# MORTALITY AND SEX-RELATED MORTALITY DIFFERENCES

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## The Model for the Loss Process

This model is described in Buhlmann, Mathematical Methods in Risk Theory, or Freifelder, A Decision Theoretic Approach to Insurance Ratemaking.

X is a random variable which is the amount of loss in the process. (In the case of pensions, X is the present value of an annuity certain for the random span of life.)

$\Lambda$ is a random variable which is a longevity parameter for individuals.

$f(x,\lambda)$ is the density of losses $x$ for an individual with parameter $\lambda$.

$u(\lambda)$ is the density of the parameter $\lambda$ in the population.

$f(x) = \int f(x,\lambda)\, u(\lambda)\, d\lambda$ is the unconditional distribution of losses for the population.

In the ratemaking problem, f and u are known functions, but $\lambda$ is unknown for the individual.

$P(\lambda) = E_X f(x,\lambda) = \int x f(x,\lambda) dx$ is the unknown true individual or risk premium.

$P = E_\lambda P(\lambda) = E_X f(x)$ is the known collective or a priori premium.

In <u>risk classification</u>, we attempt to estimate $P(\lambda)$ using known discrete information r (i.e., r=n means r is in class n) about each individual.

$p_r$ ($\sum_r p_r = 1$) is the known density of r in the population.

$U_r(\lambda)$ is the known density of the risk parameter $\lambda$ in the class r.

P(r) denotes the premium based on information r.

## Criteria for Ratemaking

1. Adequacy $\sum_r P(r)\, p_r = \underline{P}$
2. Equity
3. Not excessive

## Ratemaking Schemes

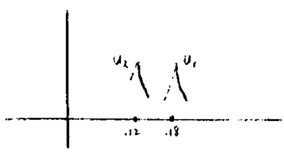
1. Non-classification $\underline{P}(r)=\underline{P}$ (Manhart, Stone)
2. Self-sufficient classes $\underline{P}(r) = \int U_r(\lambda)\; P(\lambda)d\lambda$ (Traditional)
3. Credibility rated classes (Traditional)
4. Utility Theory (Freifelder)
5. Constrained Estimation of $(\lambda|r)$ or $(P(\lambda)|r)$.
6. Minimization of Weighted Errors.
7. Analysis of Probability of Misclassification

Distribution of the Risk Parameters

The discussion so far assumes that the distribution $f(x,\lambda)$ and $u(\lambda)$ are known. In mortality theory, all we know is the unconditional $f_r(x)$, and there is disagreement on the partitioning of $f(x)$ into process randomness $F(x,\lambda)$ and parameter randomness $u(\lambda)$.

Since rates according to a standard depend on this partitioning, and not just on $f_r(\lambda)$, it would help to display some assumptions.

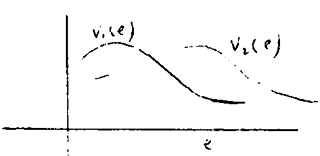
Assumption 1 (Homogeneity)



$u(\lambda)$ is a point distribution

$f_r(x) = f(x,E_\lambda u_r(x))$

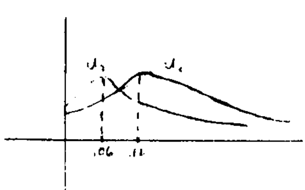
Assumption 2 (Predestination)



$f(x,\lambda)$ is a point distribution, so

$e=E_x f(x,\lambda)$ is a new risk parameter

Assumption 3 (Conjugate)



$f(x,\lambda)$ is Gompertz with $B=\lambda$

$u_r(\lambda)$ is a Gamma distribution

$f_r(x)$ is logistic