The Actuary

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Exam Seminars at Waterloo and Georgia State

Waterloo and Georgia State Universities are holding seminars to prepare students for the May 1985 exams. Waterloo: Parts 4, 5, 6, EA1, 8, 10 (U.S. & Canada) from April 20 to May 5. Georgia State: Parts 2, 3, 4, 5, 6, EA1 during April. Further information is available from Frank G. Reynolds (Waterloo) or Robert Batten (Georgia State) at their Yearbook addresses.

ACTUARIES TO OFFER SHORT COURSE TO MATHEMATICIANS

At the annual Actuarial Research Conferences, lecturers from other mathematical sciences are often invited to share their ideas with actuarial teachers and other interested actuaries. The reverse situation will happen August 10-11, 1985 in Laramie, Wyoming, when actuaries will present a short course on actuarial mathematics under the sponsorship of the American Mathematical Society (AMS). The speakers and their topics will be:

James C. Hickman: Updating Actuarial Mathematics
Harry H. Panjer: Models in Risk Theory
Stuart A. Klugman: Loss Distributions
Paul M. Kahn: Overview of Credibility Theory
Elias S. Shiu: A Survey of Graduation Theory
John A. Beekman: Actuarial Assumptions and Models for Social Security Projections

Co-organizers are James Hickman (Wisconsin, Madison, 608-262-1893), Cecil Nesbitt (Michigan, 313-764-7227), and Elias Shiu (Manitoba, 204-474-8723). It is expected that the lecture notes, edited by Professor Shiu, will proceed to publication in the AMS book series, Proceedings of Symposia in Applied Mathematics.

Interested individuals may obtain further information about the Short Course from the March Issue of Notices of the AMS, or by inquiring from the co-organizers. For preregistration and housing arrangements, one may call the toll free number 800-556-7774. □

INTEREST SENSITIVITY — A CONTINUATION

Responses to Ralph Garfield's *Skepsis Avaunt* (October), and to a lesser extent *Interest Sensitivity* offered by the Editor (December), flood our late '84 and early '85 mails. In January and February we were barely able to summarize and acknowledge your many analyses and comments, doing real justice to none.

At the risk of overdoing what has become a pretty lively exchange, we assume that there is still interest in interest, and offer below an addition to the December article and, on pages 6 and 7, an interesting letter from Cecil J. Nesbitt.

C.L.T.

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by C.L. Trowbridge

The earlier analysis shows that the interest sensitivities of many actuarial functions can be measured by -vd, where the duration is a weighted average of the years until the payments become due. This continuation looks more closely into the characteristics of d. Because we have found that d is not a constant, but in itself is a function of the interest rate i, we will hereafter refer to duration as d(i).

Definition

We have previously defined

$$d(i) = \frac{\Sigma\ t\ C_t\ v^t}{\Sigma\ C_t\ v^t}$$

where $v = 1(/1+i)$, C_t is a non-negative payment at time t, and the summation is over all t, positive or negative. Note we here allow t to be negative, representing payments in the past.

We note that d(i) is a function of the C_t, as well as a function of i; and that it can be either positive or negative, depending upon whether the payments are predominantly in the future or in the past. d(i) can also be zero, if the past and future payments are in balance. Of course, if we are dealing with the quotient of the present value functions, the average duration of the quotient will be zero if the average duration of the denominator equals the average duration of the numerator.

Calculation

Practical use of the duration concept depends partly upon our ability to calculate duration. Calculation directly from the definition is not at all difficult if the number of non-zero C_t is small, or if the interest rate is zero. Otherwise the calculation of the weighted average d(i) may be tedious. Today's electronic calculators, with power buttons and programming capabilities, make the calculation easier.

d(i) can also be computed indirectly, by calculating the original function for two closely adjacent values of i, then applying:

$$d(i) = \frac{f(i) - f(i+\Delta i)}{f(i)\cdot\Delta i}\ (1+i)$$

There will be times when this method is easier, or where one method is used as a check on the other.

Sensitivity of d(i) to i.

One of the peculiarities in this analysis is that the function d(i), used in testing the interest sensitivity of a function f(i), is itself sensitive to changes in the interest rate. This leads us to examine the derivative of d(i) with respect to i.

$$D\ d(i) = D\ \frac{\Sigma\ t\ C_t\ v^t}{\Sigma\ C_t\ v^t}$$
$$= -\ v\left[\frac{\Sigma\ t^2\ C_t\ v^t}{\Sigma\ C_t\ v^t} - \left(\frac{\Sigma\ t\ C_t\ v^t}{\Sigma\ C_t\ v^t}\right)^2\right]$$
$$= -\ v\ \sigma^2(i)$$

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Interest Sensitivity — A Continuation

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where $\sigma^2(i)$ represents the variance of the same distribution for which d(i) is the mean.

