Back testing of Investment performance by asset class

Sponsored by

Society of Actuaries Pension Section

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January 2013

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Executive Summary

Pension plan management is a difficult task. Pension plan benefits, which are related to an employee's age, earning history and other variables can result in a formidable fiduciary responsibility. Given the generally long term nature of pension plans, the behavior of the market plays a crucial role in a pension plan being able to meet its obligations. Regardless of the market performance, the structure of the benefits remains the same, unless they are negotiated to be at a different level.

In this study, we aimed to study primarily the impact of market performance on a pension plan's ability to meet its obligations. We studied periods from 1974 to 2010 and included many asset allocation strategies that varied from allocating 25% to 100% weight assigned to equity portfolios. We also studied the impact of managing plan assets more actively; from a steady move towards a fixed income (FI) portfolio as pension assets grow to a more aggressive style where the manager could move assets to and from FI as required based on assets available to fund liabilities. Finally, we also studied the impact of asset management when the plans are basically designed to wind down over time; we called them closed and frozen plans. Again, the goals were to determine which type of asset allocation system is the most efficient across all time horizons.

Our results show that it is not necessary to have a very aggressive posture to equities. Indeed, as assets become more exposed to equities, the efficiency of a portfolio (as measured by Sharpe ratio) declines. We found that an exposure to equity in the range of 35%-50% is sufficient to meet most pension obligations, provided that the plans are fully funded at the outset. We further found that a more aggressive approach, including an annual rebalancing of the portfolio provides for superior results, even if it may cost more to manage. Finally, we also found that it takes roughly an average of 7 years for a closed/frozen plan to reach a comfortable level of funding (defined as 120% of liabilities), even when the plans were forced to be less than 100% funded initially.

Acknowledgements

We acknowledge and thank the support of all members of the research committee for their valuable comments. We have tried our best to have these comments reflected in this draft version. We also would like to thank Barclay's for providing us with the data on fixed income portfolio used in this study. We are responsible for all mistakes and omissions.

Back testing of Investment performance by asset class

Introduction

Overview- Among the many uncertainties facing working United States citizens is their ability to fund and ensure a comfortable retirement stream. Given the median age of U.S. workers of 34.6 years (and expected to increase) and given the average overall balance in the 401(k) accounts of less than \$75,000 (based on Fidelity 2011 study), Americans are a long way from being able to afford a retirement that is ensured and dignified. There are several reasons for the predicament that we as a nation find ourselves in, not the least of which is our savings rate. Given a paltry savings rate of less than 5% (U.S. census 2011 estimated savings rate), it makes it increasingly difficult for Americans to secure a bright and fulfilling retirement. The proverbial "golden nest egg" looks less and less golden these days. The global financial crisis and the accompanying destruction of housing values have further exacerbated this situation. Americans have lost as much as 40 percent of their equity in homes, which has served as the largest single source of savings, and that has only added to the misery. If we add the propensity of Americans to spend significant portions of their income, it makes an already difficult situation more so.

There are several ways that Americans have been able to fund their retirement. The most basic of these is the defined benefit pension system. All other primary systems of retirement as defined in the IRS guidelines involve a larger level of individual commitment. Today, the pension system has made a rapid transition to defined contribution retirement plans, better known as 401(k), 403(b) among others. In most defined contribution plans, the person who funds the plan is solely responsible for its outcomes, good or bad, within broadly defined boundaries like types of investments. If at the end of a theoretical period, there are not enough funds to sustain the individual, then he/she either does not retire or has to alter the way of retirement. In other words, the perceived cash flow is not a specific or a certain amount. Defined benefit pensions, on the other hand, usually guarantee a fixed amount after a certain number of conditions have been met. The most basic of these conditions is the number of years one has to work to qualify for a lifetime of income from the pension provider. Like its counterpart of defined contribution plans, pension plans can and do face issues regarding whether there are sufficient assets to cover the pension liabilities. Unlike defined contribution plans, the plan sponsor is responsible for decisions such as types of investments to make, asset allocation, the amount of risk to take, and whether additional contributions are necessary. There are times when such obligations are too large for a plan. For example, the public retirement system of the United States is in deficit by roughly \$1 trillion to \$4.6 trillion dollars (depending upon different agencies reporting). How this difference will be made up is subject to debate in the future. More to the issue related to this paper, several factors may have contributed to this shortfall, including early retirement qualifications, extended life spans, and the performance of financial markets. Specifically, the focus is upon how and what type of asset allocation is most suited, under most economic scenarios, to allow a pension plan to meet its future obligations.

Types of pension plan sponsors – There are two basic types of pension plan sponsors: 1) employer based plans and 2) government based plans. In the employer based plans, the employer and sometimes the employee contribute to the pension plan trust. This contribution, for the employee's benefit, continues for a defined period. In the employer based plans, there is a direct relationship between the number of years of funding and the benefits collected; i.e., to qualify for a minimum level of benefits, a specific number for years of service is required to have been achieved. Regardless, the funds contributed to the plans are tax deferred and continue to enjoy this status until such time as funds are distributed to the plan participant. Under such a plan, and after all requisite conditions by the employee have been met, it is the employer who is liable for meeting the fixed stream of cash flows to the plan participant. The government based plans are very similar to employer based plans, and are subject to the same constraints. One of the major differences between corporate and government plans is that government plans tend to have the ability of using tax revenue as "lender of last resort" whereas private employers do not have this flexibility.

Pension plan management is a complex undertaking. There are many variables that can impact the performance of the funds, and thus its ability to meet its current and future obligations to its constituents. Among the more important variables that can impact the ability of a pension plan to deliver its promise are: 1) the initial funding levels of the plan, 2) the average salary level of the organizations, 3) the number of participants in the plan, 4) the average age of the participants, 5) the asset allocation utilized by the plan, and 6) the status of the plan as either a) ongoing, b) closed, or c) frozen. While each of the above variables can have a significant impact by itself, the main theme of this report is focused on the asset allocation strategy utilized by pension plans and its impact on both the ability of the plan to meet its obligations as well as the risk profile as a result of various asset allocation strategies.

The proposal

The proposal submitted to the Society of Actuaries was to study the performance of a pension plan under different market conditions and under varied asset allocation decisions. The primary purpose of the investigation was to determine if pension plans are viable under stresses of financial market performance as well as to determine what, if any, asset allocation can be determined to be most efficient. For the purposes of this paper, an economy can hypothetically be in one of the following three states: decline (recession or worst); stagflation, where the economy is not growing while at the same time inflation rates are high; and finally, an economy which is growing. In this paper we studied the performance of a plan over these three economic periods, plus an additional classification where the growth rate is intense. As such we chose 1974 as the starting point of our study. We chose this point because this is the period during which the U.S. economy was experiencing very low growth and high inflation rates, which were fed by a brewing oil crisis. Also, 1974 marked the passage of the Employee Retirement Income Security Act of 1974 (ERISA), which provided, for the first time, a broad framework of rules for pension plans. This period would last until the mid-1980s. It was during the middle 1980s that

tighter controls over the monetary policy by the Federal Reserve were able to reign in the higher inflation rates. This led to a prolonged period of declining interest rates. We reflect this phenomenon by studying the period from 1980-2000. We also studied two additional periods that included two recessions (2001 and 2007-2009), which, of course, are viewed as the harshest since the Great Depression of the 1930s.

