

Tables from Paper on  
 USE OF BAYES LEMMA TO CONSTRUCT  
 FREQUENCY DISTRIBUTION OF  $qx$

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Abstract

A method is presented which uses Bayes Lemma in an elementary way to construct posterior probabilities to be attached to each of several alternative prior distributions, given a few observations. The availability of efficient computer programs to numerically perform discrete convolutions for producing prior distributions makes this a practical method.

Example

A group of 3500 lives has experienced 7 deaths each year for  $n = 1, 2, 4, \text{ or } 8$  years. Determine the pure stop-loss premium at the stop-loss levels  $\ell = 7, 8, \dots, 15$ , taking into account the various possible levels of probabilities of death which might apply to the group.

Solution

$$\text{Let } A_{\ell, j} = \begin{bmatrix} 0 & 1 - k_{\ell} \cdot q \\ 1 & k_{\ell} \cdot q \end{bmatrix} \oplus 3500 = \begin{bmatrix} 0 & 1 - k_{\ell} \cdot (.002) \\ 1 & k_{\ell} \cdot (.002) \end{bmatrix} \oplus 3500$$

where

$\ell$	$k_{\ell}$	Run #1	Run #2	Run #3	Run #4
1	.25	1/7	.05	-	.05
2	.50	1/7	.15	-	.15
3	.75	1/7	.20	-	.20
4	1.00	1/7	.20	1.00	.60
5	1.25	1/7	.20	-	-
6	1.50	1/7	.15	-	-
7	1.75	1/7	.05	-	-

Please see (RISK) Notation for definition of the convolution operation  $\oplus$ .

METHODOLOGY

	Input Frequency Distributions for Period				Prior Probabilities	Observations			$r_i = q_i \cdot \prod_{j=1}^n p_{ij}$	$s_i = r_i \div \sum_{\ell=1}^m r_{\ell}$
	1	2 ...	n	n+1	$q_i$	$x_1$	$x_2$	... $x_n$		
1	$A_{11}$	$A_{12} \dots$	$A_{1n}$	$A_{1,n+1}$	$q_1$	$p_{11}$	$p_{12}$	... $p_{1n}$	$r_1$	$s_1$
2	$A_{21}$	$A_{22} \dots$	$A_{2n}$	$A_{2,n+1}$	$q_2$	$p_{21}$	$p_{22}$	... $p_{2n}$	$r_2$	$s_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	$A_{m1}$	$A_{m2} \dots$	$A_{mn}$	$A_{m,n+1}$	$q_k$	$p_{m1}$	$p_{m2}$	... $p_{mn}$	$r_m$	$s_m$
					$\Sigma = 1$					$\Sigma = 1$

n = number of observations

m = number of hypothesized distributions

$A_{ij}$  = i<sup>th</sup> hypothesized distribution for the j<sup>th</sup> period

$x_j$  = observation for the j<sup>th</sup> period

$p_{ij}$  = the probability of observing  $x_j$ , using  $A_{ij}$  as the hypothesized distribution

$$\text{Posterior distribution} = A = \bigotimes_{i=1}^m \left( [x, y, s_i \cdot p] A_{i,n+1} \right)$$

Please see (RISK) Notation for definition of the merge operation  $\bigotimes$  and the transformation operation  $\{ \}$ A.

Group of 3,500 Lives Apparently Subject  
to A Mortality Rate of .002 = 7/3500

Observation Period	Stop Loss Premiums per \$1.00 of Insurance								
	Stop Loss Level								
	7.	8.	9.	10.	11.	12.	13.	14.	15.
1 year	2.07	1.57	1.17	.85	.61	.43	.30	.21	.14
4 years	1.31	.87	.56	.34	.20	.11	.06	.03	.02
8 years	1.17	.76	.46	.27	.15	.08	.04	.02	.01
Pure Binomial	1.04	.64	.37	.20	.10	.05	.02	.01	.00

This table assumes that 7 deaths were observed during each year of the observation period. The following Bayesian approach has been used to determine the frequencies in a frequency distribution of total number of claims from which theoretical net stop-loss premiums can be determined (where M is the number of years in the observation period and x is the total number of claims):

$$\int_0^1 \left\{ \frac{\binom{3500}{7} (1-q)^{3500-7} q^7}{\int_0^1 \left\{ \binom{3500}{7} (1-q)^{3500-7} q^7 \right\}^M dq} \binom{3500}{x} (1-q)^{3500-x} q^x \right\} dq$$

$$= \frac{\binom{3500}{x} \binom{3500M}{7M} (3500M+1)}{\binom{3500+3500M}{7M+x} (3500+3500M+1)}$$

K<sub>2</sub>  
 TESTS/50/3300  
 (USING FIRST DIMENSION)

A2.

MEAN: 3.50000000  
 VARIANCE: 3.49650000  
 STANDARD DEVIATION: 1.869893045

LINE	AMOUNT	PROBABILITY	CUMULATIVE PROBABILITY	STOP-LOSS PREMIUM	STO. DEV. OF STOP-LOSS
1	0.00000000	0.03014455	0.03014455	3.50000000	1.369393045
2	1.00000000	0.10561153	0.13575608	2.530144549	1.766447395
3	1.99999999	0.18475233	0.32070841	1.665900633	1.538938496
4	2.99999999	0.21537029	0.53657870	0.986609045	1.257365633
5	4.00000000	0.18891351	0.72549221	0.523187740	0.968195430
6	5.00000001	0.13222055	0.85771276	0.240679948	0.692407904
7	6.00000002	0.07709556	0.93480832	0.106392705	0.458296020
8	7.00000003	0.03852022	0.97332854	0.041201928	0.283170143
9	8.00000004	0.01603573	0.99016427	0.014529571	0.165295281
10	9.00000005	0.00633980	0.99670307	0.004693341	0.092042692
11	10.00000007	0.00228493	0.99898800	0.001396911	0.049204229
12	11.00000009	0.00072569	0.99971374	0.000334963	0.025333150
13	12.00000013	0.00021170	0.99992494	0.00008701	0.012025131
14	13.00000012	0.00005672	0.99998167	0.000023645	0.006038319
15	14.00000014	0.00001414	0.99999581	0.000005313	0.002348006
16	15.00000015	0.00000329	0.99999910	0.000001124	0.001294147
17	16.00000017	0.00000072	0.99999982	0.000000224	0.000571829
18	17.00000021	0.00000015	0.99999997	0.000000042	0.000245689
19	18.00000023	0.00000003	0.99999999	0.000000008	0.000102327
20	19.00000026	0.00000001	1.00000000	0.000000001	0.000040863
21	20.00000029	0.00000000	1.00000000	0.000000000	0.000014959
22	21.00000032	0.00000000	1.00000000	0.000000000	0.000004383
23	22.00000029	0.00000000	1.00000000	0.000000000	0.000000000

K1  
TESTS/25/3500  
(USING FIRST DIMENSION)

A1.

MEAN: 1.75000000  
 VARIANCE: 1.749124994  
 STANDARD DEVIATION: 1.322544893

LINE	AMOUNT	PROBABILITY	CUMULATIVE PROBABILITY	STOP-LOSS PREMIUM	STO. DEV. OF STOP-LOSS
1	0.00000000	0.17369791	0.17369791	1.75000000	1.322544893
2	1.00000000	0.30412340	0.47782131	0.92369790	1.066170873
3	2.00000000	0.26616503	0.74398634	0.401519218	0.757930753
4	3.00000001	0.15525184	0.89923818	0.145505556	0.478290423
5	4.00000002	0.06749841	0.96673659	0.044743732	0.267644555
6	5.00000003	0.02374916	0.99048574	0.011880317	0.136140334
7	6.00000005	0.00692040	0.99740615	0.002766061	0.064422246
8	7.00000007	0.00172800	0.99913414	0.000572207	0.028749555
9	8.00000010	0.00037743	0.99951158	0.000106351	0.012190809
10	9.00000013	0.00007326	0.99958484	0.000017323	0.004934992
11	10.00000016	0.00001279	0.99959763	0.000002763	0.001914133
12	11.00000019	0.00000203	0.99959966	0.000000392	0.000713368
13	12.00000023	0.00000030	0.99959996	0.000000051	0.000255849
14	13.00000027	0.00000004	0.99959999	0.000000006	0.000080059
15	14.00000031	0.00000000	1.00000000	0.000000001	0.000025515
16	15.00000035	0.00000000	1.00000000	0.000000000	0.000007974
17	16.00000039	0.00000000	1.00000000	0.000000000	0.000001252
18	17.00000043	0.00000000	1.00000000	0.000000000	0.000000000

K3  
TESTS/75/3500  
(USING FIRST DIMENSION)

A3.

