Selecting Mortality Tables: A Credibility Approach

Gavin Benjamin

October 2008

Historically, pension actuaries have tended to focus their attention on the establishment of appropriate economic assumptions because changes in assumptions such as the liability discount rate can have a significant effect on the measurement of a plan's liabilities and costs.

The selection of demographic assumptions has often received less attention since changes to these assumptions tend to have a less material effect. However, the use of inappropriate demographic assumptions for valuation purposes can, over the long term, have adverse consequences if it results in a material misstatement of a pension plan's liabilities or costs.

In today's environment, the assumptions selected by pension actuaries are coming under increased scrutiny. It is becoming more and more common for actuaries to be asked to provide justification for <u>all</u> of their assumptions, including the demographic assumptions, to various stakeholders such as regulators.

As part of the ongoing valuation and plan management process, an actuary should conduct periodic reviews of the demographic experience of a pension plan and, based on these reviews and future expectations, make appropriate adjustments to the demographic assumptions. The use of credibility theory can, and perhaps should, be a key aspect in the analysis that leads the actuary to the selection of a mortality table for a pension plan valuation.

While use of credibility theory to adjust mortality rates can be very useful, the following limitations should be kept in mind with respect to the discussion:

- This article does not attempt to reflect any rules on the selection of mortality tables that may be imposed by applicable legislation or regulators. Also, it is not the intent of this article to take a position on the appropriateness of the thresholds at which a regulator may have chosen to permit a plan's mortality experience to be reflected in the selection of a mortality table.

- The focus is on the selection of the base mortality table, which reflects the mortality rates of plan members **at the time of a mortality experience study**. While assumptions regarding future mortality improvement after the study date are very important, the selection of a mortality projection assumption is beyond the scope of this article.

Definitions

For purposes of our analysis, the following definitions for a given pension plan are required:

m_x, $x = \alpha$ to τ, is the number of members and former members in a mortality experience study age x and τ is the maximum age to which a plan member can live.
It has been assumed that, for experience study purposes, integral ages are used. (For example, a member's actual age at the time of the study may be rounded to the closest integral age.)

q_x, $x = \alpha$ to τ, is the actual underlying mortality rate for members and former members age x.

q_x^E, $x = \alpha$ to τ, is the mortality rate at age x, based on the mortality table currently in use for valuation purposes.

$\hat{q}_x$, $x = \alpha$ to τ, is the estimated mortality rate at age x, which reflects the results of the experience study.

b_{xj}, $x = \alpha$ to τ, $j = 1$ to m_x, is the accrued pension for the j^{th} member or former member who is age x.

d_{xj}, $x = \alpha$ to τ, $j = 1$ to m_x, is equal to 1 if the j^{th} member or former member who is age x dies during the year, and is zero otherwise.

