1 & 2 & .3 \\ 3 & 4 & .7 \end{bmatrix}$

$n = 2$

$d = 1000$

Then:

$$B_1 = \begin{bmatrix} 1 & 2 & .3 \\ 3 & 4 & .7 \end{bmatrix}^{\oplus 2} = \begin{bmatrix} 1 & 2 & .3 \\ 3 & 4 & .7 \end{bmatrix} \oplus \begin{bmatrix} 1 & 2 & .3 \\ 3 & 4 & .7 \end{bmatrix} = \begin{bmatrix} 1+1 & 2+2 & .3 \text{ x} .3 \\ 1+1 & 2+4 & .3 \text{ x} .7 \\ 3+1 & 4+2 & .7 \text{ x} .3 \\ 3+3 & 4+4 & .7 \text{ x} .7 \end{bmatrix} = \begin{bmatrix} 2 & 4 & .09 \\ 4 & 6 & .21 \\ 4 & 6 & .21 \\ 6 & 8 & \underline{.49} \\ & & 1.00 \end{bmatrix} = \begin{bmatrix} 2 & 4 & .09 \\ 4 & 6 & .42 \\ 6 & 8 & \underline{.49} \\ & & 1.00 \end{bmatrix}$$

$$C_1 = [1000 \cdot y \div x, 0, p] \begin{bmatrix} 2 & 4 & .09 \\ 4 & 6 & .42 \\ 6 & 8 & \underline{.49} \\ & & 1.00 \end{bmatrix} = \begin{bmatrix} 1000 & 4 \cdot 2 \div .09 \\ 1000 & 6 \cdot 4 \div .42 \\ 1000 & 8 \cdot 6 \div .49 \end{bmatrix} = \begin{bmatrix} 2000 & 0 & .09 \\ 1500 & 0 & .42 \\ 1333 & 0 & \underline{.49} \\ & & 1.00 \end{bmatrix}$$

Footnote (a): the symbolism [, ,]B in this manual does not mean matrix multiplication; rather it indicates that certain operations are to be performed on the columns of the succeeding matrix, element by element. x refers to elements in the first column, y to elements in the second column and p to elements in the third. The notation $[d \cdot y \div x, 0, p]B$ indicates the replacement of each element of the first column by the result of d times the corresponding element of the second column divided by the original element of the first column, the replacement of each element of the second column by zero and the third column by itself.

Footnote (b): optionally, you can request that the system use only those ratios $\frac{y}{x}$ for which $d - j < x < d + j$, where j is a (dollar) parameter you pick; in this case, the probabilities must be "normalized" to sum to unity.

Frequency Distributions of Claim Reserves for In-Payment Claims (continued)

Thus, the frequency distributions (C_1) of claim reserves for In-Payment claims, for current incurral month β, is obtained by performing the following steps:

1. Construct the bivariate distribution T_1, by "trending" A_1.

2. Convolute T_1 for sums n-1 times, to produce the bivariate frequency distribution B_1

3. Transform B_1 into the univariate frequency distribution C_1, by multiplying the second column of B, element by element, by d and dividing by the corresponding element in the first column.

APPENDIX

Frequency Distributions of Claim Reserves for Not-In-Payment Claims

Let: β be a given current incurral month.

ℓ be the lag month, which is the number of completed months between β and the valuation date.

n be the number of In-Payment claims, for current incurral incurral month β.

m be number of claims in historical month (12 months before current month β) which have a first payment after the split date.

A_2 be the Not-In-Payment die $\left[x_i, y_i, p_i\right]$ for the incurral month 12 months before β, where

$x_i' = 0$

y_i = total payments after the split date, for the i^{th} Not-In-Payment claim

p_i = probability associated with the number pair $(0, y_i)$

($p_i = \frac{1}{m}$ if claim is considered to be complete by the valuation date; for claims still considered to be incomplete as of the valuation date, see Subappendix #1.

A_2 is "trended" to produce T_2; see Subappendix #2

N_ℓ be the die $\left[x_i, y_i, p_i;\ 1 \leqslant i\right]$ associated with lag month ℓ, where

x_i = number of claims incurred in the i^{th} historical incurral month (the earliest historical incurral month is associated with i = 1) and which have a first payment not later than lag month ℓ.

