COST OF LIVING ALLOWANCES
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Many small pension plans, perhaps designed as tax-shelters, are using Cost of Living Allowances (COLA) in calculating contributions. I want to review in this note, ways of handling the COLA in the actuarial valuation.

Assuming the COLA is given monthly and that pensions are paid monthly the present value of a straight-life annuity of $1. a month to a life age x is given by:

$$\sum_{t=0}^{\infty} (1+j)^{\frac{t}{12}} (1+i)^{-\frac{t}{12}} \frac{l_{x+\frac{t}{12}}}{l_x} \quad \ldots\ldots \quad \text{I}$$

where i is the valuation rate of interest and j is the assumed COLA. (i is assumed to be greater than j.)

In practice many actuaries would calculate this as $12\ddot{a}_x^{(12)}$ at rate (i - j). Others of more theoretical bent would calculate $12\ddot{a}_x^{(12)}$ at $\frac{(i - j)}{(1 + J)}$. The difference is slight!

For example, suppose x = 65, i = 6%, j = 3% and mortality is in accordance with the 1971 Group Annuity Table for males. At 3%, $12\ddot{a}_{65}^{(12)}$ equals $139.130; at 2.9126% (i.e. 3/1.03), $12\ddot{a}_{65}^{(12)}$ equals $140.126. The percentage error is only 0.71%.

However, it is more likely that the COLA is given once a year on the anniversary of the retirement date. Now the factor $12\ddot{a}_{65}^{(12)}$ at $\frac{(i - j)}{(1 + J)}$ is too large. The theoretically correct annuity value is given by:

$$12 \sum_{t=0}^{\infty} (1+j)^{t} (1+i)^{-t} \frac{l_{x+t}}{l_x} \ddot{a}_{x+t:\overline{1}|}^{(12)} \quad \ldots\ldots \quad \text{II}$$

where $\ddot{a}_{x+t:\overline{1}|}^{(12)}$ is computed at rate i.

Adopting the standard approximation for $\ddot{a}^{(12)}_{x+t\,\overline{1}|}$, namely $\left(\frac{13}{24} + \frac{11}{24} v\, p_{x+t}\right)$, the above expression simplifies to

$$12 \left[\frac{13}{24} \ddot{a}_x + \frac{11}{24} a_x / (1+j) \right]$$

$$\text{i.e.} \quad \left[12\, \ddot{a}^{(12)}_x \left(1 + \frac{13j}{24}\right) + \frac{143}{48} j \right] \left(\frac{1}{1+j}\right).$$

here $\ddot{a}^{(12)}_x$ is calculated at $\frac{(i - j)}{(1 + j)}$.

If x = 65, i = 6%, j = 3% and mortality is in accordance with the 1971 Group Annuity Table for Males the above value is $138.342. Use of $12\,\ddot{a}^{(12)}_{65}$ at 2.9126% gives a value 1.29% too high.

There is a more serious problem in the use of the "COLA increased" annuity values and this relates to those participants who are close to the dollar maximum pension. We know that current regulations do not allow us to claim a tax-deduction for that portion of the contribution which funds pensions in excess of that dollar maximum. However, using an annuity value at $\frac{(i - j)}{(1 + j)}$ may, in fact, implicitly allow for a pension in excess of the dollar maximum. For example, suppose a plan provides a pension equal to 100% of the final three years average compensation. A participant currently earning $100,000 will exceed the current maximum of $136,425 in 10.51 years assuming a 3% COLA and no salary scale. Ignoring this fact (as we would if we simply used $\ddot{a}^{(12)}_x$ at $\frac{(i - j)}{(1 + j)}$) would lay the plan open to the possibility of a partial disallowance of tax-deduction.

The following analysis is the correct way to value such pensions.

Define the following:

P = annual pension from normal retirement age x payable monthly.

j = COLA granted once a year on the anniversary of the retirement date.

i = valuation rate of interest

C = dollar maximum on pensions (currently in 1982 this is $136,425)

n = number of years it takes for P to reach C at rate j.

i.e. $P\,(1 + j)^n = C$

m = $[n]$ i.e. greatest integer in n.

The present value of the pension is now:

$$P \sum_{t=0}^{m} (1+j)^t (1+i)^{-t} \frac{l_{x+t}}{l_x}\, \ddot{a}^{(12)}_{x+t:\overline{1|}} + C\,(1+i)^{-(m+1)} \frac{l_{x+m+1}}{l_x}\, \ddot{a}^{(12)}_{x+m+1} \ldots$$

where all annuity values are calculated at rate i.

