Problems and Solutions Section

In ARCH 1977.1, the following problem appeared.

Problem 11, Submitted by John A. Beekman. Use demographic techniques to approximate the effects on India's population caused by recent provincial laws limiting family size.

Notes to readers. Some appreciation of the social problems created by large population growth can be gleaned by reading: Md. Humayun Kabir, "Mathematical Demography Applied to Bangladesh Population", ARCH, Issue 1975.3, expecially pages 1, 15, 16, 17, 18, 19, 20. This paper also shows some of the exciting recent demographic techniques for population projections.

Solution to 11 by: Md. Humayun Kabir
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Estimation of r from Lotka's Equation: {reference: N. Keyfitz, Introd. to the Math. of Population, Addison - Wesley, Reading, Mass., 1968, pp. 97 - 101, 111, 113}.

Let us have the following assumptions:

a. Every woman is allowed to have only 3 children {ignore the survival factor now}.

b. Sex ratio at birth is 1:1.

c. These 3 babies are born to the woman at ages 22, 25, and 28 respectively.

b. $\Rightarrow$ maternity function = $m(x) = 0.5$ which

$\Rightarrow m(22) = m(25) = m(28) = 0.5$.

If $l(x)$ is known from some source, and we assume $l(0) = 1$, then the characteristic equation

$$\psi(r) = 1 = \int_0^{\omega} e^{-rx}\, l(x)\, m(x)\, dx \text{ becomes}$$

$$1 = e^{-22r} l(22)(.5) + e^{-25r} l(25)(.5) + e^{-28r} l(28)(.5)$$

$$\text{or } e^{20r} = .5\left[e^{-2r}\, l(22) + e^{-5r}\, l(25) + e^{-8r}\, l(28)\right]$$

$$\text{or } r = \frac{\ln\left[.5\{e^{-2r}\, l(22) + e^{-5r}\, l(25) + e^{-8r}\, l(28)\}\right]}{20}$$

Iterative solution will give the value of r, the intrinsic rate of growth in the population.

One can construct one's own equation for different policy alternatives and get solutions for r.