The second part of our proposal was to determine if there exists an optimum asset allocation when it comes to managing pension assets. To study whether such an allocation exists, we created three primary and predetermined allocation strategies. These were 25% weight to equity (represented by S&P 500 index); 50% allocated to both S&P 500 and Barclay's fixed income index (FI); 65% in equity and 35% in FI. We realize that there are many other combinations of allocations, but we wanted to focus with an approach that ranges from conservative (25% allocation to equity) to fairly aggressive (65% weight in equities). In our models, the results of which we do not discuss or disclose in this section use additional allocations. The models would be able to determine which system performed the best in a broad range of economic conditions. These asset allocation designs were then implemented on three types of pension plans: an ongoing or open plan, a closed plan, and a frozen plan. In the open plan, the plan is open to new participants and is of an indefinite time period in nature. In the closed plan, as the term implies, the plan is closed to new participants; however, the benefits can continue to accrue for existing participants. Finally, in the frozen plan, not only is the plan closed to new participants, the plan benefits do not grow at all. The goal of the frozen plan is to get the funding level to a point such that all existing participants and their obligations are satisfied. After which period, the plan would cease to exist.

Applying the asset allocation to all the plans and across all economic spectrums, we hope to find a solution from which future pension fund managers could learn and implement.

Data

We used the S&P 500 as a benchmark index for equities and Barclay's total composite bond index for the fixed income component. Both data streams had a starting period in 1974 and the last stream of data was for 2010. The S&P data was collected from the Bloomberg data base while the FI data was obtained from Barclay's. The fixed income data was a composite FI index which included U.S. treasury, U.S. treasury medium term and U.S treasury long term bond data. The data obtained had returns on a monthly basis from January 1, 1973 to December 31, 2010. Table I shows the descriptive statistics for the S&P 500, while Table II summarizes the FI performance.

Table I

	1974-2010	1974-1994	1980-2000	1985-2005	1990-2010
Geometric mean	10.49%	12.76%	17.14%	13.39%	8.88%
Std. Deviation	17.8%	15.48%	13.53%	16.14%	18.58%

Descriptive Statistics for S&P 500 for 1974-2010 Period

Studying the results from Table I, the S&P 500 geometric mean is close to its historical average of 10.49% with a standard deviation of 17.8%. Looking at this data, it is noted that the 1990-2010 period was a lean period with the lowest mean and the highest volatility. Of course, this period is characterized by two recessions, one of 2001 and the other in 2007-2009. The best twenty year performance was achieved in 1980-2000 with the highest mean of 17.14% as well as with the lowest volatility of 13.53%. This period in the U.S. was characterized by falling interest rates, and interestingly, totally devoid of a slowdown in the U.S. economy.

Table II

Descriptive Statistics for Fixed Income Portfolio I for 1974-2010 Period

	1974-2010	1974-1994	1980-2000	1985-2005	1990-2010
Geometric	8.15%	9.70%	10.54%	8.94%	7.34%
mean					
Std.	6.90%	7.92%	7.83%	6.35%	5.21%
Deviation					

Contrasting the results from S&P 500 with the FI performance, there is one very obvious and somewhat surprising finding- that the overall performance of the FI portfolio was 8.15% as compared to the S&P performance of 10.49%. But what is more interesting is the volatility data. The overall volatility of the S&P was 17.8% while that of the FI portfolio was only 6.9%, which is significantly lower. These differences are highlighted in Table III, which shows the summary results of coefficient of variation computations.

Table III

Computations of Coefficient of Variations (CV) for S&P 500 and Barclay's FI portfolio

	1974-2010	1974-1994	1980-2000	1985-2005	1990-2010
CV S&P 500	1.70	1.21	0.79	1.20	2.09
CV FI	0.85	0.82	0.74	0.71	0.71

The CV data shows the overall efficiency of the FI portfolio. For example, over the entire study period of 1974-2010, the FI CV was exactly one half of the CV for the S&P 500 index. While the equities outperform the FI portfolio on an absolute basis, the efficiency of that performance is certainly in question. For the overall period, as well as each of the other four sub-periods, the FI was deemed to be more efficient. The outperformance of the FI portfolio was most stark over the 1990-2010 periods where the CV for FI was 0.79 as compared to the S&P 500 CV of 2.09. We believe that this was caused mostly as a result of the subdued equity markets performance as a result of two recessions. We can make this case, because the standard deviation of the S&P 500 index over this period was close to its historical average of roughly 18%. So the underperformance of the equities over this time period was the cause of this inefficiency.

The liabilities were discounted using Moody's AA Corporate rate, the rate often used as a benchmark when determining liabilities for the purposes of accounting for U.S. private pensions. The Moody's data as well as the S&P 500 data was provided by Nyhart while Barclay's provided the bond index data.

Descriptions of the model in the study

In this study, we explored three basic models. The first model used a basic pension plan that is ongoing or open. In the second model, we introduced a plan that would be closed to new participants. In the third and last model, we not only utilized a closed plan, but also a plan that has no benefit increases; in other words, a frozen plan. Below, we describe each model and the assumptions used in this study.

Basic model -the ongoing plan. The most basic model of a pension plan is one that is fully funded at the outset, and is ongoing. Table IV details the assumptions upon which the model was constructed. The basis for the ongoing plan is that there is a static population in the plan, meaning the demographic profile is constant for the duration of the study. As active participants either terminate or retire, the same number of actives enter into the plan. In addition, the pace of actives withdrawing matches the mortality of retirees. Finally, another assumption made was that the company would be able to have positive cash flows to fund any level of shortfall.

Table IV

Initial	Initial	Service	Average	Active	Basic	Retiree	Retiree	Retiree
Liability	Funding	Cost	Salary	Participants	Payroll	Percent	Percentage	Death
_	Level		_	_	-		Payout	Decrement
\$10M	100%	3.5%	\$20K	300	\$6.4M	30%	14%	1.8%

Assumptions made for the basic ongoing plan

Our assumption regarding the annual benefits increment is rooted in the long-term average inflation rate of 3.5%. In the plan, it is assumed that 30% of liabilities are attributed to retirees whose annual payouts start out at 14% of the retirement liabilities. The liabilities are measured at the Moody's AA Corporate rate at the beginning of the year, and the duration in the ongoing scenario is assumed to be 14. Liabilities are rolled forward each year adjusting for payouts made and incoming contributions – both of which are weighted to be mid-year cash flows. Annual contributions follow the basic Pension Protection Act funding rules where there is a normal cost component for annual benefit accruals and a 7-year amortization payment for any shortfall. For this model, the full normal cost is always contributed unless the plan is at least 110% funded, in which case the contribution is then one-half of the normal cost. If the plan is at least 120%, the contribution is assumed to be zero for that particular year. Our assumption of mortality is based upon a representative rate in the 1994 U.S. group annuity mortality table for the retiree group in the model. Since we had indicated in our proposal that we would make use of data and information on pension plans from Nyhart's pension data, we based our retiree percent of 30% as well as retiree percentage payout of 14% on similar pension plans from Nyhart's data base. For pension plans of this size (\$10M), these were common traits for the retiree and retiree percent payments. Further notes on use of assumptions used in modeling the basic and other plans are listed in Appendix A. Appendix B shows the Moody's rates that were used in this study (from 1974-2010). In this model, the only variable that is dynamic is the asset allocation. For this model, we use asset allocations that started at a very conservative 25% attributed to equity while 75% was attributed to the fixed income portion. We then gradually raise the equity portion to 50%, 65%, and then 100%. Finally, we also perform a sub-set analysis of 10 years for this model to see what would happen when the pension plan is launched over a period of sustained economic weakness.

Closed and Frozen models: These models are similar to an ongoing plan except for the fact that the plans are closed to new participants. The rationale for closing a plan to new participants is that an entity may decide to move from a defined benefit plan to a defined contribution plan. Because the plan is closed to new participants, it is anticipated that the plan will eventually run its course and come to its natural end as benefits to the last living participant are made. Because of this, we assume the duration for these plans to be lower at 12 years. The reason the duration number is lower than those for ongoing plans is because closed plans will have higher average age as the participating population ages.