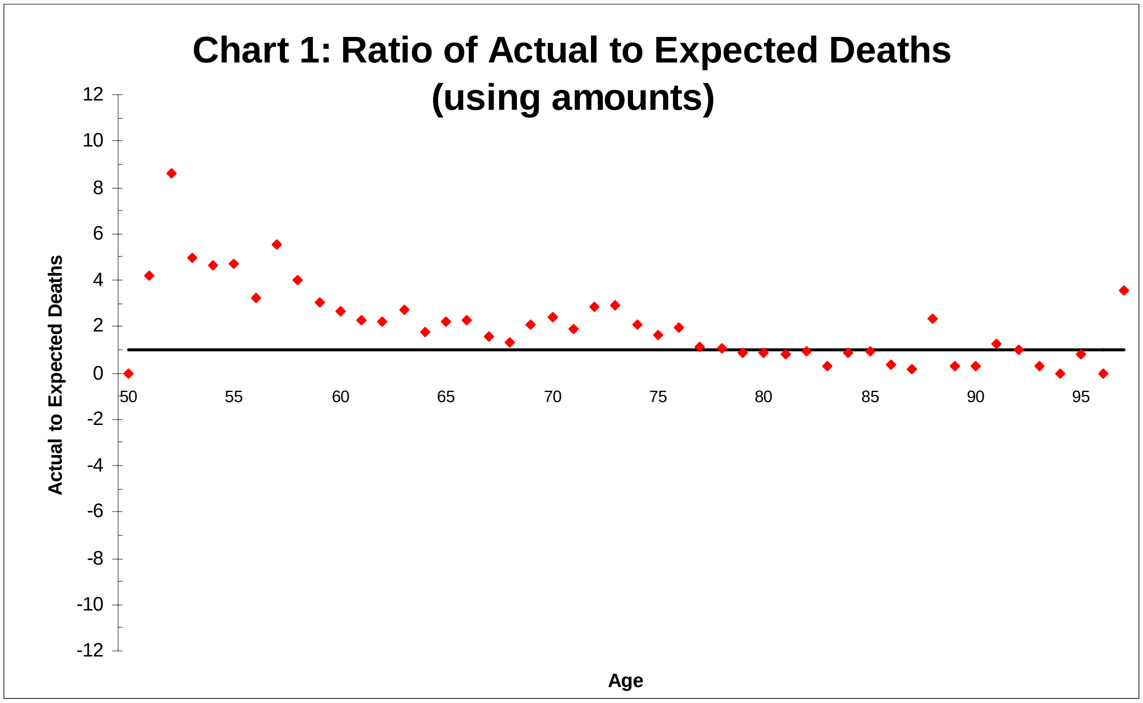
Z_p is the p$^{\text{th}}$ percentile of the standard normal distribution.

Mortality Experience Studies: Amounts versus Lives

Published mortality tables such as GAR-94, UP-94, and RP-2000 were developed using amounts rather than lives (i.e., they were determined by dividing total accrued pension amounts for those who died by total accrued pension amounts for all exposed at a given age). This approach is intended to be a proxy for weighting mortality rates by liability. Since studies have consistently shown that amount-weighted mortality rates are materially lower than lives-weighted rates, mortality experience studies should be conducted using amounts. In other words, the use of lives-weighted rates for valuation purposes will tend to result in an understatement of pension liabilities.

When Should You Consider Changing a Valuation Mortality Table?

When assessing whether to change a valuation mortality table based on the results of an experience study, begin by comparing the observed deaths weighted by benefit amount to the expected deaths weighted by benefit amount. For example, Chart 1 below shows the ratios of amounts of actual deaths ($\sum_{j=1}^{m_x} b_{xj} d_{xj}$) to the amounts of expected deaths $(\sum_{j=1}^{m_x} b_{xj} q_x^E)$ for a pension plan with 14,000 active and retired males within the age band of x = 50 to 97. Expected deaths in Chart 1 are based on the UP-94 static table projected to 2005 using mortality improvement Scale AA.



If the current mortality table is a good representation of the actual underlying mortality rates of plan members and former members, the ratios of actual to expected deaths should closely track 1 (which is shown as the black line in Chart 1). In Chart 1, actual deaths are generally greater than expected deaths before age 75, and are usually less than expected deaths after age 85. However, after age 85 there are very little data, so the credibility of the ratios after age 85 is questionable. Between ages 75 and 85, actual deaths closely track expected deaths. Overall, the differences between actual and expected deaths before age 75 seem to indicate that a revision to the current table is warranted.

In the above example, it is fairly clear that a change in the mortality table should be considered. In most situations, however, the differences between actual and expected deaths will not be so extreme. The actuary must always exercise judgment when deciding whether to revise a mortality table. One of the considerations could be the characteristics of plan members and former members (e.g., whether they are blue or white collar workers) and the potential effect on future mortality experience.

Pension plan liabilities are usually not very sensitive to the mortality assumption at younger ages. This is because mortality rates at younger ages are relatively small and many pension plans provide a death benefit to the beneficiary of a member who dies while in active employment. Therefore, as in the above example, experience studies often focus on ages 50 to 55 and above.

