A Proof of Lidstone's Theorem

Richard W. Ziock

University of Iowa

Symbols:

P = Net annual premium

i_t= Interest rate

$_tq$= Mortality rate

C_t= t-th year contribution to surplus

$_tV$= t-th year reserve

Unprimed symbols are based on assumed rates.

Primed symbols are based on experience rates

Assumptions: Level benefits, level premiums.

Two factor contribution or dividend formula:

$$({}_{t-1}V + P)(1 + i_t) - q_t(1 - {}_tV) = {}_tV \qquad (1)$$

$$({}_{t-1}V + P)(1 + i_t') - q_t'(1 - {}_tV) = {}_tV + C_t \qquad (2)$$

$$({}_{t-1}V + P)(i_t' - i_t) + ({}_tq - {}_tq')(1 - {}_tV) = C_t \qquad (2) - (1)$$

If the policy dividend equals C_t, this is the unique method of paying dividends which leaves each year's reserve unchanged. This fact is clearly seen in formula (2) since there if C_t is paid out the ending reserve is $_tV$.

Policy Comparisons:

Consider:

A: Non-Par plan based on i' and q'

B: Non-Par plan based on i and q

B': Par plan with net premiums based on i and q, which experiences i' and q', with two factor dividends paid. B' may be looked on as a non-par level benefit, decreasing premium plan based on i' and q'.

Observations:

B' has larger reserves than A.

Note:(1) Both are non par and based on i' and q'.

(2) Decreasing premiums lead to larger reserves (e.g. single premiums)

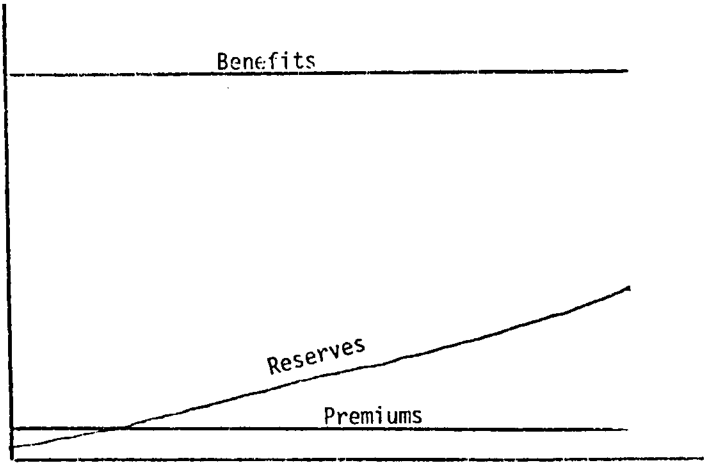
B' has same reserves as B because two factor dividends leave the reserves unchanged.

Thus B's reserves exceed A's or -

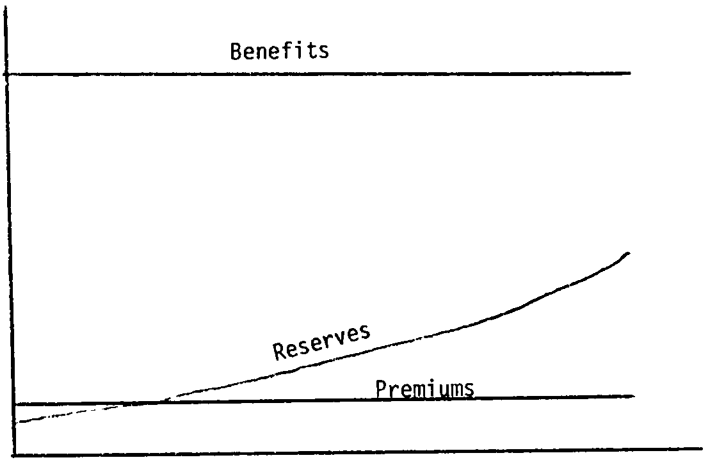
If $C_t = ({}_{t-1}V + P)(i_t' - i_t) + ({}_tq - {}_tq')(1 - {}_tV)$ always increases the reserves based on i and q exceed those based on i' and q' and vice-versa.

This heuristic proof seems to be a little easier for students than the one in the Jordan text.

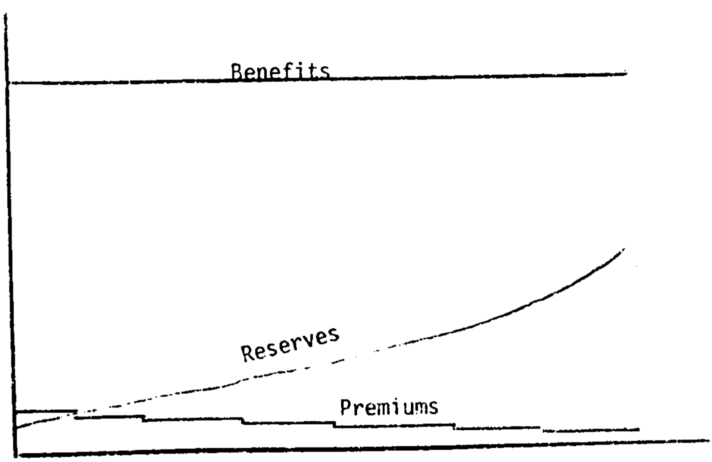
Reference: "A Simplified Illustration of Lidstone's Theorem," The Actuary, September 1969.


Benefits
Reserves
Premiums

A(i',q')


Benefits
Reserves
Premiums

B(i,q)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