Discussion of Results

Asset and liability growth rates: Figures 1 and 2 show the summary of asset and liabilities growth rates across all allocation schemes. As expected, the asset growth rate was the highest for the portfolio with 100% equity allocation, except for the periods from 1990-2010. The growth rate of liabilities was independent of the type of asset allocation.





There are four major sections of analysis that we will discuss in this paper. In the first section, we will discuss the findings from the ongoing model under various asset allocation designs. The asset allocations will range from a minimum of 25% in equity to a maximum of 100% in equity. In the second piece of the report, we will still study the ongoing plan, but vary the asset allocation based upon funding levels. The plans in this section are also not going to be fully funded. Thus, we will take a less than fully funded plan and apply asset allocations based upon the level of funding at each year of the plan. Thus there is not a pre-determined and "sticky"

level of asset allocation. The third part of the analysis will focus on varying the asset allocations based not only on the level of funding, but also will be unidirectional. That is to say, the asset allocation will be forced to move more towards fixed income weight as time moves along. In the last section of the analysis, we will study the behavior of the closed and frozen plans. The unique thing about this section is that our focus is on having the plans remain "solvent" and able to meet all its obligations. The model has a lower duration of 12 years and the asset allocation is dictated by the level of funding.

Discussion on basic and ongoing plan: An ongoing plan is one where new participants are allowed to enter the plan, with the only assumption being that the number of new participants is equivalent to participants that leave the plan. As a result, the total population in the plan remains the same. We analyzed four sub-periods (we have discussed them before): 1974-1994; 1980-2000; 1985-2005; and 1990-2010. The tables and results below will discuss four asset allocation models as well as an allocation of 100% equity over a smaller sub-period of 2000-2010. Table V- A1 through E2 shows the findings from using the various asset allocation systems, from 25% in equity to 100% in equity. Please note that our main objective in this part of the analysis is to determine the most efficient asset allocation system. Even though we will still discuss this part of the analysis in further sections, we would at least like to have a basic idea of what happens to a pension plan when a specific allocation system is imposed throughout a specified period. Therefore, it should also be noted that each section of Table V maintains the highlighted asset allocation throughout the specified sub-period.

Sub- Periods	Effective Asset Growth Rate	Effective Liability Growth Rate	Average of (A- L) Difference	Percent Funded	Std. Dev. of Funding Levels	Max Funding Percentage	Minimum Funding Percentage
1974- 1994	7.06%	3.37%	\$9,325,948	190%	56%	270%	100%
1980- 2000	7.43%	6.08%	\$1,941,528	116%	21%	171%	88%
1985- 2005	7.74%	8.37%	-\$992,824	96%	10%	122%	78%
1990- 2010	6.69%	7.25%	-\$427,751	100%	10%	127%	86%

Table V-A1: Findings from 25% equity allocation

Sub- Periods	# Periods A <l< th=""><th>Total Cash Contributions</th><th>Total Benefits Paid</th><th>Avg (A-C)/L</th><th>Percent Funded @ End</th><th>Portfolio Return</th><th>Std. Dev. Portfolio</th><th>Sharpe Ratio</th></l<>	Total Cash Contributions	Total Benefits Paid	Avg (A-C)/L	Percent Funded @ End	Portfolio Return	Std. Dev. Portfolio	Sharpe Ratio
1974-1994	0	\$1,028,000	\$15,173,000	189%	160%	10.44%	8.61%	0.79
1980-2000	4	\$10,420,000	\$28,549,000	113%	129%	11.9%	7.59%	1.09
1985-2005	15	\$21,547,000	\$34,956,000	91%	89%	10%	7.15%	0.88
1990-2010	11	\$21,542,000	\$26,280,000	95%	90%	7.88%	6.45%	0.65

Table V-A2: Findings from 25% equity allocation

Tables V-A1 and A2 show the results of allocating a 25% weighting to equity through all the four sub-periods, with the remaining weight in the FI index. One of the most important variables in managing a pension plan is the effective growth rate of its assets under management. The effective growth rate was computed by taking for each sub-period the initial amounts of assets, in this case \$10M (because the plan was assumed to be fully funded) and adding any contributions to the asset base and then subtracting the benefits paid. Appendix A shows the actual formula for determining the asset amount at the end of any specific period. Results show that growth of assets was in a very reasonable range of approximately 7%. Even at this rate of effective growth, the minimum level of funding was at 78%, and that was during the 1985-2005 sub-period. Also note the percent funded figures. These numbers are averages of percent funded, which is computed by dividing the assets by liabilities for a specific period, over an entire twenty year period. For example, for the 1974-1994 period the average level of funding was 190%. This implies that the average of assets over liabilities was 190%. The results also show the maximum level of funding achieved during any of these periods. The maximum level of funding was noted at 270%, and that was for the 1974-1994 period. It is also of interest to note that the maximum level of funding never dropped below 120%, which is good.

At a very basic level, as long as rate of growth of pension assets exceeds the rate of growth of liabilities, and all else being equal, the plan should remain liquid and solvent. We note the rate of growth of liabilities, and these results show a wide range of growth rates. The range of liability growth rates ranges from a low of 3.37% in the 1974-1994 periods while the highest rate of growth was noted as 8.37%, and that was in the 1985-2005 sub-period. Of the four sub-periods in our study, we noted that in two of the sub-periods, namely 1974-1994 and 1980-2000, the rate of growth of assets exceeded the rate of growth of liabilities. However, in the remaining two sub-periods, the rate of growth of liabilities exceeded the growth rate of assets. It is therefore no surprise that the difference of average of assets and liabilities (column 3 in A1) was negative in the corresponding sub-periods. It is also important to note that even though this difference was negative, it was not a large negative shortcoming (less than \$1M in both sub-

periods). It is also worth noting that the average funding level of the plan was in a comfortable range of more than 96% funded (in fact, the 96% was the lowest average funding level), reaching as high as 190% in the 1974-1994 period.

Since it is our main objective to determine the impact of asset allocation on pension plan viability, we also need to study the risk profile of each equity allocation. In this paper, we have several ways that speak to risk absorbed by a plan. Among them: total number of periods where the assets < liabilities; the average of (assets - contributions) as a ratio of liabilities; the percent of funding at the end of a specific period; total amount of cash contributions; and the Sharpe ratio. In this study, we do not focus on total cash contributions (even though we computed it) because it is a weaker determinant of the amount of risk a manager might assume.

Periods where assets < liabilities:

We computed the total number of periods where assets are less than liabilities for each subperiod. The results show that for a 25% asset allocation, the highest number of years in which the assets were smaller than liabilities were in the 1985-2005 period, where this number was 15 (out of the 20 total years in the period). 1990-2010 was the interval with the number of years where assets < liabilities was the next highest at 11. The most efficient period was 1974-1994, where assets were greater than liabilities in each of the twenty years. If we sum all the years where assets < liabilities, we obtain a sum of 30 years out of a cumulative 80 years. This puts the ratio of A < L to total years at 37.5%. This is one important rubric as we compare various equity weightings. The smaller this number is, the better the chances that the plan is effective. If the manager of a pension plan could invest in such a way so as to have no periods where liabilities exceed assets, it would make for a close to an ideal scenario. Of course, one way to do this is to take a very aggressive approach to investment without regard to risk. That is why we must examine the performance of other measures.

Assets-Contributions / Liabilities:

This measure is designed to capture the true nature of assets to liability ratio. The reason being that the way we compute assets, contributions by the firm become part of the investible asset base. Hence, if we take away the contributions from the total asset base, we are left with assets minus the benefits paid, and it is this number that is more meaningful as a gauge of risk. The higher this ratio, the more viable the plan. We can see that this ratio ranges from a high of 190% to a low of 91%. A 91% funding level is not all that bad, when all is said and done. Therefore, it would seem that even a conservative allocation of 25% to equity is somewhat functional.