MEAN: 5.250000000  
VARIANCE: 5.242124995  
STANDARD DEVIATION: 2.289563736

LINE	AMOUNT	PROBABILITY	CUMULATIVE PROBABILITY	STOP-LOSS PREMIUM	STO. DEV. OF STOP-LOSS
1	0.000000000	0.00522688	0.00527633	5.250000000	2.289563736
2	1.000000000	0.02748232	0.03276865	4.253226875	2.257835714
3	1.999999999	0.07222893	0.10499758	3.287936078	2.142920422
4	2.999999997	0.12651800	0.23151558	2.392374112	1.936160997
5	3.999999997	0.16616179	0.39767737	1.624330150	1.676033255
6	5.000000001	0.17453229	0.57221011	1.021947977	1.597247913
7	6.000000005	0.15272667	0.72493678	0.594098090	1.114002757
8	7.000000009	0.11452042	0.83945720	0.313974878	0.841026754
9	8.000000013	0.07511639	0.91457359	0.150372089	0.599450362
10	9.000000018	0.04378342	0.95835701	0.072835695	0.465028733
11	10.000000024	0.02296163	0.98131864	0.031162718	0.261239749
12	11.000000030	0.01094476	0.99226340	0.012441370	0.161924726
13	12.000000037	0.00478015	0.99704355	0.004844085	0.096942605
14	13.000000043	0.00192672	0.99897027	0.001626951	0.056254005
15	14.000000051	0.00072092	0.99969119	0.000536533	0.031713393
16	15.000000058	0.00025169	0.99994288	0.000167032	0.017399101
17	16.000000067	0.00008236	0.99999999	0.000049219	0.009300197
18	17.000000075	0.00002535	0.99999999	0.000013752	0.004549407
19	18.000000084	0.00000737	0.99999999	0.000003660	0.002468919
20	19.000000095	0.00000203	0.99999999	0.000000927	0.001228266
21	20.000000105	0.00000053	0.99999999	0.000000224	0.000597379
22	21.000000115	0.00000013	0.99999999	0.000000052	0.000243935
23	22.000000127	0.00000003	0.99999999	0.000000011	0.000131532
24	23.000000138	0.00000001	1.00000000	0.000000002	0.000058860
25	24.000000150	0.00000000	1.00000000	0.000000000	0.000024313
26	25.000000152	0.00000000	1.00000000	0.000000000	0.000009235
27	26.000000173	0.00000000	1.00000000	0.000000000	0.000002519
28	27.000000154	0.00000000	1.00000000	0.000000000	0.000000000

K4  
TESTS/100/3500  
(USING FIRST DIMENSION)

A4.

MEAN: 7.000000000  
 VARIANCE: 6.986000006  
 STANDARD DEVIATION: 2.643104237

LINE	AMOUNT	PROBABILITY	CUMULATIVE PROBABILITY	STOP-LOSS PREMIUM	STD. DEV. OF STOP-LOSS
1	0.000000000	0.00000551	0.00000551	7.000000000	2.643104237
2	1.000000000	0.00635129	0.00725680	6.000005513	2.634095232
3	1.999999998	0.02226770	0.02952451	5.003162318	2.590115313
4	2.999999997	0.05203235	0.08155685	4.037686325	2.479098051
5	3.999999996	0.09116088	0.17271773	3.119243579	2.291297198
6	4.999999995	0.12773484	0.30045258	2.291961411	2.049793544
7	5.999999995	0.14910931	0.44956189	1.592413986	1.793255995
8	7.000000005	0.14915200	0.59871389	1.041975867	1.506475136
9	8.000000011	0.13030300	0.72922189	0.640689750	1.226525836
10	9.000000016	0.10147716	0.83069905	0.369911634	0.955715577
11	10.000000023	0.07093374	0.90169239	0.200610680	0.711204280
12	11.000000030	0.04513876	0.94683117	0.102303069	0.506626741
13	12.000000037	0.02640090	0.97313197	0.049134235	0.347039363
14	13.000000045	0.01414170	0.98727367	0.022266204	0.229690270
15	14.000000054	0.00755870	0.99483237	0.009539871	0.147475189
16	15.000000063	0.00328746	0.99811983	0.003672242	0.092129119
17	16.000000072	0.00143497	0.99955480	0.001492073	0.056115203
18	17.000000082	0.00059935	0.99995415	0.000546473	0.033373927
19	18.000000092	0.00022853	0.99998268	0.000191022	0.019402289
20	19.000000103	0.00008993	0.99999261	0.000063704	0.011035754
21	20.000000115	0.00002923	0.99999539	0.000020319	0.006145918
22	21.000000127	0.00000972	0.99999561	0.000006208	0.003353541
23	22.000000139	0.00000308	0.99999869	0.000001320	0.001793947
24	23.000000152	0.00000093	0.99999963	0.000000512	0.000741221
25	24.000000156	0.00000027	0.99999990	0.000000139	0.000414364
26	25.000000179	0.00000009	0.99999997	0.000000036	0.000244266
27	26.000000195	0.00000002	0.99999999	0.000000009	0.000120317
28	27.000000209	0.00000001	1.00000000	0.000000002	0.000057356
29	28.000000225	0.00000000	1.00000000	0.000000000	0.000025860
30	29.000000240	0.00000000	1.00000000	0.000000000	0.000010409
31	30.000000255	0.00000000	1.00000000	0.000000000	0.000003174
32	31.000000223	0.00000000	1.00000000	0.000000000	0.000000000

K5  
TESTS/125/350J  
(USING FIRST DIMENSION)

As.

MEAN: 8.75000000  
 VARIANCE: 8.728125002  
 STANDARD DEVIATION: 2.954340028

LINE	AMOUNT	PROBABILITY	CUMULATIVE PROBABILITY	STOP-LOSS PREMIUM	STD. DEV. OF STOP-LOSS
1	0.000000000	0.00015673	0.00015673	3.750000000	2.954340029
2	1.000000000	0.00137487	0.00153160	7.750156735	2.952308351
3	1.999999999	0.00602839	0.00756000	6.751668337	2.938283401
4	2.999999999	0.01701640	0.02517680	5.759248331	2.891018556
5	3.999999999	0.03800025	0.06377702	4.784425129	2.766929177
6	4.999999994	0.06704230	0.13141932	3.848202152	2.616720576
7	5.999999994	0.09875098	0.23017030	2.979621478	2.393461543
8	6.999999994	0.12333596	0.35370626	2.207791782	2.139393906
9	7.999999996	0.13514518	0.48885144	1.563498041	1.371279995
10	9.000000000	0.13145427	0.62030571	1.052389475	1.595615610
11	10.000000015	0.11511775	0.73542346	0.672739182	1.317766399
12	11.000000021	0.09145863	0.82688209	0.408106639	1.048794479
13	12.000000028	0.06604561	0.89342770	0.234932731	0.803199228
14	13.000000034	0.04401586	0.93824356	0.123404432	0.592741335
15	14.000000042	0.02707582	0.96531938	0.066691996	0.422559228
16	15.000000049	0.01629469	0.98161407	0.032955373	0.292116116
17	16.000000057	0.00869520	0.99030927	0.015513446	0.197308473
18	17.000000066	0.00456891	0.99487826	0.008960713	0.129734754
19	18.000000074	0.00221575	0.99709401	0.004988697	0.083392544
20	19.000000083	0.00111771	0.99821172	0.00226423	0.052473047
21	20.000000093	0.00044394	0.99865566	0.001432455	0.032352505
22	21.000000103	0.00018436	0.99884002	0.000713207	0.019560183
23	22.000000113	0.00007308	0.99891310	0.000365978	0.011504003
24	23.000000124	0.00002769	0.99894079	0.000203004	0.006753500
25	24.000000135	0.00001006	0.99895085	0.000100724	0.003866686
26	25.000000147	0.00000350	0.99895435	0.000050000	0.002173936
27	26.000000159	0.00000117	0.99895552	0.000020782	0.001201493
28	27.000000172	0.00000033	0.99895585	0.000000236	0.000632838
29	28.000000185	0.00000014	0.99895600	0.000000069	0.000348560
30	29.000000198	0.00000006	0.99895606	0.000000019	0.000182530
31	30.000000212	0.00000001	1.00000000	0.000000005	0.000093282
32	31.000000226	0.00000000	1.00000000	0.000000001	0.000045966
33	32.000000240	0.00000000	1.00000000	0.000000000	0.000021249
34	33.000000255	0.00000000	1.00000000	0.000000000	0.000008634
35	34.000000269	0.00000000	1.00000000	0.000000000	0.000002567
36	35.000000283	0.00000000	1.00000000	0.000000000	0.000000000



K6  
 TESTS/150/3500  
 (USING FIRST DIMENSION)

A6.

