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Skepsis Avaunt

Sir:

Ralph Garfield in "Skepsis Avaunt" discusses the problem of what happens to P_X, the whole life premium, if the interest rate changes.

Spurred by the editor's challenge of a simpler proof I reasoned as follows:

Let B, P and I represent the present value of the benefits, premiums and earned interest under the policy where there is no discounting for the time value of money. For example, B = 1 since this is a whole life plan. Clearly,

$$B = P + I. \tag{1}$$

This is the formula underlying the general reasoning Mr. Garfield uses. Furthermore,

$$I = (P + V)i \tag{2}$$

where V is the mortality-only present value of the beginning of year reserves. (2) holds because the discounting is identical for the interest earned, premium collected, and beginning of year reserve for any single policy year. Combining (1) and (2) gives:

$$B = P(1 + i) + Vi. \tag{3}$$

Now $P = k \cdot P_X$ where k is a positive constant determined by the mortality structure and *not* the interest rate. Differentiating (3) with respect to the interest rate, i, and using (2) again:

$$0 = \frac{dB}{di} = \frac{dP}{d(P_X)} \cdot \frac{d(P_X)}{di} (1+i) + P + V + \frac{dV}{di} i ,$$

$$\frac{d(P_X)}{di} = - \frac{1}{k(1+i)} \left[\frac{I}{i} + \frac{dV}{di} i \right]$$

In the usual situation the terms inside the brackets are positive and indeed the premium decreases as the interest rate increases. However, it is possible to rig the reserves to be sufficiently negative to force $I < 0$ and $\frac{d(P_X)}{di} > 0$. Hence not only is there an error in Mr. Garfield's proof, but the proposed theorem is not true! An example is given below.

Example:

Mortality:

x	1	2	3	4	5	6	7	8	9	10
q_X	.9	.0	.0	.0	.0	.0	.0	.0	.0	1.0

Sample Values

i	$\ddot{a}_1$	A_1	P_1	I/i	$\approx \frac{d(P_1)}{di}$
.10	1.5759	.8567	.5436	-.3293	.0952
.11	1.5537	.8460	.5445	-.3145	.0798
.12	1.5328	.8358	.5452	-.2997	.0648
.13	1.5132	.8259	.5458	-.2851	.0501
.14	1.4946	.8164	.5463	-.2706	.0358
.15	1.4772	.8073	.5465	-.2562	.0220
.20	1.4031	.7662	.5460	-.1874	-.0339

Where did Mr. Garfield's proof go wrong? In his proof of $(I\ddot{a})_X < (\ddot{a}_X)^2$! He needed $\ddot{a}_{X+n} < \ddot{a}_X$ for n = 1, 2, ..., which is not always true. For instance, in the example with i = 10%, $\ddot{a}_1$ = 1.5759 while $\ddot{a}_2$ = 6.3349.

Kenneth S. Avner

Sir:

The mathematical proof by Ralph Garfield that as interest rates increase, the premium decreases, appears very interesting but unfortunately, is *not* a valid generalization.

In order to understand how the premium operates one cannot just look at the premium, but one must understand the dynamics of the flow of funds. This is the reserve calculation.

Solving a problem with calculus is letting the symbols do your thinking. This is similar to using a computer, and letting the machine do the thinking. The actuary who uses calculus and a computer does not fully appreciate the full solution unless he tries to general reason the solution. One of the best appeals to general reasoning appeared in a commentary of Lidstone's original article about changing assumptions. Although the commentator did not use 1980's terminology, he championed the general reasoning approach which one would refer to today as "Right Brain Thinking".

A level premium is a weighted average claim cost, weighted for interest and survivorship. Now charging the average claim cost is a sensible approach for a company, when the average claim is greater than the initial claims. When the average claim is greater than the initial claim cost, a fund develops. When the interest rate is raised, the fund earns additional interest income and the premium can be reduced by the additional interest earned.

If the annual claim costs are level, and, let us say, the claims occur at the beginning of the year, then the average claim cost equals the initial claim cost and no fund develops. Any change in the interest rate will have no effect on the premium.

Finally, when the initial claim cost exceeds the level premium, depending on when the premium overtakes the annual claim cost, the interest charged on the negative fund will become a source of loss; when there is an increase in interest rates, it will increase the premium.

These examples are not academic, but are realistic relationships that occur in practice. A whole life contract is one where the current claim cost increases with age. In such a situation a fund develops. In this relationship the

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