Frequency Distributions of Claim Reserves for Not-In-Payment Claims (continued)

y_i = number of claims incurred in the i^{th} (complete) historical incurral month which have a first payment after lag month ℓ; for handling of claims still Not-In-Payment as of the valuation date, see Subappendix #3.

Note that this die includes data other than for just a single incurral month.

p_i = probability associated with the number pair (x_i, y_i); this probability gives equal weight to each historical month of incurral.

C_2 be the frequency distribution of claim reserves for Not-In-Payment claims, for current incurral month β.

B_2 be the a priori bivariate frequency distribution $[0, y_i, p_i]$ of Not-In-Payment claim dollars for month of incurral β, given the distribution A_2 and the claim lag distribution N_ℓ (where ℓ is the proper lag for β); then

y_i = aggregate dollars of claims which might have been paid after the split date for claims which might have been incurred in the month 12 months before and have had a first payment date after the split date.

p_i = probability associated with the number pair $(0, y_i)$

Then: $$B = \underset{j=1}{\overset{\infty}{Ⓜ}} \left\{ [0, y, p_j \cdot p] \; T_2^{\oplus(n \cdot y_j \div x_j)} \right\}$$

where x_j, y_j and p_j are from N, and the expression $n \cdot y_j \div x_j$ is rounded to a whole number.

C_2 = $[y, 0, p]B_2$; i.e. the a priori distribution (B_2) becomes the a posteriori distribution (C_2) since there is no further relevant data.

Frequency Distributions of Claim Reserves for Not-In-Payment Claims (continued)

The following numerical example illustrates this operation:

Let $N = \begin{bmatrix} 1 & 2 & .4 \\ 3 & 4 & .6 \end{bmatrix}$ and $T_2 = \begin{bmatrix} 0 & 5 & .2 \\ 0 & 6 & .8 \end{bmatrix}$ and $n = 1$

$$\text{Then } B_2 = \underset{j=1}{\overset{2}{Ⓜ}} \left\langle \left[x,y,p_j \cdot p\right] \quad T_2^{\oplus(1 \cdot y_j \div x_j)} \right\rangle$$

$$= \left(\left[x,y,.4 \cdot p\right] \begin{bmatrix} 0 & 5 & .2 \\ 0 & 6 & .8 \end{bmatrix}^{\oplus(1 \cdot 2 \div 1)} \right) Ⓜ \left(\left[x,y,.6 \cdot p\right] \begin{bmatrix} 0 & 5 & .2 \\ 0 & 6 & .8 \end{bmatrix}^{\oplus(1 \cdot 4 \div 3)} \right)$$

$$= \left(\left[x,y,.4 \cdot p\right] \begin{bmatrix} 0 & 5 & .2 \\ 0 & 6 & .8 \end{bmatrix}^{\oplus 2} \right) Ⓜ \left(\left[x,y,.6 \cdot p\right] \begin{bmatrix} 0 & 5 & .2 \\ 0 & 6 & .8 \end{bmatrix}^{\oplus 1} \right)$$

$$= \left(\left[x,y,.4 \cdot p\right] \begin{bmatrix} 0 & 10 & .04 \\ 0 & 11 & .32 \\ 0 & 12 & .64 \end{bmatrix} \right) Ⓜ \left(\left[x,y,.6 \cdot p\right] \begin{bmatrix} 0 & 5 & .2 \\ 0 & 6 & .8 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 & 10 & .016 \\ 0 & 11 & .128 \\ 0 & 12 & .256 \end{bmatrix} Ⓜ \begin{bmatrix} 0 & 5 & .120 \\ 0 & 6 & .480 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 10 & .016 \\ 0 & 11 & .128 \\ 0 & 12 & .256 \\ 0 & 5 & .120 \\ 0 & 6 & .480 \end{bmatrix} = \begin{bmatrix} 0 & 5 & .120 \\ 0 & 6 & .480 \\ 0 & 10 & .016 \\ 0 & 11 & .128 \\ 0 & 12 & \underline{.256} \\ & & 1.000 \end{bmatrix}$$

$$C_2 = \left[y,0,p\right] B_2$$

$$= \left[y,0,p\right] \begin{bmatrix} 0 & 5 & .120 \\ 0 & 6 & .480 \\ 0 & 10 & .016 \\ 0 & 11 & .128 \\ 0 & 12 & .256 \end{bmatrix} = \begin{bmatrix} 5 & 0 & .120 \\ 6 & 0 & .480 \\ 10 & 0 & .016 \\ 11 & 0 & .128 \\ 12 & 0 & .256 \end{bmatrix}$$