Building a Table from Scratch—Full Credibility

If a decision is made to change the mortality assumption, one approach is to create completely new tables using the data from the experience study. In this case, for each age x, the estimate of q_x (i.e., $\hat{q}_x$) would be determined as:

$$\hat{q}_x = \frac{\sum_{j=1}^{m_x} b_{xj} d_{xj}}{\sum_{j=1}^{m_x} b_{xj}}$$

An obvious question is "How credible is my estimate of q_x?". One way of defining full credibility is to select a small constant *r* and large probability *p*, and say that $\hat{q}_x$ is fully credible if:

$$\Pr[\left|\hat{q}_x - q_x\right| \le r q_x] \ge p$$

Assuming that deaths are independent and using the Central Limit Theorem, it can be shown that full credibility is achieved when the expected <u>number</u> of deaths is greater than or equal to:

$$\lambda_0 \frac{m_x \sum_{j=1}^{m_x} b_{xj}^2}{\left(\sum_{j=1}^{m_x} b_{xj}\right)^2}, \text{ where } \lambda_0 = \left(\frac{Z_{1-(1-p)/2}}{r}\right)^2$$

Alternatively, full credibility is achieved when the expected <u>dollars</u> of death are greater than or equal to:

$$\lambda_0 \frac{\sum_{j=1}^{m_x} b_{xj}^2}{\sum_{j=1}^{m_x} b_{xj}}$$

For example, consider males age 70 and assume that a mortality experience study is based on number of lives, in which case $b_{70,j}$ is set equal to 1 for all *j*. With *r* = 0.05 and *p* = 0.90, full credibility is achieved when the expected number of deaths is greater than or equal to:

$$\lambda_0 \frac{m_{70}\sum_{j=1}^{m_{70}} b_{70,j}^2}{\left(\sum_{j=1}^{m_{70}} b_{70,j}\right)^2} = \lambda_0 = \left(\frac{1.645}{0.05}\right)^2 = 1{,}082$$

For credibility calculations, r = 0.05 and p = 0.90 are common values used for those variables and are based on achieving a typical desired level of confidence.

If the actual underlying mortality table is UP-94 static, projected to 2005 using Scale AA, then $q_{70} = 0.022$ and more than 49,000 exposures for males age 70 are needed to expect the 1,082 deaths required for the estimate of q_{70} to be fully credible. For an experience study based on amounts rather than lives, the number of exposures needed would be even greater.

Extrapolating from the above example for males age 70 to other ages, it is clear that a large amount of data is required to build a fully credible mortality table from scratch. Only a few of the largest pension schemes in the world can build a fully credible mortality table from scratch using their own experience data.

In addition, creating a mortality table from scratch is not a trivial exercise. For example, once the raw mortality rates have been determined, they must then be smoothed so that the rates progress in a reasonable pattern from age to age. Graduation techniques are often used for purposes of smoothing these rates. The process can be time-consuming and challenging.