Sharpe ratio:

Perhaps the most efficient way of risk testing is to make use of the Sharpe ratio. The Sharpe ratio is computed as follows: (return – risk free rate)/Std. deviation. The risk free rate was defined by the 10-year Treasury bond yield, and this is the rate we used in our study. Although

there is debate regarding what is a true "risk free" rate, corporate finance practice has been to use the 10-year Treasury yield. The higher the ratio, the more efficient the investment would be in relation to its risk profile. For the 25% weight to equity, we see that the Sharpe ratio ranges from a low of 0.65 to a high of 1.09. To determine how efficient this investment profile is, we will wait to discuss results from other allocations.

Discussion on 50% allocation:

The results of imposing a 50% equity weighting to assets are depicted in Tables V-B1 and B2. As one might expect, the rubrics on effective rate of return and average (A-L) difference, percent funded and maximum funded are all better than their 25% counterparts. The difference of (A-L) was not negative in any of the four sub-periods. However, because higher equity weight puts more risk on the overall portfolio, we noted that the amount for minimum level of funding was generally better for the 25% equity weighting. This greater risk is also evident by looking at the standard deviation of the funding, which is higher in each of the four sub-periods. Clearly, there is a price to pay for higher allocation to equity.

When we look at the total number of periods where assets were less than liabilities, we come up with 25 years from a possible 80 years. This gives a better percent ranking at 31.25% as compared to 37.25% in the 25% allocation. Another interesting way to contrast these results is to look at the percent funded at the end. We note that even though the percent funded figures are higher for the first two sub-periods, they are essentially the same for the last two sub-periods. Even when they are superior, after some point, it becomes meaningless to look at plans and decipher the difference between say 200% funded from one that is 190% funded. As far as we are concerned, this turned out to be not a very meaningful statistic.

The overall portfolio performance was better under the 50% weighting, but so was the standard deviation. When we compared the Sharpe ratios of the two weightings, we noted that in general, the Sharpe ratio was superior under the 25% weighting. Thus the results so far are mixed with respect to efficiency of higher equity weighting.

Sub-Periods	Effective Asset Growth Rate	Effective Liability Growth Rate	Average of (A- L) Difference	Percent Funded	Std. Dev. of Funding Levels	Max Funding Percentage	Minimum Funding Percentage
1974-1994	8.12%	3.37%	\$11,745,036	210%	70%	290%	99%
1980-2000	8.80%	6.08%	\$4,560,437	126%	23%	178%	95%
1985-2005	8.19%	8.37%	\$1,895,695	104%	17%	153%	80%
1990-2010	6.57%	7.25%	\$419,002	104%	16%	149%	82%

Table V-B1: Findings from 50% equity allocation

Sub- Periods	# Periods A <l< th=""><th>Total Cash Contributions</th><th>Total Benefits Paid</th><th>Avg (A-C)/L</th><th>Percent Funded @ End</th><th>Portfolio Return</th><th>Std. Dev. Portfolio</th><th>Sharpe Ratio</th></l<>	Total Cash Contributions	Total Benefits Paid	Avg (A-C)/L	Percent Funded @ End	Portfolio Return	Std. Dev. Portfolio	Sharpe Ratio
1974-1994	1	\$1,169,000	\$15,173,000	209%	199%	11.38%	10.37%	0.74
1980-2000	1	\$5,672,000	\$28,549,000	125%	166%	13.68%	8.76%	1.14
1985-2005	13	\$15,201,000	\$34,956,000	101%	97%	11.31%	9.56%	0.80
1990-2010	10	\$18,625,000	\$26,280,000	100%	88%	8.66%	9.97%	0.50

Table V-B2: Findings from 50% equity allocation

Discussion on 65% allocation:

Tables V-C1 and C2 depict the results from imposing a 65% weighting to equities. As for the 50% allocation, the results in the effective rate category are superior. There was only one subperiod where the assets rate of growth was smaller than the rate for liability. As before, the average of (A-L) differences are all positive and increasingly so as compared to the 25% and 50% weighting, as are the rubrics for percent funded. But because of the higher weighting to equity, the standard deviation of the funding levels is higher, indicating the higher volatility in the levels of funding. The minimum levels of funding, though, are on par with the previous equity weightings indicating that higher equity weighting, even if it helps on the positive side of distribution, the lower side of distribution remains relatively untouched. In other words, the worst outcomes are present with the same frequency as before.

Of course from a manager's point of view and her/his peace of mind, one rubric that is important is the number of years A < L. In this case, this number was negative for a total of 18 years out of 80 years cumulative. This puts the percentage at 22.5%, which is much improved from the 37.5% in the 25% equity weighting. Finally, the portfolio returns and standard deviation numbers are all higher, as expected, but the same could not be said for the Sharpe ratio, which continues to favor the 25% allocation model.

Sub-Periods	Effective Asset Growth Rate	Effective Liability Growth Rate	Average of (A- L) Difference	Percent Funded	Std. Dev. of Funding Levels	Max Funding Percentage	Minimum Funding Percentage
1974-1994	8.67%	3.37%	\$13,132,608	221%	79%	308%	94%
1980-2000	9.80%	6.08%	\$6,623,615	133%	26%	199%	99%
1985-2005	8.67%	8.37%	\$4,258,379	111%	24%	178%	81%
1990-2010	6.55%	7.25%	\$1,461,214	108%	22%	170%	79%

Table V-C1: Findings from 65% equity allocation

Table V-C2: Findings from 65% equity allocation

Sub- Periods	# Periods A <l< th=""><th>Total Cash Contributions</th><th>Total Benefits Paid</th><th>Avg (A-C)/L</th><th>Percent Funded @ End</th><th>Portfolio Return</th><th>Std. Dev. Portfolio</th><th>Sharpe Ratio</th></l<>	Total Cash Contributions	Total Benefits Paid	Avg (A-C)/L	Percent Funded @ End	Portfolio Return	Std. Dev. Portfolio	Sharpe Ratio
1974-1994	1	\$1,239,000	\$15,173,000	220%	223%	11.95%	11.70%	0.70
1980-2000	1	\$4,018,000	\$28,549,000	133%	199%	14.70%	10.00%	1.10
1985-2005	7	\$13,058,000	\$34,956,000	108%	106%	12.10%	11.40%	0.74
1990-2010	9	\$17,162,000	\$26,280,000	105%	88%	9.12%	12.40%	0.44

Discussion on 100% allocation:

As indicated before, and even though we understand that no manager would likely assign a 100% weight to equity, we were curious to see what our analysis would look like under this scenario. There are plenty of reasons to not use 100% allocation, not the least of which is the extreme risk of equity underperforming, such as in the case of a recession. The fact that benefits have to be paid from the asset base, a depletion of asset base at the "wrong" time could put the plan in a potential "insolvent" state. Nevertheless, we wanted to see if we could make a model case for this allocation. Tables V-D1 and D2 show the findings.

By now a few things are becoming obvious and expected. First, the effective asset rate is better, though it is interesting that no matter what the allocation, the sub-period of 1990-2010, or an economic period that mimics this sub-period, the asset growth rate is less than its liability counterpart. So, no matter what, we have shown that if macro-economic conditions are such that they include two recessions with equity markets dropping by roughly 40% on each occasion (2001 and 2009), it is likely difficult to recover from such lagging performance. As expected,

the assets generally exceed liabilities and do so in a more meaningful way. Also, the number of years where the assets < liabilities is also reduced to a mere 11 out of a possible 80 years. That puts this percentage at only 13.75%, which is far superior to any other equity model. Clearly, if this is the metric that a manager wants to use, he/she should assign a 100% weight to equity. As discussed previously, this strategy is clearly not without its own challenges. For example, the standard deviation of funded and standard deviation of the portfolio is much higher under this weighting. Further, the Sharpe ratio continues its trend of deteriorating as the equity weights increase.

