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THE ENGINEER AND THE ACTUARY: A FABLE

by Oakley E. Van Slyke

The city office building of the City of Foresight was built in 1937. The building was worn by winds, burned by small fires, and shaken by earthquakes for over 40 years. By 1983, the City Council was not sure the building was still sound. Also, the operating costs—maintenance, heat, and air conditioning—had increased ten-fold in those 40 years.

The Council hired a consulting engineer to determine the structural soundness of the building. The engineer was also asked to recommend changes in the building that would reduce operating costs.

The engineer made a number of tests and calculations. He tested supporting beams and joints to determine their load-bearing strength. He tested heat flow in and out of the building through critical doors and windows. He calculated the costs and projected the savings from possible design changes.

The engineer presented his findings in a report to the City Council. The Council adopted a plan that eventually resulted in a stronger building and a saving of $20,000 per year.

The City's self-insurance program was established July 1, 1978. The City contributed to the self-insurance fund in 1979, 1980 and 1981, but no additional funding was added in 1982. The fund began strong, but suffered through inflation, changes in benefit levels and several large claim settlements. The City Council was not sure the fund was still sound in 1983. Also, claims and administrative costs increased several-fold in those five years.

The Council hired a consulting actuary to determine the financial soundness of the risk management program. The consultant was also asked to recommend changes in the program that would reduce claims and administrative costs.

The actuary made a number of tests and calculations. He tested loss reserves to determine the adequacy of the fund's surplus. He compared the frequency and cost of claims in Foresight to the frequency and cost of claims in similar cities. He calculated the costs and projected the yearly savings from possible design changes.

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OVERLAP THEORY

James P. Walsh's request (October issue) for an explanation of how the 15% figure is calculated in the contention by unisex-pricing advocates that "15% (of women) live longer than men (but) 100% of women are penalized", yielded five fine responses, from George H. Andrews, Donald C. Baillie, Daniel F. Case, Robert J. Myers and James D. Reid. This attempt at summarization starts with Prof. Baillie's letter which makes greatest use of actuarial symbols.

Baillie Analysis

If you plot the two sexes' histograms of d_x above age 65 (using the same ℓ_{65}) on top of each other, you will find that they have a common area of about 85% of all the ℓ_{65} deaths. This overlap is then interpreted as representing the deaths of the great majority of individual men and women who "die at the same ages". The 15% of the male histogram lying above, and to the left, of the female histogram represents those individual men who "die before women", and vice versa for the 15% of the female deaths lying above, and to the right, of the male histogram.

Actuaries thinking in terms of pairs of lives may find this a strange argument, since the 15% is entirely different from $_{\infty}q^{\;\;\;2}_{\underset{m}{65}\,\underset{f}{65}}$. But at least one non-actuary in Canada and one in the United States have used the argument successfully; the latter published his victory in The Journal of Risk and Insurance, Vol. XLVI, No. 4 (1979).

If T is the cross-over duration, where $\ell_{65+T}\mu_{65+T}$ (or d_{65+T}) is the same for both sexes, then the 15% figure is just $_Tq_{65}$ male *less* $_Tq_{65}$ female. These 15% of men who "die early" are deemed to be matched by the 15% of women who "die late". One can make this idea less fuzzy by trying to measure the average duration between the "early male" deaths and the "late female" deaths, using

$$\int_T^{\infty}\left({}_tp_{\underset{f}{65}}\,\mu_{\underset{f}{65}+t} - {}_tp_{\underset{m}{65}}\,\mu_{\underset{m}{65}+t}\right)t\,dt$$

less

$$\int_0^{T}\left({}_tp_{\underset{m}{65}}\,\mu_{\underset{m}{65}+t} - {}_tp_{\underset{f}{65}}\,\mu_{\underset{f}{65}+t}\right)t\,dt$$

This difference is just $\mathring{e}_{\underset{f}{65}} - \mathring{e}_{\underset{m}{65}}$, a figure that seems more directly relevant to the debate about annuity values than does $_{\infty}q^{\;\;\;2}_{\underset{m}{65}\,\underset{f}{65}}$. The average span between the 15% of men who "die early" and the matching women who "die late" is then $(\mathring{e}_{\underset{f}{65}} - \mathring{e}_{\underset{m}{65}})/0.15$

As a vastly simplified example, suppose each of the two curves is flat, with uniform annual density of 1/34 for men aged 65 to 99, and 1/40 for women at ages 65 to 105. Taking T as 34, we find the "early men" probability to be

$$34((1/34) - (1/40)) = 0.15$$

and the "late women" probability to be

$$(40 - 34)((1/40) - 0) = 0.15$$

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Overlap Theory

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The respective values of $\mathring{e}_{65}$ are 17 and 20 years, their 3-year difference being 15% of 20. Here $_{\infty}q^{\;\;2}_{\underset{m}{65}\,\underset{f}{65}}$ equals. 0.575.

