Article From:

The Actuary

December 1984 – Volume No. 18, Issue No. 10

DEATHS

Yvon Boucher	FSA 1981
Michael R. Carrigan	FSA 1954
Robert P. Coates	FSA 1940
Stephen S. Cooper	ASA 1984
J. Craig Davidson	FSA 1947
Daniel F. Drennan	ASA 1952
Forrest S. Ockels	ASA 1950

REPORT OF THE SECRETARY

The Board of Governors met October 13 and 14. The significant non-routine actions taken included the following:

1. The Board accepted the report of the Planning Committee entitled "Serving the Needs of Pension Actuaries" and voted to forward the report to the Pension Section with a request that the Pension Section comment on the report and on actions that the Pension Section may take to implement it.
2. The Board approved for circulation to the Fellows of the Society three proposed amendments to the Constitution and one proposed amendment to the By-Laws. These proposed amendments would change disciplinary procedures, would allow Sections to make public expressions of professional opinion in the same manner as Committees, and would increase the number of Society Vice-Presidents from the present four to six. You will soon receive these proposed amendments in the mail to vote on.
3. The Board approved, with certain modifications, the Final Report of the Joint Committee on the Role of the Valuation Actuary. This report recommends significant changes in the role of the valuation actuary and in the general principles underlying the valuation of life insurance companies.
4. The Board adopted the new budget, including an increase in dues of $10 for all Fellows and for all individuals who have been Associates for 10 or more years, and of $5 for all other Associates.
5. The Board adopted the Report of the Committee on Theory of Dividends and Other Non-Guaranteed Elements in Life Insurance and Annuities, and authorized the committee to publicly express an opinion with respect to the report. □

INTEREST SENSITIVITY

by C.L. Trowbridge

Past issues of this newsletter have carried two articles by Ralph Garfield (April '83 and October '84) and one by Robert Myers (November '84) that attack special cases of a common problem. Each explores the behavior of some actuarial function as the interest rate changes.

In his earlier article Mr. Garfield proves that the ratio of the single life annuity to the joint-and-survivor form moves in the same direction as the interest rate, while his more recent proof is that $P_x = A_x/\ddot{a}_x$ varies in the opposite direction. Mr. Myers shows us that the ratio $\bar{a}_{\overline{e_x}|}$ (long ago proved to be no less than unity) moves *with* the interest rate as i increases through the lower end of its range, but at some level changes direction, and thereafter decreases.

Could all of these interesting relationships be demonstrated by means of a unifying theory? Do James Ramenda's observations (December '83) about the first Garfield article give us a clue? He tells us that a_x is shorter, and hence less interest sensitive, than $a_{\overline{xy}}$; which means that as i increases the numerator falls off less than the denominator, and the ratio increases. We seem to need what might be called an "index of interest sensitivity". Would such an index be useful in analyzing a wide range of interest sensitivity problems?

As a start along these lines, think of a generalized present value function of the discrete type

$$f(i) = \sum_{t=0}^{\infty} C_t v^t$$

where $v = 1/1+i$, i and C_t are non-negative and mutually independent, and C_t represents the amount of a payment at some future time t (including, perhaps, a probability as to its being paid).

We see immediately that the first derivative of the log of f(i) with respect to i is

$$D \log f(i) = \frac{f'(i)}{f(i)} = -v \frac{\Sigma t C_t v^t}{\Sigma C_t v^t} = -vd$$

where d (which we will now call the "duration") is the weighted average of the various t, with the $C_t v^t$ series acting as the weighting function.

We are not surprised to find that D log f(i) is negative (v and d are positive under the imposed limitations), and hence that f(i) decreases as the interest rate rises, because we are used to the idea that functions like A_x or a_x are inversely related to the interest rate. We may be a bit more surprised to realize that — vd is a measure of interest rate change, and hence is a satisfactory index of interest sensitivity, indicating not only direction but magnitude as well.

We do well to pause at this point to take a close look at the characteristics of d. Students of immunization theory and readers of Mr. Ramenda's letter will recognize d as the "duration" useful in testing the matching of assets and liabilities.

For a single payment at the end of n years, d is n; but for multiple payments d is a weighted average. Moreover, d is a function of i, because the v^ts are a part of the weighting function. As interest rates rise, less weight is given to the higher values of t, so the duration shortens. As a simple example, the duration of $\ddot{a}_{\overline{5}|}$ at zero interest is clearly 2, but at 10% it is only 1.81. As the interest rate approaches infinity, the duration of $\ddot{a}_{\overline{5}|}$ (indeed any present value function) approaches 0. Because both the v and the d terms of our index are functions of i, interest sensitivity changes as we move up the interest rate range.