MEAN: 10.500000000  
 VARIANCE: 10.468499985  
 STANDARD DEVIATION: 3.235506141

LINE	AMOUNT	PROBABILITY	CUMULATIVE PROBABILITY	STOP-LOSS PREMIUM	STD. DEV. OF STOP-LOSS
1	0.000000000	0.00002711	0.00002711	10.50000000	3.235506141
2	1.000000000	0.00023546	0.00031257	9.50002711	3.235506141
3	2.000000000	0.00150276	0.00181533	8.50003967	3.231059696
4	2.999999999	0.00527246	0.00708779	7.50215500	3.214204611
5	3.999999999	0.01366994	0.02075773	6.50924277	3.167595510
6	4.999999999	0.02918113	0.05013886	5.53020350	3.072572283
7	5.999999999	0.05114748	0.10128634	4.58033935	2.919339227
8	6.999999999	0.07632015	0.17760649	3.68162569	2.713858037
9	9.000000000	0.10092758	0.27853407	2.85973215	2.473109929
10	9.000000000	0.11753320	0.39606727	2.13876622	2.213977651
11	10.000000000	0.12377805	0.52064532	1.53563346	1.945240045
12	11.000000000	0.11616870	0.63681402	1.05627979	1.670172299
13	12.000000011	0.10339280	0.74020682	0.67509279	1.392388374
14	13.000000016	0.08346560	0.82367242	0.43728960	1.126586275
15	14.000000020	0.06255435	0.88622677	0.26295207	0.880834025
16	15.000000024	0.04374412	0.93097089	0.15116374	0.665956879
17	16.000000029	0.02967006	0.96064095	0.08312962	0.490236415
18	17.000000035	0.01768027	0.97832122	0.04376055	0.350767545
19	18.000000041	0.01029416	0.98861538	0.02237156	0.244942035
20	19.000000046	0.00567665	0.99429203	0.01067672	0.167283309
21	20.000000052	0.00297293	0.99726496	0.00495854	0.111927520
22	21.000000059	0.00148244	0.99874740	0.00221334	0.073451620
23	22.000000066	0.00070540	0.99945280	0.00095058	0.047320137
24	23.000000074	0.00032097	0.99977377	0.00039323	0.025947925
25	24.000000081	0.00013992	0.99991369	0.00015635	0.018629610
26	25.000000089	0.00005554	0.99996923	0.00006038	0.011396304
27	26.000000097	0.00002354	0.99999277	0.00002246	0.006858572
28	27.000000105	0.00000911	0.99999988	0.00000303	0.004062400
29	28.000000114	0.00000340	0.99999999	0.00000031	0.002363973
30	29.000000123	0.00000123	0.99999999	0.00000005	0.001360455
31	30.000000132	0.00000043	0.99999999	0.00000003	0.000769476
32	31.000000142	0.00000014	0.99999999	0.00000001	0.000428572
33	32.000000151	0.00000005	0.99999999	0.00000000	0.000234711
34	33.000000161	0.00000001	0.99999999	0.00000000	0.000126065
35	34.000000171	0.00000000	1.00000000	0.00000000	0.000065944
36	35.000000182	0.00000000	1.00000000	0.00000000	0.000033097
37	36.000000192	0.00000000	1.00000000	0.00000000	0.000015454
38	37.000000202	0.00000000	1.00000000	0.00000000	0.000006278
39	38.000000210	0.00000000	1.00000000	0.00000000	0.000001859
40	39.000000219	0.00000000	1.00000000	0.00000000	0.000000000

RUN #1  
N = 4

NDRM OUTPUT  
(USING FIRST DIMENSION)

MEAN: 7.25  
VARIANCE: 9.34  
STANDARD DEVIATION: 3.01

LINE	AMOUNT	PROBABILITY	CUMULATIVE PROBABILITY	STOP-LOSS PREMIUM	STD. DEV. OF STOP-LOSS
1	0.00	0.00154495	0.00154495	7.25	3.01
2	1.00	0.00897909	0.01052404	6.25	2.99
3	2.00	0.02694682	0.03747086	5.26	2.93
4	3.00	0.05566227	0.09313312	4.30	2.86
5	4.00	0.08900775	0.18214087	3.39	2.80
6	5.00	0.11746615	0.29960703	2.57	2.35
7	6.00	0.13316424	0.43277127	1.87	2.39
8	7.00	0.13323974	0.56601101	1.31	1.82
9	8.00	0.11998340	0.68599442	0.97	1.54
10	9.00	0.09808070	0.78407520	0.56	1.27
11	10.00	0.07496427	0.85903947	0.34	1.01
12	11.00	0.05313442	0.91217389	0.20	0.70
13	12.00	0.03533231	0.94750620	0.11	0.59
14	13.00	0.02228374	0.96979094	0.06	0.43
15	14.00	0.01334519	0.98313613	0.03	0.31
16	15.00	0.00703432	0.99017045	0.02	0.22
17	16.00	0.00418759	0.99435804	0.01	0.15
18	17.00	0.00221011	0.99656815	0.00	0.10
19	18.00	0.00112521	0.99769336	0.00	0.07
20	19.00	0.00055397	0.99824733	0.00	0.05
21	20.00	0.00026428	0.99851161	0.00	0.03
22	21.00	0.00012239	0.99863400	0.00	0.02
23	22.00	0.00005510	0.99868910	0.00	0.01
24	23.00	0.00002414	0.99871324	0.00	0.01
25	24.00	0.00001031	0.99872355	0.00	0.01
26	25.00	0.00000429	0.99872784	0.00	0.00
27	26.00	0.00000174	0.99872958	0.00	0.00
28	27.00	0.00000059	0.99872997	0.00	0.00
29	28.00	0.00000027	0.99872999	0.00	0.00
30	29.00	0.00000010	0.99872999	0.00	0.00
31	30.00	0.00000004	0.99872999	0.00	0.00
32	31.00	0.00000001	0.99872999	0.00	0.00
33	32.00	0.00000000	1.00000000	0.00	0.00

K7  
TESTS / 175 / 3500  
(USING FIRST DIMENSION)

A7.

