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Professor Ralph Garfield, Chairman
Actuarial Science Division
College of Insurance
123 William Street
New York, New York 10038

Dear Professor Garfield:

I have just read your contribution to the 1978.1 issue of ARCH. I believe that I have the answer to your question concerning the non-applicability of the identity

$$DU_x = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{U_{x+nh} - U_{x-nh}}{nh}$$

Consider the following solution to this problem:

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{E^{nh} - E^{-nh}}{nh} = \frac{1}{h} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(E^h)^n}{n} - \frac{1}{h} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(E^{-h})^n}{n}$$

$$\overset{*}{=} \frac{1}{h} \ln(1+E^h) - \frac{1}{h} \ln(1+E^{-h})$$

$$= \frac{1}{h} \ln(E^h)$$

$$= D$$

The step at * does not satisfy Kellison's requirement for applying infinite series of operators. Upon expanding E^{hn} in terms of Δ_h, we see that Δ_h^k will appear infinitely often in the summation, regardless of the value of k. Thus, even for polynomials there will be no n beyond which the summand is guaranteed to vanish. The summability of the series will depend on the function, as you have observed.

It does appear from your example that the following relationship will hold for all polynomials (and perhaps a more general class of functions).

$$DU_x = \lim_{h \to 0} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{U_{x+nh} - U_{x-nh}}{nh}.$$

I have not attempted to verify this statement.

Sincerely yours,

Stuart Klugman
Assistant Professor
Department of Statistics

SK:ivp

c.c. Arnold Shapiro
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