Sub-Periods	Effective Asset Growth Rate	Effective Liability Growth Rate	Average of (A- L) Difference	Percent Funded	Std. Dev. of Funding Levels	Max Funding Percentage	Minimum Funding Percentage
1974-1994	9.86%	3.37%	\$ 16,562,561	248%	103%	381%	84%
1980-2000	12.29%	6.08%	\$ 12,926,678	156%	46%	312%	100%
1985-2005	9.67%	8.37%	\$ 10,965,502	131%	44%	260%	73%
1990-2010	6.43%	7.25%	\$ 4,595,918	121%	38%	232%	71%

Table V-D1: Findings from 100% equity allocation

Table V-D2: Findings from 100% equity allocation

Sub- Periods	# Periods A <l< th=""><th>Total Cash Contributions</th><th>Total Benefits Paid</th><th>Avg (A-C)/L</th><th>Percent Funded @ End</th><th>Portfolio Return</th><th>Std. Dev. Portfolio</th><th>Sharpe Ratio</th></l<>	Total Cash Contributions	Total Benefits Paid	Avg (A-C)/L	Percent Funded @ End	Portfolio Return	Std. Dev. Portfolio	Sharpe Ratio
1974-1994	2	\$1,515,000	\$ 15,173,000	247%	338%	13%	16%	0.62
1980-2000	0	\$1,080,000	\$ 28,549,000	155%	312%	14%	14%	0.99
1985-2005	3	\$8,280,000	\$ 34,956,000	129%	127%	17%	16%	0.63
1990-2010	6	\$14,977,000	\$ 26,280,000	118%	86%	10%	19%	0.35

The results of the various asset allocations and their impact on rates of return, standard deviation and Sharpe ratios are depicted in Figures 3-5 and further discussed in tables that follow.







The figures show that even though the rate of return obtained on a pension portfolio is superior (as might be expected) for 100% equity allocation, so is the risk that goes with this aggressive style. Figure 4 clearly shows that the standard deviation for a portfolio with 100% equity allocation is far larger than under any other asset allocation strategy. Finally, Figure 5 shows that the Sharpe ratio is far lower for 100% equity allocation, bringing in question whether this type of risk-reward is warranted.

Discussion on 2000-2010 Period:

Thus far, we are able to say that generally, as the equity weighting increases, the asset performance becomes better and the number of years that assets are smaller than liabilities becomes smaller. Clearly, these are two rubrics that a manager would be interested in. After all, what is more important to a pension manager than to ensure that asset performance rate exceeds liability growth rate and that assets under management are greater than liabilities with as great of a frequency as possible? There is only one problem with such a premise. In our model, we have thus far taken a twenty year window for the analysis. However, we wondered what would happen if the plan were to start in a year such as 2000. What is unique about starting a plan during this period is that immediately after starting the plan, the U.S. economy went through a modest recession, but a more than modest correction in the S&P 500. Later within this period, the economy went through a more severe recession and an equally greater correction in the equities market (though FI performed relatively better). If a pension plan were started during this period, and even if it were fully funded, what would be the impact of heavy equity weighting? This is a question that we examined and we report these results in Tables V-E1 and E2.

The results are startling and eye-opening. First, we examine the performance of the plan using all four equity allocations. It is clear from looking at these panels that the pension plan was under water no matter what the weighting. So, if the macro situation is bad enough, there is likely nothing a pension manager can do. However, there is also something very obvious from these findings - that the results become worse with heavier weighting towards equity. For example, all measures (rate on assets, average of A-L, funded standard deviation, and others) are meaningfully worse off. Most of all, the Sharpe ratio is actually negative with respect to the 100% equity bond. The message from this analysis is therefore that while a more aggressive stance would be tolerable under a "normal" economic period, it would be problematic under a negative economic scenario. Since it is difficult to forecast a recession, as it was in the 2001 and 2007, it is best to avoid a very heavy weight to equities. Therefore, it would seem that an equity weighting between 25% and 50% is sufficient to deliver the results under most economic circumstances. In fact, it is a 25% weighting to equity when a portfolio of a pension plan is most efficient, based on this analysis.

Asset Allocation	Effective Asset Growth Rate	Effective Liability Growth Rate	Average of (A- L) Difference	Percent Funded	Std. Dev. of Funding Levels	Max Funding Percentage	Minimum Funding Percentage
25% equity	8.44%	9.94%	-\$2,356,268	88%	6.18%	100%	81%
50% equity	7.92%	9.94%	-\$3,122,746	84%	8.48%	100%	71%
65% equity	7.53%	9.94%	-\$3,593,497	81%	10.20%	100%	64%
100% equity	6.40%	9.94%	-\$4,709,998	75%	14.34%	100%	51%

Table V-E1: Findings from 2000-2010 Period

Table V-E2: Findings from 2000-2010 Period

Sub- Periods	# Periods A <l< th=""><th>Total Ccash Contributions</th><th>Total Benefits Paid</th><th>Avg (A- C)/L</th><th>Percent Funded @ End</th><th>Portfolio Return</th><th>Std. Dev. Portfolio</th><th>Sharpe Ratio</th></l<>	Total Ccash Contributions	Total Benefits Paid	Avg (A- C)/L	Percent Funded @ End	Portfolio Return	Std. Dev. Portfolio	Sharpe Ratio
25% equity	10	\$15,271,000	\$9,795,000	80%	87%	5.40%	4.10%	0.41
50% equity	10	\$16,702,000	\$9,795,000	75%	83%	4.40%	9.90%	0.07
65% equity	10	\$ 17,580,000	\$9,795,000	72%	80%	3.80%	12.00%	0.01
100% equity	10	\$19,663,000	\$9,795,000	65%	72%	2.50%	19.30%	-0.06

Discussion of results from dynamic allocation:

Another issue a pension manager faces is how often to rebalance the investment portfolio. There are three basic ways to resolve this. One is to adopt a pre-determined asset allocation and stick to it, no matter what the performance in the market. This was the approach that we demonstrated in Table V through various equity allocations. The second approach is to rebalance the portfolio based on some pre-determined funding level triggers as well as specific asset allocations. The third approach is to rebalance the portfolio based on specific funding levels, but with focused attempts towards a more conservative model. We will discuss this approach and the results in a later section of this paper. In this section we adopted the second approach wherein we assume that the manager has the flexibility to change the allocation from equity to FI or vice-versa annually based upon funding levels. Appendix C discusses our assumptions for this model, but the basic premise is that we adopted five funding levels- from 90% at the lowest level to greater than 120% at the highest level. We are most aggressive at the lower range (75% equity allocation) and become increasingly conservative at higher levels of plan funding. For example, at funding levels greater than 120%, the entire portfolio gets shifted to fixed income. The manager therefore has the flexibility to increase or decrease the weight of equity each and every year depending upon the funding levels. As a result of this assumption, the manager always starts out the year with the most aggressive weight assigned to equity- at 75%. Table VI (with panels A and B) demonstrates these results.

The variables that we want to focus on are effective rate of return on assets; average of (A-L) difference; standard deviation of the funded plans; minimum percent funded; number of years A<L; portfolio statistics; and the Sharpe ratio. Specifically, we should be interested in comparing these results with the pre-determined asset allocation system discussed in the previous section.

The results of effective rate of return on assets show that generally speaking, a more dynamic approach to portfolio management produces higher rates of return. For example, if we compare the results from Table VI-A1, we note that these results are generally better than those for 25%, 50%, and 65% equity allocation, except for the 1974-1994 period. Only the 100% equity based model produces superior results. So, if a manager wants a higher rate of return, it was shown to be beneficial to be more "active" in financial markets; more specifically in equities markets. Even though the plan was underfunded significantly (and therefore not directly comparable to the traditional model of 100% funded plans), the average of (A-L) was found to be positive during all the sub-periods. The minimum level of funding, though in the range of 70%-80% was reasonable as the plan was underfunded at the outset to begin with. In that context, a minimum of 70% is an impressive result anyway.