MEAN: 12.25000000  
 VARIANCE: 12.207125002  
 STANDARD DEVIATION: 3.493869631

LINE	AMOUNT	PROBABILITY	CUMULATIVE PROBABILITY	STOP-LOSS PREMIUM	STD. DEV. OF STOP-LOSS
1	0.000000000	0.00000463	0.00000463	12.25000000	3.493869631
2	1.000000000	0.00005757	0.00006220	11.25000468	3.492760833
3	1.999999999	0.00005377	0.00011597	10.25000694	3.492726086
4	2.999999997	0.00014882	0.00026479	9.25004297	3.487400669
5	3.999999997	0.00044877	0.00071356	8.25234782	3.469571335
6	4.999999996	0.00102520	0.00173876	7.25865143	3.425542334
7	5.999999995	0.00224521	0.00398397	6.27590031	3.339363172
8	6.999999994	0.00391363	0.00789760	5.31549130	3.202520270
9	7.999999994	0.00609434	0.01399194	4.39426359	3.014745680
10	8.999999994	0.00818947	0.02218141	3.53314022	2.788442474
11	9.999999995	0.01004154	0.03222295	2.73390658	2.539034477
12	10.999999997	0.01119742	0.04342037	2.07509749	2.277500892
13	12.000000001	0.01142695	0.05484732	1.50816582	2.009261443
14	13.000000009	0.01078880	0.06563612	1.05551370	1.733536432
15	14.000000014	0.00942039	0.07505651	0.71054640	1.458566541
16	15.000000019	0.00763946	0.08269597	0.45978303	1.192847060
17	16.000000023	0.00580260	0.08849857	0.28591428	0.947467931
18	17.000000028	0.00423437	0.09273294	0.17087156	0.731542947
19	18.000000033	0.00287780	0.09561074	0.09317259	0.550064671
20	19.000000038	0.00182367	0.09743441	0.05425167	0.403630833
21	20.000000043	0.00112379	0.09855820	0.02885441	0.277500892
22	21.000000048	0.00065908	0.09921728	0.01478095	0.203029582
23	22.000000054	0.00036071	0.09957799	0.00729835	0.140694960
24	23.000000060	0.00019427	0.09977226	0.00347546	0.095405847
25	24.000000065	0.00009933	0.09987159	0.00159684	0.063008156
26	25.000000072	0.00004314	0.09991473	0.00071055	0.041721730
27	25.000000078	0.00002280	0.09993753	0.00030541	0.026936347
28	27.000000085	0.00010249	0.09994002	0.00012706	0.017124665
29	28.000000092	0.00004465	0.09998467	0.00005121	0.010724386
30	29.000000099	0.00001873	0.09999340	0.00002001	0.006618126
31	30.000000106	0.00000763	0.09999503	0.00000759	0.004025707
32	31.000000113	0.00000350	0.09999853	0.00000279	0.002414392
33	32.000000120	0.00000114	0.09999967	0.00000100	0.001427946
34	33.000000128	0.00000042	0.09999977	0.00000035	0.000632841
35	34.000000136	0.00000015	0.09999992	0.00000012	0.000478864
36	35.000000144	0.00000005	0.09999997	0.00000004	0.000271132
37	36.000000153	0.00000002	0.09999999	0.00000001	0.000150761
38	37.000000162	0.00000001	1.00000000	0.00000000	0.000081835
39	38.000000170	0.00000000	1.00000000	0.00000000	0.000042832
40	39.000000180	0.00000000	1.00000000	0.00000000	0.000021075
41	40.000000198	0.00000000	1.00000000	0.00000000	0.000009216
42	41.000000195	0.00000000	1.00000000	0.00000000	0.000003072
43	42.000000177	0.00000000	1.00000000	0.00000000	0.000000000

RUN #1  
N=8

NUM4/OUTPUT  
(USING FIRST DIMENSION)

MEAN: 7.14  
VARIANCE: 7.93  
STANDARD DEVIATION: 2.82

LINE	AMOUNT	PROBABILITY	CUMULATIVE PROBABILITY	STOP-LOSS PREMIUM	STD. DEV. OF STOP-LOSS
1	0.00	0.00116705	0.00116705	7.14	2.82
2	1.00	0.00740693	0.00857387	6.14	2.81
3	2.00	0.02400322	0.03257710	5.15	2.76
4	3.00	0.05208272	0.08465981	4.19	2.64
5	4.00	0.08896085	0.17362066	3.27	2.44
6	5.00	0.12108802	0.29470868	2.45	2.20
7	6.00	0.14079217	0.43550085	1.74	1.93
8	7.00	0.14156680	0.57706765	1.18	1.66
9	8.00	0.12619907	0.70326672	0.76	1.38
10	9.00	0.10129141	0.80455814	0.46	1.11
11	10.00	0.07411743	0.87867557	0.27	0.86
12	11.00	0.04995624	0.92863181	0.15	0.64
13	12.00	0.03128786	0.96001967	0.08	0.46
14	13.00	0.01834512	0.97836479	0.04	0.33
15	14.00	0.01013430	0.98849909	0.02	0.22
16	15.00	0.00530338	0.99380247	0.01	0.15
17	16.00	0.00264111	0.99644358	0.00	0.10
18	17.00	0.00125643	0.99769999	0.00	0.06
19	18.00	0.00057285	0.99827284	0.00	0.04
20	19.00	0.00025095	0.99852379	0.00	0.03
21	20.00	0.00010587	0.99862966	0.00	0.02
22	21.00	0.00004310	0.99867276	0.00	0.01
23	22.00	0.00001696	0.99868972	0.00	0.01
24	23.00	0.00000646	0.99869618	0.00	0.00
25	24.00	0.00000239	0.99869857	0.00	0.00
26	25.00	0.00000085	0.99869942	0.00	0.00
27	26.00	0.00000030	0.99869972	0.00	0.00
28	27.00	0.00000010	0.99869982	0.00	0.00
29	28.00	0.00000003	0.99869985	0.00	0.00
30	29.00	0.00000001	1.00000000	0.00	0.00
31	30.00	0.00000000	1.00000000	0.00	0.00
32	31.00	0.00000000	1.00000000	0.00	0.00
33	32.00	0.00000000	1.00000000	0.00	0.00
34	33.00	0.00000000	1.00000000	0.00	0.00
35	34.00	0.00000000	1.00000000	0.00	0.00
36	35.00	0.00000000	1.00000000	0.00	0.00
37	36.00	0.00000000	1.00000000	0.00	0.00
38	37.00	0.00000000	1.00000000	0.00	0.00
39	38.00	0.00000000	1.00000000	0.00	0.00
40	39.00	0.00000000	1.00000000	0.00	0.00
41	40.00	0.00000000	1.00000000	0.00	0.00
42	41.00	0.00000000	1.00000000	0.00	0.00
43	42.00	0.00000000	1.00000000	0.00	0.00

NORM/CUTPUT  
(USING FIRST DIMENSION)

Run #1  
n=1

MEAN: 7.64  
VARIANCE: 13.23  
STANDARD DEVIATION: 3.64

LINE	AMOUNT	PROBABILITY	CUMULATIVE PROBABILITY	STOP-LOSS PREMIUM	STD. DEV. OF STOP-LOSS
1	0.00	0.00407865	0.00407865	7.64	3.64
2	1.00	0.01634395	0.02042260	6.64	3.60
3	2.00	0.03639545	0.05731805	5.66	3.50
4	3.00	0.06128907	0.11860712	4.72	3.32
5	4.00	0.08409453	0.20360175	3.84	3.19
6	5.00	0.10127029	0.30517204	3.04	2.93
7	6.00	0.10908111	0.41505314	2.35	2.57
8	7.00	0.10926784	0.52462099	1.76	2.30
9	8.00	0.10216966	0.62679065	1.29	2.02
10	9.00	0.08998323	0.71677388	0.91	1.75
11	10.00	0.07536624	0.79214011	0.63	1.48
12	11.00	0.06030737	0.85244748	0.42	1.22
13	12.00	0.04624262	0.89869017	0.27	0.99
14	13.00	0.03404109	0.93273126	0.17	0.76
15	14.00	0.02498422	0.95671547	0.11	0.61
16	15.00	0.01838733	0.97320280	0.06	0.46
17	16.00	0.01372746	0.98397026	0.04	0.34
18	17.00	0.00975799	0.99038825	0.02	0.25
19	18.00	0.00409809	0.99478634	0.01	0.19
20	19.00	0.00219256	0.99717919	0.01	0.13
21	20.00	0.00134550	0.99837499	0.00	0.09
22	21.00	0.00072939	0.99925438	0.00	0.06
23	22.00	0.00038113	0.99963551	0.00	0.04
24	23.00	0.00019211	0.99982752	0.00	0.03
25	24.00	0.00009347	0.99992100	0.00	0.02
26	25.00	0.00004393	0.99996533	0.00	0.01
27	26.00	0.00001995	0.99998533	0.00	0.01
28	27.00	0.00000877	0.99999375	0.00	0.00
29	28.00	0.00000373	0.99999748	0.00	0.00
30	29.00	0.00000154	0.99999901	0.00	0.00
31	30.00	0.00000061	0.99999962	0.00	0.00
32	31.00	0.00000024	0.99999986	0.00	0.00
33	32.00	0.00000009	0.99999995	0.00	0.00
34	33.00	0.00000003	0.99999998	0.00	0.00
35	34.00	0.00000001	0.99999999	0.00	0.00
36	35.00	0.00000000	1.00000000	0.00	0.00
37	36.00	0.00000000	1.00000000	0.00	0.00
38	37.00	0.00000000	1.00000000	0.00	0.00
39	38.00	0.00000000	1.00000000	0.00	0.00
40	39.00	0.00000000	1.00000000	0.00	0.00
41	40.00	0.00000000	1.00000000	0.00	0.00
42	41.00	0.00000000	1.00000000	0.00	0.00
43	42.00	0.00000000	1.00000000	0.00	0.00