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Interest Sensitivity

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To this point we have only examined the interest rate sensitivity of a present value function, and we have learned little that we did not already know. We have nonetheless laid a background for examining the sensitivities of more complicated actuarial functions, specifically those that can be expressed as the ratio of two present values. Let $f_1(i)$ and $f_2(i)$ be two present value functions with durations d_1 and d_2 respectively; and let us direct our attention to the interest sensitivity of the ratio $f_1(i)/f_2(i)$

$$
\begin{aligned}
D \log \frac{f_1(i)}{f_2(i)} &= D \log f_1(i) - D \log f_2(i) \\
&= -vd_1 + vd_2 \\
&= -v(d_1 - d_2) \\
&= -v\ d_c
\end{aligned}
$$

We now see that the index of interest sensitivity of a ratio is v multiplied by a composite duration d_c which is equal to the difference between the durations of numerator and denominator. The sign depends on which of these is the larger. The ratio behaves as a present value function if the composite duration d_c is positive. The duration concept can be extended to function of the accumulation type (e.g. $s_{\overline{n}|}$) by letting negative values of t represent payments in the past, thereby admitting negative (or zero) values of d_c. The ratio behaves as an accumulation function if the composite duration is negative. A present value function moves in the opposite direction of interest rate; an accumulation function moves with interest rate.

Though calculation of the d's may be difficult, it is often possible to determine by general reasoning which is the larger. If so, the direction of interest sensitivity becomes immediately evident. Going back to the two Garfield proofs, we can arrive at his results by noting that:

(1) a_x represents a shorter annuity than $a_{\overline{xy}}$, $d_1 < d_2$, and the ratio moves *with* the interest rate since d_c is negative.
(2) A_x, representing a payment at the end of life, clearly is of longer duration than $\ddot{a}_x$, representing a series of payments throughout life. $d_c = d_1 - d_2$ is positive, and the ratio, $P_x = A_x/\ddot{a}_x$ moves in the oppositive direction.

Obviously the ratio that Myers examines is a bit more complicated. At the lower values of i he finds the $a_{\overline{\mathring{e}_x}|} / \bar{a}_x$ ratio increasing with interest, but at higher levels of i he finds the opposite. This must mean that the duration of $\bar{a}_{\overline{\mathring{e}_x}|}$ is less than that of $\bar{a}_x$ at $i = 0$, and for higher values of i up to a point; but at even higher levels the reverse must be true. Both durations shorten as i increases, but the faster shortening of the originally longer duration eventually results in its becoming the shorter. As Myers suggests, further investigation seems indicated.

Interested readers may wish to carry further the development started here. Surely more can be done with the matter of magnitude, as opposed to only the direction, of interest sensitivity; and with easier methods of determining d. For the present at least, we leave this further development to others. □

A SUCCESSFUL FIRST

"Executive Management for the Acturary" was the first all-day management seminar sponsored by the Committee on Continuing Education. The objective was to show that analytically oriented actuaries can become good managers.

Michael J. Vopatek, whose dynamic presentation style was highly rated by participants, led a series of discussions and group activities on productivity, versatility, communication strategies, motivation and styles of interacting.

Two more presentations of this seminar are scheduled in conjunction with the San Francisco and St. Louis meetings in the Spring. Watch your mail for further information.

ADDRESS

Note the new masthead address for *The Actuary.* The use of a PO Box number in place of a street address is for the editor's convenience only. His personal address remains the same as it was.

THE 19th ACTUARIAL RESEARCH CONFERENCE

The 19th Actuarial Research Conference was held October 8 and 9 at the University of California, Berkeley, under the joint sponsorship of the SOA Committee on Research and the Operations Research Center of UC. Local arrangements and program were efficiently and expertly handled by Professor William Jewell. The general topic of the meeting was credibility theory and Bayesian approximation and it attracted speakers from the fields of statistics and engineering as well as the usual complement of actuaries. This provided for considerable interaction and a fruitful sharing of problems and solutions.

The 27 presentations may be loosely divided into four groups. They were: (1) theoretical analyses of various "pure" Bayesian approaches from obtaining prior distributions to posterior prediction, (2) empirical Bayes methods, (3) linear Bayes (credibility) and modifications thereof, and (4) applications of the above to specific problems from assumptions in pension plans to experience rating and the estimation of outstanding claims.

Copies of most of the presentations will be appearing in a future issue of *ARCH.* Readers who cannot wait may obtain a copy of the titles and abstracts of the presentations by writing Stuart Klugman at his yearbook address. This list also contains the addresses of the authors, many of whom would be pleased to supply copies of their paper upon request. □

1985 — ASTIN COMPETITION FOR YOUNG RESEARCHERS

ASTIN (Actuarial Studies in Non-Life Insurance) is a section of the International Actuarial Association that has as its main objective the promotion of actuarial research, particularly in non-life insurance.

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