The other significant rubric would be the total number of years that the assets are less than liabilities. Based on this measure, the cumulative number of years where A < L was 26. Again, that is from a cumulative of 80 years. That puts this percentage at roughly 32.5%, which is on

par with 50% equity allocation. Again, it is important to bear in mind that the plan was started at a 20% disadvantage. Finally, the Sharpe ratio was better, but only marginally so. So, the small degree of efficiency has to be put in context on the increased number of trades and other institutional expenses incurred. Finally, based upon the periods weighted average equity allocation, it turns out that the average equity allocation across all sub-periods is 36%, which is on par with our finding based on 100% plan funding. The average equity allocation in any sub-period was no more than 44%, which is well within limits of findings previously made.

Sub-Period	Effective Asset Growth Rate	Effective Liability Growth Rate	Average of (A- L) Difference	Percent Funded	Std. Dev. of Funding Levels	Max Funding Percentage	Minimum Funding Percentage
1974-1994	5.62%	3.51%	\$ 5,784,522	158%	49%	241%	76%
1980-2000	8.90%	6.08%	\$ 2,408,332	115%	19%	154%	80%
1985-2005	10.01%	8.37%	\$ 2,507,564	103%	19%	137%	70%
1990-2010	8.73%	7.25%	\$ 1,555,314	104%	15%	131%	78%

Table VI-A1: 80%-Dynamic allocation

Table VI-A2: 80% - Dynamic allocation

Sub- Periods	# Periods A <l< th=""><th>Total Cash Contributions</th><th>Tot</th><th>tal Benefits Paid</th><th>Avg (A- C)/L</th><th>Percent Funded @ End</th><th>Portfolio Return</th><th>Std. Dev. Portfolio</th><th>Sharpe Ratio</th><th>Weighted Equity %</th></l<>	Total Cash Contributions	Tot	tal Benefits Paid	Avg (A- C)/L	Percent Funded @ End	Portfolio Return	Std. Dev. Portfolio	Sharpe Ratio	Weighted Equity %
1974- 1994	5	\$ 2,485,000	\$	15,173,000	157%	150%	12.65%	9.53%	0.94	21%
1980- 2000	3	\$ 9,753,000	\$	28,549,000	113%	135%	13.05%	9.49%	0.99	35%
1985- 2005	11	\$ 14,914,000	\$	34,956,000	100%	108%	11.73%	8.98%	0.89	44%
1990- 2010	7	\$ 15,905,000	\$	26,280,000	100%	105%	9.86%	9.90%	0.62	44%

The data on annual dynamic strategy with 70% funding merely confirm what one would ordinarily expect. That is, the effective growth rate is lower, as are the differences in (A-L), maximum percentage funding, and minimum percentage funding. As expected, the total cumulative years where A < L are greater at 31. One would expect this because the plan started out at 70%. Interestingly enough, the equity proportion of assets was similar to its counterpart at 80% funding. We performed this part of the analysis to study the relative impact of a less than 100% plan funding. Clearly, the smaller the degree of funding at the beginning, the more stress it puts on all other important rubrics such as the ability to fund benefits paid. What is important

to note thus far from this analysis is that even though a more aggressive posture is helpful at the margin, the level of initial funding is what drives the model.

Sub-Period	Effective Asset Growth Rate	Effective Liability Growth Rate	Average of (A- L) Difference	Percent Funded	Std. Dev. of Funding Levels	Max Funding Percentage	Minimum Funding Percentage
1974-1994	5.19%	3.51%	\$ 4,717,852	148%	47%	239%	69%
1980-2000	9.45%	6.08%	\$ 1,232,629	107%	17%	146%	70%
1985-2005	10.67%	8.37%	\$ 1,766,463	100%	21%	136%	64%
1990-2010	9.46%	7.25%	\$ 1,211,202	102%	17%	129%	70%

Table VI-B1: 70% - Dynamic Allocation

Table VI-B2-70% - Dynamic allocation

Sub- Periods	# Periods A <l< th=""><th>Total Cash Contributions</th><th>Total Benefits Paid</th><th>Avg (A- C)/L</th><th>Percent Funded @ End</th><th>Portfolio Return</th><th>Std. Dev. Portfolio</th><th>Sharpe Ratio</th><th>Weighted Equity %</th></l<>	Total Cash Contributions	Total Benefits Paid	Avg (A- C)/L	Percent Funded @ End	Portfolio Return	Std. Dev. Portfolio	Sharpe Ratio	Weighted Equity %
1974- 1994	6	\$ 3,212,000	\$ 15,173,000	147%	138%	12.57%	9.44%	0.94	22%
1980- 2000	7	\$ 12,314,000	\$ 28,549,000	107%	131%	14.82%	9.17%	1.21	42%
1985- 2005	11	\$ 17,041,000	\$ 34,956,000	96%	107%	12.17%	9.37%	0.90	47%
1990- 2010	7	\$ 17,746,000	\$ 26,280,000	97%	105%	9.81%	10.16%	0.60	44%

One directional balancing:

The results on balancing the asset portfolio on an annual basis were clearly shown to be a superior strategy. However, we also pointed out that such a strategy needs to be put in context of costs imposed in executing this strategy. Because we were concerned about the cost of the model, we studied a variant of this plan by limiting what a manager could do. In this part of the study, we allow the manager to become more conservative only. That is to say, once a certain plan funding threshold is met, the manager shifts the future allocations to a more conservative stance, and maintains that specific allocation even if the plan funding levels drop below the threshold that would require a more aggressive posture. For example, say we started the plan at 80%. In such a case, we allow the manager to adopt the most aggressive (in our model) allocation of 75% to equities. Subsequently, assume that the plan funding level jumps to 105%. In our model, that would require a shift to 50% equity. Now, if during the next year the funding

level drops below 105%, the manager would still maintain the 50% allocation. The only way for this allocation to change is if the plan funding level becomes higher than 110%.

When applying this model across all four sub-periods at 80% initial funding level, we note that the performance is notably weak when measured by average of (A-L), which are minimal to actually negative in two of the sub-periods. Also worthy of noting is the cumulative number of years that the A < L. That number jumps all the way to 40 years, which is a whopping 50% of the total cumulative years. However, we also note that though the average of (A-L) does become negative, these numbers are quite minimal in scope. Also, the maximum and minimum levels of funding are within range of previous findings, as is the Sharpe ratio. So, from the efficiency point of view, having a less aggressive trading stance does not increase the efficiency of the asset management, even as it improves the overall economic viability of the plan. The average equity allocation was found to be approximately 25% under this model, which is, as expected, more conservative than annual rebalancing.

When we apply the same model to an initial 70% funding level, we noted that all positive variables are slightly less positive and almost all negative variables are more so. For example, the cumulative number of years has now jumped to 43, which is better than half of all years. So, even though the efficiency of asset management is relatively similar to its more dynamic model, it is clear that shifting to a unidirectional asset allocation in an open plan is something that we would not recommend. If the manager were so inclined, he/she could simply have adopted a more conservative approach of 25%-35% allocation to equity and have ended up with a much lower number of years where the assets were smaller than liabilities. For example, even under a 25% allocation to equity, the cumulative number of years where A < L was 30. If the manager goes to 50% equity allocation, the results drop to 25 total cumulative years. Either way, it would seem that any reasonable asset allocation is capable of providing efficient and better economic outcomes. However, these findings must be tempered with the fact that our reference is with respect to a 100% funded plan against one that starts out at a significant disadvantage at 80% funding level. So, what value does this analysis actually serve? We selected underfunded plans to see not only the impact it would have on the entire matrix, but also as a way to justify a fairly aggressive potential stance with respect to up to 75% equity weighting. It is likely that if a plan is fully funded at the outset that it may never need such an aggressive posture, and our results thus far bear this out.