RUN #2

n=4

NORM/OUTPUT  
(USING FIRST DIMENSION)

MEAN: 7.21  
 VARIANCE: 8.87  
 STANDARD DEVIATION: 2.98

LINE	AMOUNT	PROBABILITY	CUMULATIVE PROBABILITY	STOP-LOSS PREMIUM	STD. DEV. OF STOP-LOSS
1	0.00	0.00154516	0.00154516	7.21	2.98
2	1.00	0.0092033	0.01056549	6.21	2.96
3	2.00	0.02714245	0.03770795	5.22	2.91
4	3.00	0.05614322	0.09385117	4.26	2.77
5	4.00	0.08961443	0.18366561	3.35	2.57
6	5.00	0.11948854	0.30215415	2.54	2.32
7	6.00	0.13417797	0.43633212	1.84	2.06
8	7.00	0.13400435	0.57033647	1.28	1.78
9	8.00	0.12033743	0.69067391	0.95	1.51
10	9.00	0.09858775	0.78926166	0.54	1.24
11	10.00	0.07452123	0.86378289	0.33	0.98
12	11.00	0.05244350	0.91622639	0.19	0.75
13	12.00	0.03401499	0.95024138	0.11	0.56
14	13.00	0.02156030	0.97200168	0.06	0.41
15	14.00	0.01273743	0.98513911	0.03	0.29
16	15.00	0.00716816	0.99230727	0.01	0.20
17	16.00	0.00385661	0.99616388	0.01	0.14
18	17.00	0.00198983	0.99815372	0.00	0.09
19	18.00	0.00098718	0.99914090	0.00	0.06
20	19.00	0.00047201	0.99961291	0.00	0.04
21	20.00	0.00021796	0.99983087	0.00	0.03
22	21.00	0.00009738	0.99992825	0.00	0.02
23	22.00	0.00004216	0.99997041	0.00	0.01
24	23.00	0.00001772	0.99998817	0.00	0.01
25	24.00	0.00000723	0.99999536	0.00	0.00
26	25.00	0.00000287	0.99999823	0.00	0.00
27	26.00	0.00000111	0.99999934	0.00	0.00
28	27.00	0.00000042	0.99999976	0.00	0.00
29	28.00	0.00000015	0.99999992	0.00	0.00
30	29.00	0.00000005	0.99999997	0.00	0.00
31	30.00	0.00000002	0.99999999	0.00	0.00
32	31.00	0.00000001	1.00000000	0.00	0.00
33	32.00	0.00000000	1.00000000	0.00	0.00
34	33.00	0.00000000	1.00000000	0.00	0.00
35	34.00	0.00000000	1.00000000	0.00	0.00
36	35.00	0.00000000	1.00000000	0.00	0.00
37	36.00	0.00000000	1.00000000	0.00	0.00
38	37.00	0.00000000	1.00000000	0.00	0.00
39	38.00	0.00000000	1.00000000	0.00	0.00
40	39.00	0.00000000	1.00000000	0.00	0.00
41	40.00	0.00000000	1.00000000	0.00	0.00
42	41.00	0.00000000	1.00000000	0.00	0.00
43	42.00	0.00000000	1.00000000	0.00	0.00

NORM/CUTPUT  
(USING FIRST DIMENSION)

RUN #2  
n=8

MEAN: 7.14  
VARIANCE: 7.92  
STANDARD DEVIATION: 2.81

LINE	AMOUNT	PROBABILITY	CUMULATIVE PROBABILITY	STOP-LOSS PREMIUM	STD. DEV. OF STOP-LOSS
1	0.00	0.00116800	0.00116800	7.14	2.81
2	1.00	0.00741311	0.00858111	6.14	2.80
3	2.00	0.02402362	0.03260474	5.15	2.75
4	3.00	0.05292656	0.08553129	4.18	2.63
5	4.00	0.08903051	0.17456191	3.27	2.44
6	5.00	0.12177447	0.29633637	2.44	2.20
7	6.00	0.14007635	0.43721273	1.74	1.93
8	7.00	0.14162693	0.57884075	1.18	1.66
9	8.00	0.12622355	0.70506430	0.76	1.36
10	9.00	0.10127679	0.80634110	0.46	1.11
11	10.00	0.07407174	0.88041284	0.27	0.86
12	11.00	0.04909290	0.93030573	0.15	0.64
13	12.00	0.03122063	0.96152636	0.08	0.46
14	13.00	0.01829416	0.97981053	0.04	0.32
15	14.00	0.01000500	0.98981554	0.02	0.22
16	15.00	0.00526714	0.99508268	0.01	0.15
17	16.00	0.00281647	0.99789915	0.00	0.10
18	17.00	0.00124000	0.99913915	0.00	0.06
19	18.00	0.00056355	0.99970270	0.00	0.04
20	19.00	0.00024573	0.99994743	0.00	0.02
21	20.00	0.00010309	0.99995052	0.00	0.02
22	21.00	0.00005169	0.99999221	0.00	0.01
23	22.00	0.00001629	0.99999850	0.00	0.01
24	23.00	0.00000615	0.99999435	0.00	0.00
25	24.00	0.00000223	0.99999212	0.00	0.00
26	25.00	0.00000090	0.99999122	0.00	0.00
27	26.00	0.00000037	0.99999085	0.00	0.00
28	27.00	0.00000019	0.99999066	0.00	0.00
29	28.00	0.00000009	0.99999057	0.00	0.00
30	29.00	0.00000004	1.00000000	0.00	0.00
31	30.00	0.00000000	1.00000000	0.00	0.00
32	31.00	0.00000000	1.00000000	0.00	0.00
33	32.00	0.00000000	1.00000000	0.00	0.00
34	33.00	0.00000000	1.00000000	0.00	0.00
35	34.00	0.00000000	1.00000000	0.00	0.00
36	35.00	0.00000000	1.00000000	0.00	0.00
37	36.00	0.00000000	1.00000000	0.00	0.00
38	37.00	0.00000000	1.00000000	0.00	0.00
39	38.00	0.00000000	1.00000000	0.00	0.00
40	39.00	0.00000000	1.00000000	0.00	0.00
41	40.00	0.00000000	1.00000000	0.00	0.00
42	41.00	0.00000000	1.00000000	0.00	0.00
43	42.00	0.00000000	1.00000000	0.00	0.00

NORM/TESTS/100/3500  
 (USING FIRST DIMENSION)

RUN # 3  
 n = 4

MEAN: 7.00  
 VARIANCE: 6.99  
 STANDARD DEVIATION: 2.64

LINE	AMOUNT	PROBABILITY	CUMULATIVE PROBABILITY	STOP-LOSS PREMIUM	STD. DEV. OF STOP-LOSS
1	0.00	0.00090551	0.00090551	7.00	2.64
2	1.00	0.00635129	0.00725680	6.00	2.63
3	2.00	0.02226770	0.02952451	5.01	2.59
4	3.00	0.05203235	0.08155685	4.04	2.48
5	4.00	0.09116083	0.17271773	3.12	2.29
6	5.00	0.12773484	0.30045259	2.29	2.05
7	6.00	0.14910931	0.44956189	1.59	1.78
8	7.00	0.14915200	0.59871389	1.04	1.51
9	8.00	0.13050800	0.72922189	0.64	1.23
10	9.00	0.10147716	0.83069905	0.37	0.96
11	10.00	0.07099334	0.90169239	0.20	0.71
12	11.00	0.04513878	0.94683117	0.10	0.51
13	12.00	0.02630080	0.97313197	0.05	0.35
14	13.00	0.01414170	0.98727367	0.02	0.23
15	14.00	0.00705870	0.99433237	0.01	0.15
16	15.00	0.00328740	0.99761973	0.00	0.09
17	16.00	0.00143497	0.99905470	0.00	0.06
18	17.00	0.00058935	0.99964415	0.00	0.03
19	18.00	0.00022853	0.99987268	0.00	0.02
20	19.00	0.00008393	0.99995661	0.00	0.01
21	20.00	0.00002928	0.99998589	0.00	0.01
22	21.00	0.00000972	0.99999561	0.00	0.00
23	22.00	0.00000308	0.99999869	0.00	0.00
24	23.00	0.00000093	0.99999963	0.00	0.00
25	24.00	0.00000027	0.99999990	0.00	0.00
26	25.00	0.00000008	0.99999997	0.00	0.00
27	26.00	0.00000002	0.99999999	0.00	0.00
28	27.00	0.00000001	1.00000000	0.00	0.00
29	28.00	0.00000000	1.00000000	0.00	0.00
30	29.00	0.00000000	1.00000000	0.00	0.00
31	30.00	0.00000000	1.00000000	0.00	0.00
32	31.00	0.00000000	1.00000000	0.00	0.00



