DISCONTINUITY AT THE BOUNDARY

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To solve mutiple choice problems by elimination, one approach is to study values of functions at the boundary. [1] [2] *In this note, we show by examples that the method of boundary values can be risky if the tacit assumption of continuity at boundary is not checked.*

EXAMPLE 1 (Compare [2,p.4,S76M21])

If $s(x) = v^x$ *, simplify* $\bar{A}_{x_1 x_2 \cdots x_m}$.

(A) $\frac{1}{m\delta}$ (B) $\frac{m}{m+1}$ (C) $\frac{m-1}{m}$ (D) $\frac{1-v^m}{1-v^{m-1}}$ (E) $\frac{v-v^{m+1}}{1-v^{m+1}}$

Boundary Value Method:

Let $v = 1$. *Then* $i = 0$ *and everyone lives forever.*

Since $\bar{A}_{x_1\ x_2\ \cdots\ x_m} = 0$, *this eliminates (A), (B), and (C).*

Let $v = 0$. *Then* $i = \infty$ *and everyone dies immediately.*

If $\bar{A}$ *is taken to mean payment an instant after death,*

then $\bar{A}_{x_1\ x_2\ \cdots\ x_m} = 0$ *and agrees with (E).*

If $\bar{A}$ *is taken to mean payment immediately at death,*

then $\bar{A}_{x_1\ x_2\ \cdots\ x_m} = 1$ *and agrees with (D).*

Conventional Method:

$$\bar{A}_{x_1\ x_2\ \cdots\ x_m} = \int_0^\infty v^t\ {}_tp_{x_1 x_2 \cdots x_m}\ \mu_{x_1+t:\cdots:x_m+t}\ dt$$

$$= \int_0^\infty v^t\ v^{mt}\ (\ -m\ \log_e v\ )\ dt$$

$$= \frac{m}{m+1}\ (\ 1\ -\ \lim_{t\to\infty} v^{(m+1)t}\ )$$

$$= \frac{m}{m+1} \qquad \text{for} \quad 0 \le v < 1\ .$$

The correct answer, (B), is eliminated by the boundary value method!

Explanation:

$\bar{A}_{x_1\ x_2\ \cdots\ x_m} = 0$ *for* $v = 1$ *from the conventional method aomputation. Since* $\bar{A}_{x_1\ x_2\ \cdots\ x_m}$, *as a function of* v , *is not continuous at* $v = 1$, *using that boundary value eliminates the correct answer.*

EXAMPLE 2 (Compare [2,p.2,F75M1])

Which of the following is exactly equal to

$$\frac{\mathring{e}_x - \mathring{e}_{x:n}}{\mathring{e}_{x+n}}$$

(A) 0 (B) ½ (C) 1 (D) ${}_np_x$ (E) ${}_nq_x$

Boundary Value Method:

As suggested in [2] *, we consider a life table in which everyone lives to the exact age* ω *and dies. Under this assumption, for* $n < \omega - x$,

$$\frac{\mathring{e}_x - \mathring{e}_{x:n}}{\mathring{e}_{x+n}} = \frac{(\omega - x) - n}{\omega - (x+n)} = 1 .$$

This eliminates (A), (B), and (E). Next, consider

$$\lim_{n \to \omega - x} \frac{\mathring{e}_x - \mathring{e}_{x:n}}{\mathring{e}_{x+n}} = 1 .$$

Since ${}_{\omega-x}p_x = 0$ *, (D) is eliminated and (C) appears to be the correct answer.*

Conventional Method:

$$\mathring{e}_x - \mathring{e}_{x:n} = \int_n^\infty {}_tp_x \, dt = {}_np_x \int_0^\infty {}_sp_{x+n} \, ds = {}_np_x \, \mathring{e}_{x+}$$

Thus, the correct answer is (D).

Explanation:

Under the assumption of guaranteed survival until the doomed age of ω *, the given expression*

$$\frac{\overset{o}{e}_x - \overset{o}{e}_{x:n}}{\overset{o}{e}_{x+n}} ,$$

as a function of n *, is not defined at* $n = \omega - x$ *. Taking the limit at that boundary value eliminates the correct answer.*

Bambrough [1] *and Silver* [2] *have given excellent accounts and examples for nonstandard methods. These methods extend imagination into specific cases and deepen understanding of the mathematical model. For the students, the hazards illustrated in this note would enhance the educational value of these methods and would be a catalyst for thorough studies.*

REFERENCES

1. *Bambrough, Brian:* Problem Solving in Life Contingencies, *published by the author, 1979.*
2. *Silver, Murray: Nonstandard Solutions to Problems in Life Contingency Examinations,* ARCH, *1979.2, 1-5.*