Sub-Period	Effective Asset Growth Rate	Effective Liability Growth Rate	Average of Differen	r (A- L) Percen nce Funded	Std. Dev. of Funding Levels	Max Funding Percentage	Minimum Funding Percentage
1974-1994	5.30%	3.51%	\$ 5,11	9,559 152%	46%	233%	76%
1980-2000	7.31%	6.08%	\$ (439	9,687) 105%	22%	165%	80%
1985-2005	9.80%	8.37%	\$ 1,88	3,305 102%	18%	134%	70%
1990-2010	8.17%	7.25%	\$ 49	101%	15%	131%	78%

Table VII-A1: 80% - Step allocation

Table VII-A2-80% - Step Allocation

Sub- Periods	# Periods A <l< th=""><th>Total Cash Contributions</th><th>Total Benefits Paid</th><th>Avg (A- C)/L</th><th>Percent Funded @ End</th><th>Portfolio Return</th><th>Std. Dev. Portfolio</th><th>Sharpe Ratio</th><th>Weighted Equity %</th></l<>	Total Cash Contributions	Total Benefits Paid	Avg (A- C)/L	Percent Funded @ End	Portfolio Return	Std. Dev. Portfolio	Sharpe Ratio	Weighted Equity %
1974- 1994	5	\$\$ 2,508,000	\$ 15,173,000	151%	141%	12.30%	9.20%	0.93	20%
1980- 2000	13	\$ 16,323,000	\$ 28,549,000	102%	101%	11.29%	8.42%	0.90	8%
1985- 2005	11	\$ 15,842,000	\$ 34,956,000	98%	104%	11.61%	8.80%	0.90	43%
1990- 2010	11	\$ 17,802,000	\$ 26,280,000	97%	95%	11.05%	8.75%	0.84	31%

Table VII-B1: 70% - Step allocation

Sub-Period	Effective Asset Growth Rate	Effective Liability Growth Rate	Average of (A- L) Difference	Percent Funded	Std. Dev. of Funding Levels	Max Funding Percentage	Minimum Funding Percentage
1974-1994	4.08%	3.51%	\$ 2,716,314	130%	38%	204%	69%
1980-2000	7.97%	6.08%	\$ (1,335,647)	99%	19%	146%	70%
1985-2005	10.45%	8.37%	\$ 1,049,233	98%	18%	131%	64%
1990-2010	8.90%	7.25%	\$ 144,330	99%	17%	129%	70%

Sub- Periods	# Periods A <l< th=""><th>(</th><th>Total Cash Contributions</th><th colspan="2">Total Benefits Paid</th><th>Avg (A- C)/L</th><th>Percent Funded @ End</th><th>Portfolio Return</th><th>Std. Dev. Portfolio</th><th>Sharpe Ratio</th><th>Weighted Equity %</th></l<>	(Total Cash Contributions	Total Benefits Paid		Avg (A- C)/L	Percent Funded @ End	Portfolio Return	Std. Dev. Portfolio	Sharpe Ratio	Weighted Equity %
1974- 1994	6	\$	3,347,000	\$	15,173,000	125%	112%	10.16%	11.32%	0.57	23%
1980- 2000	15	\$	19,095,000	\$	28,549,000	95%	100%	10.89%	8.64%	0.83	8%
1985- 2005	11	\$	18,012,000	\$	34,956,000	94%	102%	11.68%	8.92%	0.89	39%
1990- 2010	11	\$	19,615,000	\$	26,280,000	94%	95%	11.05%	8.75%	0.84	31%

Table VII-B2-70%- Step allocation

The Impact on Closed and Frozen plans:

In this section, we study the impact on two additional types of plans: closed and frozen. The two are similar in that they are both closed to new participants. However, the frozen plan also differs in that the benefits cease to accrue further. The idea under both plans is that they are designed to phase out after all retiree obligations have been met. Because of the nature of the plan, one assumption in our model that is different from other models is the duration. In this model, we use a duration of 12 years as opposed to 14 years. Further, we expect that the retiree percent is higher because there are no new entries which keep the average age at a relatively higher level (and thus the lower duration).

As a result of these assumptions, we expect that the liabilities growth rate is going to be smaller. In fact, we also expect the asset growth rate to be smaller as the manager eventually looks to wind the plans down. This in fact was the finding. Note that for both closed and frozen plans, both the asset and growth rates are lower. For the frozen plan, in fact, they are even negative (1974-1994). This makes sense as the liabilities no longer accrue, and thus have to grow at a much smaller rate to meet their obligations. For both types of plans, the total cumulative number of years where A < L are also low at 20 and 23, respectively. Perhaps the most important statistics is the number of years it takes either plan to reach what we define to be fully funded at 120%. It is to be noted that it only takes approximately seven years (the average over all 80 periods, though the range is between one to 12 years) for either plan to reach 120% funding level. At this level, the manager can simply choose to allocate all assets to FI and still have the plan meet all its obligations. This finding is verified by the fact that the funding percentage at the end of either plans is well above 100%.

Table VIII-B1: 80% - Closed

Sub-Period	Effective Asset Growth Rate	Effective Liability Growth Rate	Average of (A- L) Difference		Percent Funded	Std. Dev. of Funding Levels	Max Funding Percentage	Minimum Funding Percentage
1974-1994	5.32%	2.99%	\$	3,625,014	136%	36%	197%	71%
1980-2000	5.57%	3.43%	\$	1,670,972	113%	15%	181%	80%
1985-2005	7.08%	5.10%	\$	2,385,134	109%	16%	131%	76%
1990-2010	4.70%	2.21%	\$	2,361,301	114%	17%	133%	80%

Table VIII-B2-80%- Closed

Sub- Periods	# Periods A <l< th=""><th colspan="2">Total Cash Contributions</th><th>Avg (A- C)/L</th><th>Percent Funded @ End</th><th>Retirement Percent at End</th><th># Years till 120%</th></l<>	Total Cash Contributions		Avg (A- C)/L	Percent Funded @ End	Retirement Percent at End	# Years till 120%
1974- 1994	6	\$	3,027,000	134%	125%	43%	6
1980- 2000	3	\$	9,301,000	111%	120%	43%	1
1985- 2005	6	\$	11,431,000	105%	116%	43%	12
1990- 2010	5	\$	9,802,000	111%	129%	43%	7

Table VIII-B1: 80% - Frozen

Sub-Period	Effective Asset Growth Rate	Effective Liability Growth Rate	Aver E	age of (A- L) Difference	Percent Funded	Std. Dev. of Funding Levels	Max Funding Percentage	Minimum Funding Percentage
1974-1994	5.45%	-0.79%	\$	5,603,058	185%	72%	281%	70%
1980-2000	4.82%	0.39%	\$	5,371,798	151%	20%	190%	80%
1985-2005	5.62%	2.56%	\$	2,228,377	113%	26%	153%	76%
1990-2010	3.32%	0.05%	\$	2,112,818	119%	25%	154%	78%

Sub- Periods	# Periods A <l< th=""><th colspan="2">Total Cash Contributions</th><th>Avg (A- C)/L</th><th>Percent Funded @ End</th><th>Retirement Percent at End</th><th># Years till 120%</th></l<>	Total Cash Contributions		Avg (A- C)/L	Percent Funded @ End	Retirement Percent at End	# Years till 120%
1974- 1994	6	\$	1,222,000	185%	271%	43%	6
1980- 2000	1	\$	384,000	151%	190%	43%	1
1985- 2005	9	\$	2,405,000	112%	144%	43%	12
1990- 2010	7	\$	2,102,000	118%	152%	43%	9

Table VIII-B2-80% - Frozen

Summary

The results of this study have allowed us to draw a few concrete conclusions. One, it is not necessary to adopt a very aggressive equity posture when it comes to managing plan assets. Our findings have shown that a range of 36%-50% weight in equity can provide ample return to allow a plan to meet its obligations, as long as service costs are funded each year. We should emphasize that if a plan is run like a typical corporate plan, where service costs are not being continuously funded, the plan most likely runs the risk of being severely underfunded, which might necessitate higher equity allocations and the accompanying inefficiencies mentioned before. Even when the portfolios were rebalanced on an annual basis, the average equity weight was close to 36%. So, it is clear that in most "normal" economic scenarios, an asset allocation in this range will do the job. We would clearly advise any manager not to assign a 100% weight to equities because as shown in the analysis, this kind of weighting can produce disastrous results under the strain of a severe economic downturn. The risk that is associated with such a weighting is difficult to justify.