NORM/CUTPUT  
(USING FIRST DIMENSION)

Run #2  
n=1

MEAN: 7.34  
VARIANCE: 11.49  
STANDARD DEVIATION: 3.39

LINE	AMOUNT	PROBABILITY	CUMULATIVE PROBABILITY	STOP-LOSS PREMIUM	STD. DEV. OF STOP-LOSS
1	0.00	0.00351391	0.00351391	7.34	3.39
2	1.00	0.01542326	0.01893717	6.35	3.36
3	2.00	0.03695897	0.05589614	5.36	3.27
4	3.00	0.06415578	0.12005192	4.42	3.09
5	4.00	0.09034970	0.21040163	3.54	2.67
6	5.00	0.10954637	0.31994800	2.75	2.61
7	6.00	0.11845568	0.43840368	2.07	2.34
8	7.00	0.11684426	0.55524794	1.51	2.07
9	8.00	0.10674485	0.66199279	1.06	1.79
10	9.00	0.09127048	0.75326327	0.73	1.52
11	10.00	0.07357991	0.82684318	0.48	1.26
12	11.00	0.05622524	0.88306842	0.31	1.01
13	12.00	0.04088367	0.92395209	0.19	0.80
14	13.00	0.02837475	0.95232684	0.11	0.61
15	14.00	0.01854266	0.97116950	0.07	0.46
16	15.00	0.01199799	0.98316749	0.04	0.34
17	16.00	0.00733653	0.99050402	0.02	0.25
18	17.00	0.00431601	0.99482003	0.01	0.19
19	18.00	0.00244591	0.99726594	0.01	0.12
20	19.00	0.00133694	0.99860288	0.00	0.09
21	20.00	0.00070565	0.99930853	0.00	0.06
22	21.00	0.00036091	0.99966944	0.00	0.04
23	22.00	0.00017770	0.99984714	0.00	0.03
24	23.00	0.00008493	0.99993207	0.00	0.02
25	24.00	0.00003934	0.99997141	0.00	0.01
26	25.00	0.00001757	0.99998884	0.00	0.01
27	26.00	0.00000770	0.99999614	0.00	0.00
28	27.00	0.00000320	0.99999934	0.00	0.00
29	28.00	0.00000134	0.99999999	0.00	0.00
30	29.00	0.00000054	0.99999999	0.00	0.00
31	30.00	0.00000021	0.99999999	0.00	0.00
32	31.00	0.00000009	0.99999999	0.00	0.00
33	32.00	0.00000003	0.99999999	0.00	0.00
34	33.00	0.00000001	0.99999999	0.00	0.00
35	34.00	0.00000000	1.00000000	0.00	0.00
36	35.00	0.00000000	1.00000000	0.00	0.00
37	36.00	0.00000000	1.00000000	0.00	0.00
38	37.00	0.00000000	1.00000000	0.00	0.00
39	38.00	0.00000000	1.00000000	0.00	0.00
40	39.00	0.00000000	1.00000000	0.00	0.00
41	40.00	0.00000000	1.00000000	0.00	0.00
42	41.00	0.00000000	1.00000000	0.00	0.00
43	42.00	0.00000000	1.00000000	0.00	0.00

NGRM/TESTS/100/3500  
 (USING FIRST DIMENSION)

RUN #3  
 n=1

MEAN: 7.00  
 VARIANCE: 6.99  
 STANDARD DEVIATION: 2.64

LINE	AMOUNT	PROBABILITY	CUMULATIVE PROBABILITY	STOP-LOSS PREMIUM	STD. DEV. OF * STOP-LOSS
1	0.00	0.00090551	0.00090551	7.00	2.64
2	1.00	0.00645129	0.00725630	6.00	2.63
3	2.00	0.02246770	0.02952451	5.01	2.59
4	3.00	0.05203235	0.08155685	4.04	2.48
5	4.00	0.09116089	0.17271773	3.12	2.29
6	5.00	0.12773484	0.30045258	2.29	2.05
7	6.00	0.14910931	0.44956189	1.59	1.78
8	7.00	0.14915200	0.59871389	1.04	1.51
9	8.00	0.13050800	0.72922189	0.64	1.23
10	9.00	0.10147716	0.83069905	0.37	0.96
11	10.00	0.07099334	0.90169239	0.20	0.71
12	11.00	0.04513873	0.94683117	0.10	0.51
13	12.00	0.02630080	0.97313197	0.05	0.35
14	13.00	0.01414170	0.98727367	0.02	0.23
15	14.00	0.00705870	0.99433237	0.01	0.15
16	15.00	0.00328740	0.99761983	0.00	0.09
17	16.00	0.00143497	0.99905480	0.00	0.06
18	17.00	0.00058935	0.99964415	0.00	0.03
19	18.00	0.00022853	0.99987268	0.00	0.02
20	19.00	0.00008393	0.99995661	0.00	0.01
21	20.00	0.00002928	0.99998589	0.00	0.01
22	21.00	0.00000972	0.99999561	0.00	0.00
23	22.00	0.00000309	0.99999859	0.00	0.00
24	23.00	0.00000093	0.99999953	0.00	0.00
25	24.00	0.00000027	0.99999980	0.00	0.00
26	25.00	0.00000008	0.99999997	0.00	0.00
27	26.00	0.00000002	0.99999999	0.00	0.00
28	27.00	0.00000001	1.00000000	0.00	0.00
29	28.00	0.00000000	1.00000000	0.00	0.00
30	29.00	0.00000000	1.00000000	0.00	0.00
31	30.00	0.00000000	1.00000000	0.00	0.00
32	31.00	0.00000000	1.00000000	0.00	0.00

NUM/TESTS/100/3500  
 (USING FIRST DIMENSION)

Run #3  
 $n = 8$

MEAN: 7.00  
 VARIANCE: 6.99  
 STANDARD DEVIATION: 2.64

LINE	AMOUNT	PRGBABILITY	CUMULATIVE PROBABILITY	STOP-LOSS PREMIUM	STD. DEV. OF STOP-LOSS
1	0.00	0.00090551	0.00090551	7.00	2.64
2	1.00	0.00635129	0.00725680	6.00	2.63
3	2.00	0.02226770	0.02952451	5.01	2.59
4	3.00	0.05263235	0.08155685	4.04	2.48
5	4.00	0.09116088	0.17271773	3.12	2.29
6	5.00	0.12773484	0.30045257	2.29	2.05
7	6.00	0.14910931	0.44956189	1.59	1.78
8	7.00	0.14915200	0.59871389	1.04	1.51
9	8.00	0.13050800	0.72922189	0.64	1.23
10	9.00	0.10147716	0.83069905	0.37	0.96
11	10.00	0.07099334	0.90169239	0.20	0.71
12	11.00	0.04513878	0.94683117	0.10	0.51
13	12.00	0.02630080	0.97313197	0.05	0.35
14	13.00	0.01414170	0.98727367	0.02	0.23
15	14.00	0.00705370	0.99432737	0.01	0.15
16	15.00	0.00328746	0.99761483	0.00	0.09
17	16.00	0.00143497	0.99904980	0.00	0.06
18	17.00	0.00058915	0.99963895	0.00	0.03
19	18.00	0.00022853	0.99986748	0.00	0.02
20	19.00	0.00008393	0.99995141	0.00	0.01
21	20.00	0.00002928	0.99998069	0.00	0.01
22	21.00	0.00000972	0.99999041	0.00	0.00
23	22.00	0.00000303	0.99999344	0.00	0.00
24	23.00	0.00000093	0.99999437	0.00	0.00
25	24.00	0.00000027	0.99999464	0.00	0.00
26	25.00	0.00000008	0.99999472	0.00	0.00
27	26.00	0.00000002	0.99999474	0.00	0.00
28	27.00	0.00000001	1.00000000	0.00	0.00
29	28.00	0.00000000	1.00000000	0.00	0.00
30	29.00	0.00000000	1.00000000	0.00	0.00
31	30.00	0.00000000	1.00000000	0.00	0.00
32	31.00	0.00000000	1.00000000	0.00	0.00