For a bit more realistic model, we can try a male density rising linearly, such as .020 + .001t, peaking at t = 20, and then declining linearly as .080 − .002t, to reach zero at t = 40. Female density: .010 + .0015t, peaking at t = 20 also (for simplicity), declining as .072 − .0016t, to zero at t = 45. Here T = 20, and the difference between the $_{T}q_{65}$'s is 0.6 − 0.5 = 0.1. The $\mathring{e}_{65}$'s work out (I hope!) to 17 1/3 and 20 1/6, a difference of 2.83 years, implying an average span of 28.3 years between the 10% of deaths that are "early" and "late". Here, $_{\infty}q^{\;\;2}_{\underset{m}{65}\,\underset{f}{65}}$ is 0.578, I believe.

Statements that look absurd to us actuaries sometimes have at least a small core of actuarial respectability if we take the trouble to dig it out. Let's always try!, says Prof. Baillie.

Observations By Others

None of our other correspondents held out an olive branch. Prof. Andrews said:

> The hollow nature of the (overlap) argument is emphasized when one notes that in an entirely analogous way one could argue that even though only 15% of 60-year-old males retiring early live longer than the 65-year-old males retiring at normal retirement age, 100% of the early retirees are penalized through lower monthly payments. Quite clearly, if enough people buy such arguments, age-based actuarial tables are vulnerable to attack. It would be funny if it weren't so serious.

Mr. Case had worked out what figures corresponding to the 15% would apply on two annuity tables. Using the 1971 Individual Annuity Table, the total of unmatched deaths was 14.41% of the starting number at age 65; for the 1971 Group Annuity Table it was much higher, 19.7%. He emphasized the disparity in average ages of the unmatched deaths because of its risk classification implications—for the 14.41% case, it was 92.59 years for women, 72.27 for men.

Mr. Myers identified the theory's developer as Professor of Economics Barbara Bergmann, University of Maryland. She and a colleague published it in the Fall 1975 issue of *Civil Rights Digest;* "I thought", says Mr. Myers, "that I had demolished (it) in a paper (in the same magazine, Winter 1977 issue)". He considers the theory as erroneous an application of actuarial science as, say, that if the expectation of life at birth for a particular category is 64, then none of this category will survive to age 65 and receive Social Security benefits.

Says Mr. Reid: "Of course, the logic underlying (the theory) is quite stupefying . . . I know of no rational argument that will cause (its) advocates to reconsider since it is not the reasoning but the conclusion which they consider important." *E.J.M.*

Engineer and Actuary

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During the course of their work, the engineer and the actuary happened to meet. They found they had a lot in common. They talked about the calculus courses they had had in college, compared their training in regulatory law, and noted the differences and similarities in their other academic courses. They were amused to note that, although their academic training was important, both now used the insights gained from ten years of practical problem-solving for clients to guide them to the best solutions for their clients.

The actuary presented his findings in a report to the City Council. The Council adopted a plan that eventually resulted in a stronger self-insurance fund and a savings of $20,000 per year.

Moral: The actuary, like the engineer, can increase your security, and often save you money.

AN ACTUARIAL GUIDE TO JAI-ALAI

by David M. Lipkin

(Second of two articles. The first) was in our November 1983 issue)

This article will first examine the results which would be expected if all teams had equal ability, and then contrast these expected results with actual experience.

Expected Results

In 1982, Carl Anderson, an actuarial student, wrote an APL program to simulate jai-alai. Each team's skill level can be input to the program, but, for these Monte Carlo tests, equal skill levels were assumed. The percentages below came from a sample of 5,000 simulated games:

Team	*Winning Percentage*
1	16.78%
2	16.92
3	14.28
4	12.54
5	10.38
6	9.32
7	8.82
8	10.96
Standard deviation	2.99%

This pattern is similar to what might be expected, as the low-numbered teams have a large advantage over the others. Team 8's is a special case described in the first article.

Actual Results

This next table was compiled from the programs sold at performances.

	Winning Percentages		
	Hartford		*Tampa*
	1982	*1983*	*1983*
1	13.64%	14.56%	14.00%
2	16.78	14.66	13.77
3	14.12	12.82	12.47
4	11.54	12.58	12.04
5	11.99	11.78	12.22
6	10.71	11.22	11.05
7	9.47	10.27	11.73
8	11.75	12.11	12.72
Number of games	2,903	2,122	1,620
Standard deviation	2.13%	1.43%	.92%

How can we reconcile these actual results with the expected? Upon preliminary comparison, it seems that:

1. The difference in actual results between teams one and two in Hartford's 1982 season is surprisingly large.
2. The data for these samples follow the general pattern of the expected results.

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