This is indeed an interesting relationship. We see at once that the duration decreases as the interest rate rises (σ^2 must be >0), and that the change is measured by $v\sigma^2$. For a wide-spread series of C_t, $d_{(i)}$ will be highly sensitive to changes in i, but a compact series will be less so. Should there be only one non-zero C_t, i.e. payments all concentrated at a single point of time, $\sigma^2 = 0$, and duration is independent of i.

Limiting Values of d(i).

We have seen that d(i) decreases as the interest rate increases. This of course means that if d is originally positive, it becomes less so; and that if d is first negative it becomes more so. What is the limit of d(i) as i becomes infinite?

Dividing numerator and denominator by v^k where k is the smallest value of t for which a non-zero C_t exists shows that the limit in question is k. Thus the earliest payment is controlling for extreme values of i.

Illustration

We conclude with an example that illustrates many of the relationships here considered. We assume a payment of 1 a year ago, and a payment of 2 five years hence. The C_t series becomes $C_1 = 1$, $C_5 = 2$, and all other $C_t = 0$. $f(i) = (1+i) + 2v^5$, and $d(i) = \frac{-(1+i) + 10\ v^5}{(1+i) + 2\ v^5}$. The following table shows values of f(i), d(i), and σ^2 (i) for a selection of interest rates.

i	f(i)	d(i)	σ^2(i)
0	3	3	8
.03	2.7552176	2.7569829	8.426977
.06	2.5545164	2.5102919	8.7396026
.060001	2.5545103	2.5102837	8.7396107
.09	2.3898628	2.2634412	8.9305989
.12	2.2548537	2.0197623	8.9996093
.4678	1.7613591	0	5.0
1.0	2.0625	−0.8181818	1.0578505
10.0	11.000006	−0.9999938	.0000395
∞	∞	−1	0

Notes:

(a) i = .060001 included to permit check by indirect calculation.

(b) f(i) decreases up to point where d(i) crosses zero, then increases.

(c) d(i) decreases, becoming asymptotic to −1.

(d) σ^2 (i) increases, then decreases. Although not previously discussed, the derivative of σ^2 (i) is −v times the third moment. At i = 0 the third moment is −16 and the derivative of σ^2(i) is positive. Somewhere above 12% the third moment turns positive. In the limit all the weight is at −1, and the variance approaches zero. ☐

Disability Valuation Tables

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The timetable is very short! The Committee needs your responses quickly in order to address them in the May 22 Report. We hope to be able to address subsequent responses received prior to July 31. Any major concerns you may have should either be exposed at PD46 in San Francisco, or addressed to the Committee Co-chairmen, William J. Taylor (Massachusetts Mutual) and Duane Kidwell (The Paul Revere Companies) with a copy to Mark Doherty, Director of Research (The Society Office). ☐

LETTERS

Study Material

Sir:

In light of the many recent syllabus changes, I feel that each exam should be published and included in the study material for the following exam period, along with suitable sample questions for any new material. *All* students would then have access to the prior exam, which access only *some* students now enjoy.

I have confidence in the exam committee's ability to design unique questions for each exam period. Since CPA and bar exams become common knowledge shortly after their administration, why not the exams of the actuarial profession?

Dean L. Taylor

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Sir:

In October Rick Edwards asked why students should be tested on material unavailable until a few months before the test. Another question is why students should be tested on material that has not been updated in four years, while the related laws and procedures have been modified in each of those years? (See Part 7P - EA1&2).

Cathy Drown

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Reserve Bases and Professional Standards

Sir:

The tax turbulence of 1983 and 1984 has given rise to some interesting tax strategies by various companies. Much has been done and not discussed openly. The 1983 audits by the IRS will be done in a couple of years. Then, there will be some interesting decisions.

In practice, many actuaries have discovered that many State Insurance Departments do not police the raising of reserves. Thus, in effect, there is no check on a company which chooses to raise reserves for some tax purpose unless the IRS strengthens its enforcement procedures.

By reading recommendations 7 and 9 of the American Academy of Actuaries,

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