NORM/OUTPUT  
(USING FIRST DIMENSION)

Run #4  
n=4

MEAN: 6.82  
VARIANCE: 7.10  
STANDARD DEVIATION: 2.66

LINE	AMOUNT	PROBABILITY	CUMULATIVE PROBABILITY	STOP-LOSS PREMIUM	STD. DEV. OF STOP-LOSS
1	0.00	0.00138283	0.00138233	6.82	2.66
2	1.00	0.00804180	0.01002464	5.82	2.65
3	2.00	0.02701158	0.03763622	4.83	2.60
4	3.00	0.05992102	0.09755724	3.86	2.47
5	4.00	0.09903719	0.19659443	2.96	2.26
6	5.00	0.13259307	0.32918750	2.16	2.01
7	6.00	0.14941279	0.47860029	1.49	1.75
8	7.00	0.14544991	0.62405020	0.97	1.47
9	8.00	0.12404968	0.74869988	0.59	1.19
10	9.00	0.09539872	0.84409860	0.34	0.92
11	10.00	0.06594315	0.91004176	0.18	0.68
12	11.00	0.04154793	0.95158969	0.09	0.48
13	12.00	0.02404274	0.97563243	0.04	0.33
14	13.00	0.01206075	0.98869318	0.02	0.22
15	14.00	0.00639434	0.99488752	0.01	0.14
16	15.00	0.00296932	0.99785685	0.00	0.09
17	16.00	0.00129325	0.99915010	0.00	0.05
18	17.00	0.00053026	0.99968036	0.00	0.03
19	18.00	0.00020537	0.99988573	0.00	0.02
20	19.00	0.00007535	0.99996108	0.00	0.01
21	20.00	0.00002620	0.99998728	0.00	0.01
22	21.00	0.00000872	0.99999600	0.00	0.00
23	22.00	0.00000276	0.99999876	0.00	0.00
24	23.00	0.00000084	0.99999960	0.00	0.00
25	24.00	0.00000024	0.99999984	0.00	0.00
26	25.00	0.00000007	0.99999991	0.00	0.00
27	26.00	0.00000002	0.99999993	0.00	0.00
28	27.00	0.00000000	1.00000000	0.00	0.00
29	28.00	0.00000000	1.00000000	0.00	0.00
30	29.00	0.00000000	1.00000000	0.00	0.00
31	30.00	0.00000000	1.00000000	0.00	0.00
32	31.00	0.00000000	1.00000000	0.00	0.00

NORM/OUTPUT  
(USING FIRST DIMENSION)

RUN #4  
n=8

MEAN: 6.93  
VARIANCE: 7.03  
STANDARD DEVIATION: 2.65

LINE	AMOUNT	PROBABILITY	CUMULATIVE PROBABILITY	STOP-LOSS PREMIUM	STD. DEV. OF STOP-LOSS
1	0.00	0.00107291	0.00107291	0.93	2.65
2	1.00	0.00716963	0.00824254	5.93	2.64
3	2.00	0.02420221	0.03244475	4.94	2.59
4	3.00	0.05491607	0.08736083	3.97	2.43
5	4.00	0.09406424	0.18142505	3.06	2.28
6	5.00	0.12934615	0.31077121	2.24	2.04
7	6.00	0.14924893	0.46022019	1.55	1.77
8	7.00	0.14701107	0.60803126	1.01	1.49
9	8.00	0.12836354	0.73539480	0.62	1.21
10	9.00	0.09924369	0.83563849	0.36	0.94
11	10.00	0.06913396	0.90477244	0.19	0.70
12	11.00	0.04301507	0.94859751	0.10	0.50
13	12.00	0.02546773	0.97405524	0.05	0.34
14	13.00	0.01306885	0.98772409	0.02	0.23
15	14.00	0.00601337	0.99453746	0.01	0.14
16	15.00	0.00316993	0.99770741	0.00	0.09
17	16.00	0.00138261	0.99909002	0.00	0.06
18	17.00	0.00056752	0.99965753	0.00	0.03
19	18.00	0.00021997	0.99987751	0.00	0.02
20	19.00	0.00008070	0.99995827	0.00	0.01
21	20.00	0.00002816	0.99998643	0.00	0.01
22	21.00	0.00000935	0.99999573	0.00	0.00
23	22.00	0.00000296	0.99999874	0.00	0.00
24	23.00	0.00000090	0.99999964	0.00	0.00
25	24.00	0.00000026	0.99999990	0.00	0.00
26	25.00	0.00000007	0.99999997	0.00	0.00
27	26.00	0.00000002	0.99999999	0.00	0.00
28	27.00	0.00000001	1.00000000	0.00	0.00
29	28.00	0.00000000	1.00000000	0.00	0.00
30	29.00	0.00000000	1.00000000	0.00	0.00
31	30.00	0.00000000	1.00000000	0.00	0.00
32	31.00	0.00000000	1.00000000	0.00	0.00

NORMAL OUTPUT  
(USING FIRST DIMENSION)

RUN #4  
n = 1

MEAN:	6.49
VARIANCE:	7.42
STANDARD DEVIATION:	2.72

LINE	AMOUNT	PROBABILITY	CUMULATIVE PROBABILITY	STOP-LOSS PREMIUM	STD. DEV. OF STOP-LOSS
1	0.00	0.00329728	0.00329728	6.49	2.72
2	1.00	0.01551116	0.01880844	5.49	2.70
3	2.00	0.04067075	0.05887920	4.51	2.61
4	3.00	0.07453878	0.13341798	3.57	2.44
5	4.00	0.11044585	0.24386383	2.70	2.22
6	5.00	0.13694159	0.38080542	1.94	1.96
7	6.00	0.14618752	0.52699294	1.33	1.68
8	7.00	0.13693164	0.66392458	0.85	1.41
9	8.00	0.11413100	0.77805558	0.52	1.13
10	9.00	0.08559060	0.86364617	0.29	0.87
11	10.00	0.05828188	0.92192805	0.16	0.64
12	11.00	0.03631313	0.95824118	0.08	0.45
13	12.00	0.02033894	0.97908012	0.04	0.31
14	13.00	0.01107744	0.99015746	0.02	0.20
15	14.00	0.00548188	0.99563934	0.01	0.13
16	15.00	0.00253664	0.99817598	0.00	0.08
17	16.00	0.00110188	0.99927786	0.00	0.05
18	17.00	0.00045090	0.99972876	0.00	0.03
19	18.00	0.00017437	0.99990313	0.00	0.02
20	19.00	0.00006391	0.99996703	0.00	0.01
21	20.00	0.00002226	0.99998929	0.00	0.01
22	21.00	0.00000735	0.99999664	0.00	0.00
23	22.00	0.00000234	0.99999898	0.00	0.00
24	23.00	0.00000071	0.99999972	0.00	0.00
25	24.00	0.00000021	0.99999992	0.00	0.00
26	25.00	0.00000006	0.99999998	0.00	0.00
27	26.00	0.00000002	0.99999999	0.00	0.00
28	27.00	0.00000000	1.00000000	0.00	0.00
29	28.00	0.00000000	1.00000000	0.00	0.00
30	29.00	0.00000000	1.00000000	0.00	0.00
31	30.00	0.00000000	1.00000000	0.00	0.00
32	31.00	0.00000000	1.00000000	0.00	0.00

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(RISK)NOTATION

The following notation is used in this manual and should be thoroughly mastered before proceeding further.