Frequency Distributions of Claim Reserves for Not-In-Payment Claims (continued)

Thus, the frequency distribution (C_2) of claim reserves for Not-In-Payment claims, for current incurral month β, is obtained by performing the following steps:

1. Construct the univariate distribution T_2, designated $\left[0,y_i,p_i\right]$, by "trending" A
2. Construct the bivariate distribution N_ℓ, designated $\left[x_j,y_j,p_j\right]$.
3. For each row j of N_ℓ, perform the following steps:
   (a) convolute T_2 for sums $(n \cdot y_j \div x_j)-1$ times;
   (b) transform the resulting univariate distribution by multiplying each of the probabilities by p_j. The result is a fragment of a distribution.
4. Merge the univariate distribution fragments from step 3, to form the univariate distribution B_2
5. Transform B_2 into univariate distribution C_2, by substituting $(y_i,0)$ for $(0,y_i)$ in each row of B_2.

Noncredible current months of incurral (the starred months) are handled differently, because the value of n is so small as to be considered not a reliable predictor for these months. For these months the single point estimate #3 for number of Not-In-Payment claims is used as an estimate of the mean value of the frequency distribution of number of Not-In-Payment claims. A frequency distribution of number of Not-In-Payment claims for each lag month ℓ (where ℓ corresponds to a particular starred month) is built by multiplying this estimated mean value figure by the ratios of $\dfrac{x_i + y_i}{x + y}$ taken from the frequency distributions

$N_\ell = \left[x_i,y_i,p_i\right]$ for each value of $\ell \geq t$, where t = the most recent credible (nonstarred) lag month.

Frequency Distributions of Claim Reserves Combining In-Payment and Not-In-Payment Claims

For each current incurral month β

$$D = C_1 \oplus C_2$$

is the frequency distribution $\left[x_i, 0, p_i\right]$ of aggregate claim reserves.

Frequency Distributions of Claim Reserves Combining All Months of Incurral

$D_1 = \bigoplus_{\mathrm{all}_\beta} {}_\beta C_1$ is the frequency distribution $\left[x_i, 0, p_i\right]$ of In-Payment claim reserves for all current months of incurral combined.

$D_2 = \bigoplus_{\mathrm{all}_\beta} {}_\beta C_2$ is the frequency distribution $\left[x_i, 0, p_i\right]$ of Not-In-Payment reserves for all current months of incurral combined.

$D = D_1 \oplus D_2 = \bigoplus_{\mathrm{all}_\beta} {}_\beta D$ is the frequency distribution $\left[x_i, 0, p_i\right]$ of claim reserves for all current months of incurral combined.

Listings of any of the frequency distributions of claim reserves can be obtained by running the program LISTDICE.

SUBAPPENDIX #1

Adjustment of Dollar Dice for Incomplete Historical Months

Fix an incomplete historical incurral month.

The split date is the date exactly one year prior to the valuation date.

For the i^{th} claim let

x_i = dollars paid before split date

y_i = dollars paid after split date but before the valuation date, if $x_i \neq 0$ (i.e. claim was In-Payment as of split date); and $y_i = 0$ if $x_i = 0$.

z_i = dollars paid after split date but before the valuation date, if $x_i = 0$ (i.e. claim was Not-In-Payment as of split date); and $z_i = 0$ if $x_i \neq 0$.

Also, let

n_1 = number of claims with $x \neq 0$ (In-Payment claims)

n_2 = number of claims with $x = 0$ (known* Not-In-Payment claims)

IPCF\$ = the Method 3 completion factor for determining future claim dollars on claims already in course of payment as of the valuation date.

NIPCF\$ = the Method 3 completion factor for determining future claim dollars on claims not yet in course of payment as of the valuation date.

CF# = Method 3 completion factor for number of claims, as of the valuation date.

s = total dollars paid = $\Sigma(x_i + y_i + z_i)$

n = total number of eventual claims

u = number of unknown* claims

k = total dollars on known* claims

t = total dollars on unknown* claims

Estimate n by $(n_1 + n_2) \div$ CF#

Estimate u by $n - n_1 - n_2$

Estimate k by $\dfrac{\sum_i (x_i + y_i + z_i)}{\text{IPCF\$}}$

* i.e. "known" or "unknown" as of the valuation date; Not-In-Payment and In-Payment are determined as of the split date.