This expression simplifies to:

$$P\left[\frac{13}{24}\,\ddot{a}_{x:\overline{m+1|}} + \frac{11}{24}\,a_{x:\overline{m+1|}}\left(\frac{1}{1+j}\right)\right] + C\cdot {}_{m+1}E_x \cdot \ddot{a}^{(12)}_{x+m+1}$$

where $\ddot{a}_{x:\overline{m+1|}}$ and $a_{x:\overline{m+1|}}$ are both calculated at rate $\frac{(i - j)}{(1 + J)}$ and ${}_{m+1}E_x\, \ddot{a}^{(12)}_{x+m+1}$

calculated at rate i. An alternate way to write this expression is:

$$\frac{P}{(1+j)}\left[\left(1 + \frac{13j}{24}\right)\ddot{a}^{(12)}_{x:\overline{m+1|}} + \frac{143}{576}\,j\left(1 - {}_{m+1}E_x\right)\right] + C\cdot\frac{N^{(12)}_{x+m+1}}{D_x} \ldots$$

where $\ddot{a}^{(12)}_{x:\overline{m+1|}}$ and ${}_{m+1}E_x$ are both calculated at rate $\frac{(i - j)}{(1 + J)}$ and $\frac{N^{(12)}_{x+m+1}}{D_x}$ i

calculated at rate i.

Some examples will show the kind of error we introduce by ignoring the "levelling off" in the pension. In all cases i = 6%; j = 3%; C = 136,425.

P	60,000		100,000		110,000	
m	27		11		8	
x	55	65	55	65	55	65
Exact Value .e. Formula V.	939,739	Not calculated	1,433,460	1,100,774	1,511,384	1,173,655
alue ignoring Levelling off".	956,919	700,629	1,594,864	1,167,717	1,754,351	1,284,487
'ercentage Error	1.8%		11.1%	6.1%	16.1%	9.4%

The above value confirm what is intuitively reasonable, namely:

(a) the closer the current pension is to the dollar maximum the greater is the percentage error in ignoring the levelling off because there are greater "excess" pensions.

(b) the older the normal retirement age the smaller is the percentage error since the "excess" pension payments have a smaller probability of payment.

It is clear that the value of this "COLA increased annuity" is dependent on m whic is a function of P and C. Provided commutation functions at rate $\frac{(i - j)}{(1 + j)}$ are available it is a simple matter to compute the correct present value. Of course, a sophisticated computer program will handle the calculations quite easily. If neither commutation func tions nor a computer program is available the calculation is difficult. For such a situ tion I have devised an ad hoc approximate formula which gives results close enough to t true figures for all practical purposes.

Let us consider two cases:

case 1: $m < e_x$ i.e. the life age x is exp ected to live past the point in time wh the COLA causes the dollar maximum to be reached.

Here we will treat the annuity in the first (m + 1) years as an annuity certain a derive a present value of: $P \sum_{t=0}^{m} (1+j)^t (1+i)^{-t} \ddot{a}_{\overline{1}|}^{(12)}$

where $\ddot{a}_{\overline{1}|}^{(12)}$ is calculated at rate i.

This simplifies to $P \ddot{a}_{\overline{1}|}^{(12)} \ddot{a}_{\overline{m+1}|}$ where $\ddot{a}_{\overline{1}|}^{(12)}$ is at rate i and $\ddot{a}_{\overline{m+1}|}$ is

rate $\frac{(i - j)}{(1 + j)}$.

For the period past time (m + 1), the pension payable is C p.a. and at age x const a deferred annuity. Instead of valuing this by a factor of $\left(\ddot{a}_x^{(12)} - \ddot{a}_{x:\overline{m+1}|}^{(12)}\right)$ at rat we will change the $\ddot{a}_{x:\overline{m+1}|}^{(12)}$ to $\ddot{a}_{\overline{m+1}|}^{(12)}$, thinking that we treated the annuity as an an certain for the first (m + 1) years in the factor $P\ddot{a}^{(12)}_{\overline{n}|}\ddot{a}_{\overline{m+1}|}$ and hence overestimate the value so we will similarly treat it in the deferred portion of the annuity, and hence overvalue also. Hopefully the two errors will balance out. The value thus becomes:

$$P\ddot{a}^{(12)}_{\overline{n}|}\ddot{a}_{\overline{m+1}|} + C\left(\ddot{a}^{(12)}_{x} - \ddot{a}^{(12)}_{\overline{m+1}|}\right) \quad \ldots \quad \underline{\overline{\text{VI}}}.$$

where $\ddot{a}^{(12)}_{\overline{n}|}$, $\ddot{a}^{(12)}_{x}$ and $\ddot{a}^{(12)}_{\overline{m+1}|}$ are all calculated at rate i and $\ddot{a}_{\overline{m+1}|}$ is calculated at rate $\frac{(i - j)}{(1 + j)}$.

Applying formula $\underline{\overline{\text{V}}}$ to the above examples for P = 100,000 and P = 110,000 we get the following values:

P	100,000		110,000	
m	11		8	
x	55	65	55	65
Exact Value Using Formula V	1,433,460	1,100,774	1,511,384	1,173,655
Approximate Value Using Formula VI	1,427,853	1,085,562	1,508,925	1,166,635
Percentage Error	- 0.4%	- 1.4%	- 0.2%	- 0.6%

These errors are certainly acceptable in the whole context of an actuarial valuation.

Case 2: $m > \mathring{e}_x$

Here it is reasonable to ignore the levelling off; after all the percentage error is only 1.8% for normal retirement age 55 and virtually zero for a normal retirement age of 65.