Making investment decisions simply based upon one factor is not wise. As a result, even though a 25% weighting towards equity produced the most efficient investment performance (as measured by Sharpe ratio), we found that other measures of investment performance should be taken into account. We would recommend that asset managers look at not only the Sharpe ratio, overall standard deviation of the plan, and the plan funding levels, but also focus on the total number of years that a plan's assets might be less than plan liabilities. The risk is that based on cumulative number of years where a plan's assets < liabilities, a manager might be tempted to allocate a 100% weight to equity, though we caution in the strongest possible terms that such a weighting be avoided. Again, the reason being that a manager must look at the overall riskiness of the plan rather than years "under water" scenario.

The second conclusion is that annual rebalancing of the portfolio seems advantageous, and indeed such rebalancing can produce robust results. For example, with the 80% initial funding

level, the overall average number of periods (out of a total 80 periods) where the assets < liabilities was 26. If we compare this absolute number with the 100% funding level of a simple plan, we note that it took an equity allocation of 50% to achieve this number. Clearly, it can be argued that annual rebalancing produces superior outcomes. It is also clear that annual rebalancing is superior to a unidirectional balancing, which produces far too many years where the pension plan might be "under water". However, our view is that this posture should be adopted if and only if the plans are most severely underfunded at the outset. If they are funded fully at the outset, a more conservative approach can result in sufficient levels of funding.

So the issues to resolve for pension providers are as follows: What type of plan to offer? How much to fund initially? Should the service costs be funded on an ongoing basis? Based on these parameters, an appropriate level of asset allocation can then be applied. We have tried to address two of these basic questions. We were able to demonstrate that when a pension plan is fully funded at the beginning, there is no need to have an aggressive posture. However, if a pension plan starts the initial funding at 80%, then other means must be used to get to a comfortable level of funding. It should also be noted that if plans are underfunded at the beginning, and even though the time weighted allocation to equity is generally low, there can be periods where the equity exposure may reach as high as 75% in order to achieve pension objectives. Of course, it goes without saying that starting a plan at lower funding levels does increase the overall risk that such a plan may not be able to achieve fully funded status as it is more subject to the timing issues of market behavior. An underfunded plan, starting out at the wrong period (say 2000) would find it nearly impossible to achieve a fully funded status without meaningful additional contributions by the plan sponsor.

The third and final conclusion that we can draw is that when plans are either closed and/or frozen, a gradual move towards FI asset allocation is sufficient to fully fund a pension plans requiring only about seven years to reach a fully funded level of 120%. Therefore, for closed and frozen plans, it is best and quite suitable to have the manager move towards a more conservative stance without the fear of not being able to meet plan obligations.

Appendix A: Calculations

Asset computations:

Ending period assets = Beginning period assets*(1+r) + Contributions*(1+sqrt(1+r)) – benefits paid*(1+sqrt(1+r))

Where r = periodic rate of return on a specific portfolio

Liability Methodology and Assumptions

Initial liabilities set at \$10 Million in base year (1974)

Liability amounts divided into two groupings with percentages: Actives / Term Vested (70%), Retired (30%) with static population (for ongoing plan) and no new actives (for closed and frozen plans)

Actives:	Base year payroll – \$6.4 Million (from 320 active count and average salary \$20,000)			
	Service Cost – 3.5% of payroll (to approximate actuarial cost due to the annual benefit accruals from a 1% final average pay plan)			
	Payrolls increase using the US CPI-W Multiplier			
Retirees:	Base year annual payouts set at 14% of retiree liabilities			
	Subsequent annual payouts of 14% of retiree liabilities are normalized for changes in interest rates			
	Mortality assumption -2.5% annually, represented rate based on 1994 US GAM blend table for the retiree group as a whole			
Liabilities:	Changes in liabilities beyond base year are valued at the Moody's AA Corporate rate			
	Duration – 14 (for ongoing plan); 12 (for closed and frozen plans)			
	Annual liability valuations employ a roll-forward method with retiree payouts weighted to be at mid-year			
	Liabilities adjusted annually for changes in the Moody's AA Corporate rate using the assumed duration			

Contribution Methodology and Assumptions

Annual contributions follow a methodology similar to the current Pension Protection Act rules:

Service Cost plus a 7-year amortization payment of any shortfall

Amortization interest rate is the current year's Moody's AA Corporate rate

Years with funding percentage below 100% - Service Cost + 7-yr amortization payment

Years with funding percentage at least 100% but below 110% - Service Cost

Years with funding percentage at least 110% but below 120% - One-half of Service Cost

Years with funding percentage at least 120% - no contribution

Service Cost for frozen plan is \$0

	Moodv's		US CPI-
	AĂ		W*
1974	7.92%	1973	6.20%
1975	9.20%	1974	10.96%
1976	9.25%	1975	9.07%
1977	8.24%	1976	5.73%
1978	8.40%	1977	6.47%
1979	9.33%	1978	7.72%
1980	11.15%	1979	11.43%
1981	13.78%	1980	13.41%
1982	15.00%	1981	10.25%
1983	12.44%	1982	6.02%
1984	12.76%	1983	2.99%
1985	12.50%	1984	3.51%
1986	10.63%	1985	3.48%
1987	9.02%	1986	1.59%
1988	10.33%	1987	3.59%
1989	9.81%	1988	4.00%
1990	9.11%	1989	4.79%
1991	9.39%	1990	5.22%
1992	8.61%	1991	4.11%
1993	8.24%	1992	2.90%
1994	7.12%	1993	2.82%
1995	8.62%	1994	2.46%
1996	6.99%	1995	2.88%
1997	7.41%	1996	2.87%
1998	6.99%	1997	2.27%
1999	6.65%	1998	1.33%
2000	7.78%	1999	2.19%
2001	7.48%	2000	3.49%
2002	7.19%	2001	2.72%
2003	6.63%	2002	1.38%
2004	6.02%	2003	2.22%
2005	5.69%	2004	2.61%
2006	5.50%	2005	3.52%
2007	5.58%	2006	3.19%
2008	5.91%	2007	2.88%
2009	5.80%	2008	4.09%
2010	5.44%	2009	-0.67%

Appendix B: Historical Rates for Liabilities

* The prior year US CPI-W is used in the roll-forward process due to using prior year compensations for a typical beginning of year actuarial valuation, i.e., 1974 Valuation is based on 1973 compensations.

Appendix C: Assumptions for dynamic model asset allocation

Plan funding level	Percent allocated to equity		
<90%	75%		
90-99%	65%		
100-109%	50%		
110-119%	25%		
>120%	0%		

Initial pre-determined asset allocation:

* 120% funding level is the assumed level to be fully funded on a plan termination basis.