Adjustment of Dollar Dice for Incomplete Historical Months (continued)

Estimate t by $\dfrac{\sum_i (x_i + y_i + z_i)}{\text{NIPCF\$}}$

Let $f = 1 + \dfrac{k - s}{\sum_i (y_i + z_i)}$

Inflate y_i values to y_i' by letting $y_i' = y_i \cdot f$

Inflate z_i values to z_i' by letting $z_i' = z_i \cdot f$

If $u \neq 0$, for each line $\left[0, z_j', p_j\right]$

write an additional line $\left[0, \dfrac{t}{\sum_i z_i'} \cdot z_j', \dfrac{u}{n_2} \cdot p_j\right]$

Normalize the die so that the probabilities sum to unity.

Trending

The Claim Reserve System provides the following default option to handle trends; but, the trend rates are in a Cande file which can be modified to show whatever trends you wish.

Let ${}_{\beta}A = {}_{\beta}A_1 \text{ (M) } {}_{\beta}A_2$

Mean of $[x+y,0,p]_{\beta}A$, denoted by $\overline{[x+y,0,p]_{\beta}A}$, is calculated for each value of β not exceeding the valuation month minus 12. For example, suppose $[x+y,0,p]_{\beta}A_2$ is available for 36 months prior to the valuation date; then the following mean values are available:

$\overline{[x+y,0,p]\ {}_{7401}A}$	$\overline{[x+y,0,p]\ {}_{7501}A}$	$\overline{[x+y,0,p]\ {}_{7601}A}$
$\overline{[x+y,0,p]\ {}_{7402}A}$	$\overline{[x+y,0,p]\ {}_{7502}A}$	$\overline{[x+y,0,p]\ {}_{7602}A}$
$\vdots$	$\vdots$	$\vdots$
$\overline{[x+y,0,p]\ {}_{7412}A}$	$\overline{[x+y,0,p]\ {}_{7512}A}$	$\overline{[x+y,0,p]\ {}_{7612}A}$

Then:

12 months trend factor for ${}_{7401}A$ is taken to be ${}_{7401}t = \dfrac{\overline{[x+y,0,p]\ {}_{7501}A}}{\overline{[x+y,0,p]\ {}_{7401}A}}$

$\vdots \qquad \vdots \qquad \vdots$

12 months trend factor for ${}_{7512}A$ is taken to be ${}_{7512}t = \dfrac{\overline{[x+y,0,p]\ {}_{7612}A}}{\overline{[x+y,0,p]\ {}_{7512}A}}$

12 months trend factor for each of ${}_{7601}A$ thru ${}_{7612}A$ is taken to be

$$t = \frac{1}{24}\left(\frac{\overline{[x+y,0,p]\ {}_{7501}A}}{\overline{[x+y,0,p]\ {}_{7401}A}} + \cdots + \frac{\overline{[x+y,0,p]\ {}_{7612}A}}{\overline{[x+y,0,p]\ {}_{7512}A}}\right)$$

Then

$[0,{}_{\beta}t\cdot y,p]_{\beta}A_1$ is the "trended" ${}_{\beta}A_1$, designated ${}_{\beta}T_1$

$[0,{}_{\beta}t\cdot y,p]_{\beta}A_2$ is the "trended" ${}_{\beta}A_2$, designated ${}_{\beta}T_2$

See Footnote (a) on Page A-2 for description of this notation.

Adjustment of Numbers Dice for Incomplete Historical Months

For any particular lag month t_0, the numbers dice are of the form $\left[x_i, y_i, p_i\right]$

where

$$x_i = C_{t_0, i_0}$$

$$y_i = \left(C_{s_{i_0}, i_0} \div CF_{s_{i_0}, i}\right) - x_i$$

where

s_i = greatest lag for which data is available for the i^{th} incurred month

$$CF_{s_{i_0}, i} = C_{s_{i_0}, i} \div \left(C_{s_i, i} \div CF_{s_i, i}\right)$$

where

$CF_{s_i, i}$ is based on Completion Factor Method #3.

Each i_0 is assumed to be equally likely, and each i ($\leqslant i_0$) is assumed equally likely, in attaching probabilities to the number pairs (x_i, y_i).

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