Frequency Distributions

Consider a game consisting of 1 flip of an unbiased coin, where \$1 is won if the coin turns up heads and \$1 is lost if the coin turns up tails. An example of a frequency distribution representing the profits realizable in this game is:

$$\begin{bmatrix} -1 & .5 \\ +1 & .5 \end{bmatrix}$$

The general form of this distribution is designated as

$$\left[ x_i^{(1)}, p_i^{(1)}; 1 \leq i \leq 2 \right].$$

A frequency distribution may have any number of lines. The superscripts on  $x$ ,  $p$  and  $n$  merely designate to which frequency distribution the elements belong, and represent neither exponents nor finite difference notation.

Whenever possible, we will reserve capital letters to represent frequency distributions. Lower case letters denote real numbers. The letters  $x$  and  $p$  will be restricted to the amount and the probability, respectively, in a frequency distribution; the letter  $y$  will be restricted to the second amount in a bivariate frequency distribution (see below). Any exceptions will be specifically noted as they occur.

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Convolution for Sums

If two coins are flipped (with the same payoff as above), then the frequency distribution of combined profits is:

$$\begin{bmatrix} -2 & .25 \\ 0 & .25 \\ 0 & .25 \\ +2 & .25 \end{bmatrix} \quad \text{or, equivalently,} \quad \begin{bmatrix} -2 & .25 \\ 0 & .50 \\ 2 & .25 \end{bmatrix}$$

This "two coin" distribution can be derived as follows:

$$\begin{bmatrix} -1 & .5 \\ +1 & .5 \end{bmatrix} \oplus \begin{bmatrix} -1 & .5 \\ +1 & .5 \end{bmatrix} = \begin{bmatrix} (-1)+(-1)=-2 & .5 \times .5 = .25 \\ (-1)+(+1)= 0 & .5 \times .5 = .25 \\ (+1)+(-1)= 0 & .5 \times .5 = .25 \\ (+1)+(+1)= 2 & .5 \times .5 = .25 \end{bmatrix},$$

taking each combination of a line from the first frequency distribution and a line from the second frequency distribution.

The operator  $\oplus$  is called "convolute for sums". We say that  $A \oplus A$  is the convolution of A and A for sums. In the above example,

$$A = \begin{bmatrix} -1 & .5 \\ +1 & .5 \end{bmatrix}.$$

Suppose  $A = \begin{bmatrix} -1 & .5 \\ 1 & .5 \end{bmatrix}$  and  $B = \begin{bmatrix} 0 & .2 \\ 2 & .8 \end{bmatrix}$ .

$$\text{Then } A \oplus B = \begin{bmatrix} -1 & .5 \\ 1 & .5 \end{bmatrix} \oplus \begin{bmatrix} 0 & .2 \\ 2 & .8 \end{bmatrix} = \begin{bmatrix} -1+0=-1 & .5 \times .2 = .1 \\ -1+2= 1 & .5 \times .8 = .4 \\ 1+0= 1 & .5 \times .2 = .1 \\ 1+2= 3 & .5 \times .8 = .4 \end{bmatrix} = \begin{bmatrix} -1 & .1 \\ 1 & .5 \\ 3 & .4 \end{bmatrix}$$

Alternatively, we may designate A as  $\left[ x_i^{(1)}, p_i^{(1)}; 1 \leq i \leq 2 \right]$   
and B as  $\left[ x_j^{(2)}, p_j^{(2)}; 1 \leq j \leq 2 \right]$ .



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Then  $A \oplus B$  would be designated as

$$\left[ x_i^{(1)} + x_j^{(2)}, p_i^{(1)} \cdot p_j^{(2)}; 1 \leq i \leq 2, 1 \leq j \leq 2 \right]$$

or as  $\left[ x_i^{(3)}, p_i^{(3)}; 1 \leq i \leq 3 \right]$ .

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Transformation Operations

If we wish to create  $C = \begin{bmatrix} -2 & .5 \\ 2 & .5 \end{bmatrix}$  from  $\begin{bmatrix} -1 & .5 \\ 1 & .5 \end{bmatrix}$ , we can

represent the operation as follows:

$$C = [2 \cdot x, 1 \cdot p] A = \begin{bmatrix} 2 \cdot (-1) & .5 \\ 2 \cdot (1) & .5 \end{bmatrix} = \begin{bmatrix} -2 & .5 \\ 2 & .5 \end{bmatrix}.$$

In this case, a bracketed expression is placed in direct conjunction with the symbol for a distribution (i.e., no operator such as  $\oplus$  intervenes). This represents a transformation of the distribution, where  $x$  and  $p$  are formal symbols to indicate that the 2 is multiplied by each  $x_i^{(1)}$  and that the  $p_i^{(1)}$  are to be left unchanged.

Other transformation operations are represented analogously. For example,

$$D = [2/x, .4 \cdot p] A = \begin{bmatrix} 2/(-1) & .4 \cdot .5 \\ 2/(+1) & .4 \cdot .5 \end{bmatrix} = \begin{bmatrix} -2 & .2 \\ 2 & .2 \end{bmatrix}$$

It is also possible to represent a transformation where one or more factors must be calculated. For example, if  $\bar{x}$  is the mean of the  $x_i$  values in the distribution, then

$$\left[ \frac{1}{\bar{x}} \cdot x, 1 \cdot p \right] A = \left[ \frac{x_i^{(1)}}{\sum_{j=1}^n x_j^{(1)} \cdot p_j^{(1)}}, p_i^{(1)}; 1 \leq i \leq n^{(1)} \right]$$

Convolution for Products

The operation  $\odot$  is called "convolute for products", and the result of  $A \odot B$  is called the "convolution of A and B for products".

$$A \odot B = \left[ x_i^{(1)}, p_i^{(1)}; 1 \leq i \leq n^{(1)} \right] \odot \left[ x_j^{(2)}, p_j^{(2)}; 1 \leq j \leq n^{(2)} \right]$$

$$= \left[ x_i^{(1)} \cdot x_j^{(2)}, p_i^{(1)} \cdot p_j^{(2)}; 1 \leq i \leq n^{(1)}, 1 \leq j \leq n^{(2)} \right]$$

$$\text{Thus, } A \odot B = \begin{bmatrix} -1 & .5 \\ 1 & .5 \end{bmatrix} \odot \begin{bmatrix} 0 & .2 \\ 2 & .8 \end{bmatrix} = \begin{bmatrix} -1 \cdot 0 = 0 & .5 \cdot .2 = .1 \\ -1 \cdot 2 = -2 & .5 \cdot .8 = .4 \\ 1 \cdot 0 = 0 & .5 \cdot .2 = .1 \\ 1 \cdot 2 = 2 & .5 \cdot .8 = .4 \end{bmatrix} = \begin{bmatrix} -2 & .4 \\ 0 & .2 \\ 2 & .4 \end{bmatrix}$$

Convolution to Merge

The third operator used is represented by the symbol  $\odot$  and is called a "merge convolution". The effect of  $A \odot B$  is to append distribution B to distribution A. To illustrate a common use of this operator:

$$\left( [x, .6 \cdot p] A \right) \odot \left( [x, .4 \cdot p] B \right) = \begin{bmatrix} -1 & .3 \\ 1 & .3 \end{bmatrix} \odot \begin{bmatrix} 0 & .08 \\ 2 & .32 \end{bmatrix} = \begin{bmatrix} -1 & .3 \\ 0 & .08 \\ 1 & .3 \\ 2 & .32 \end{bmatrix}.$$

Convolution to a Power

The operation  $A \oplus A$  can also be written as  $A^{\oplus 2}$ . In the general case,  $A^{\oplus n}$  is defined for integer values of  $n \geq 0$ .

$$A^{\oplus n} = A \underbrace{\oplus A \oplus \dots \oplus A}_{n-1 \text{ operations}} = \left( \left( \left( A \oplus A \right) \oplus A \right) \oplus \dots \right) \oplus A$$

$$A^{\oplus 1} = A, \text{ of course.}$$

$A^{\oplus 0}$  is defined as the ZERO distribution,  $[0,1]$ , a 100% probability of a zero amount.

$$\text{Similarly, } A^{\ominus n} = \underbrace{A \ominus A \ominus \dots \ominus A}_{n-1 \text{ operations}}$$

$$\text{Again, } A^{\ominus 1} = A$$

$A^{\ominus 0}$  is the UNITY distribution,  $[1,1]$ , a 100% probability of a unit amount.