Adjusting a Standard Table—Full Credibility

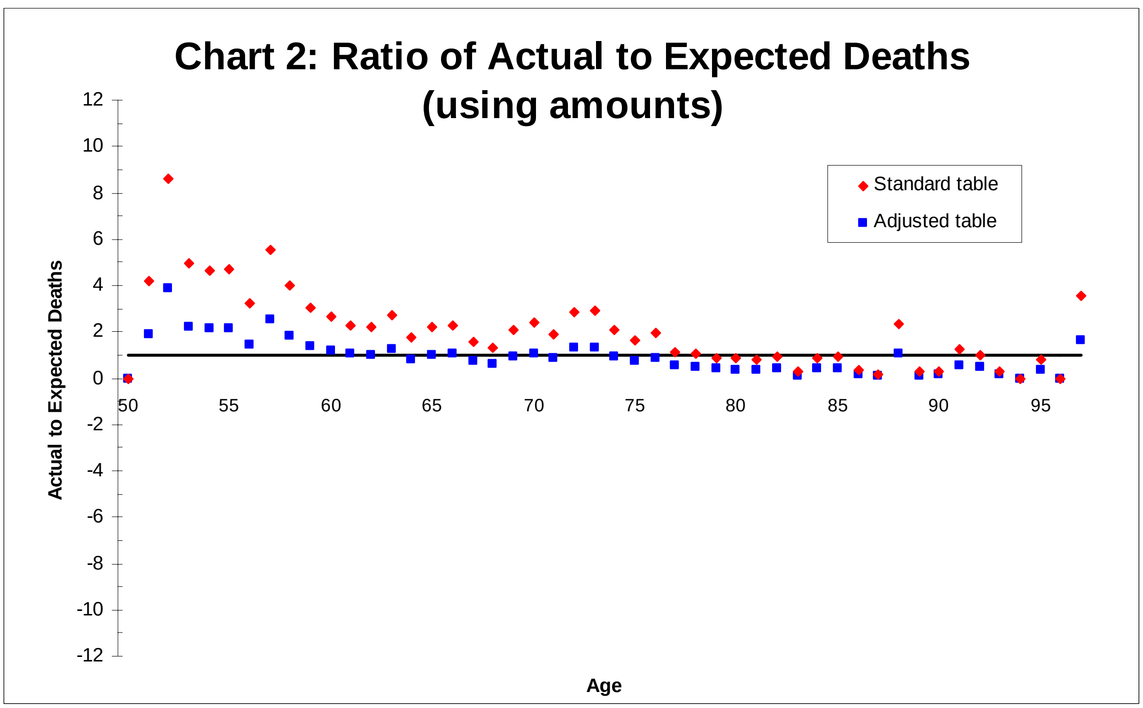
As can be seen above, building a mortality table from scratch is a non-starter for most pension plans. A more practical approach is often to rate a version of a standard table, such as the UP-94 or RP-2000, up or down based on the total death amounts from the experience study. More precisely, define $\hat{f}$ as the ratio of actual to total expected death amounts for all ages, and then, assuming this value is fully credible, set the mortality rate at each age *x* to $\hat{f}\, q_x^E$.

For example, in the experience study used to develop Chart 1 above:

$$\hat{f} = \frac{\sum_{x=50}^{97}\sum_{j=1}^{m_x} b_{xj} d_{xj}}{\sum_{x=50}^{97}\sum_{j=1}^{m_x} b_{xj} q_x^E} = 2.20 \text{, and } \hat{q}_{70} \text{ would be set equal to } \hat{f}\, q_{70}^E = 2.20 \text{ x } 0.022 = 0.0484.$$

For ages close to τ, the maximum age to which a plan member is assumed to live, mortality rates should be adjusted, if necessary, so that there is a reasonable progression from the mortality rates close to τ to the mortality rate of 1 at age τ.

Chart 2 shows in blue actual to expected deaths using the adjusted mortality rates.



It is reasonable to ask whether the resulting mortality table created by adjusting a variation of a standard table as described above will be credible. If credibility is assumed to be achieved when:

$$\Pr[|\hat{f} - f| \le rf] \ge p \text{, where } f = \frac{\sum_{x=a}^{b}\sum_{j=1}^{m_x} b_{xj} q_x}{\sum_{x=a}^{b}\sum_{j=1}^{m_x} b_{xj} q_x^E}$$ and *a* and *b* are the minimum and

maximum ages, respectively, of the plan members included in the experience study, then it can be shown that the table is fully credible when the expected number of deaths, *N,* is greater than or equal to:

$$\lambda_0 \frac{\left(\sum_{x=a}^{b} m_x q_x\right)\left(\sum_{x=a}^{b} q_x \sum_{j=1}^{m_x} b_{xj}^2\right)}{\left(\sum_{x=a}^{b} q_x \sum_{j=1}^{m_x} b_{xj}\right)^2}$$

Alternatively, full credibility is achieved when the expected dollars of death, *D*, are greater than or equal to:

$$\lambda_0 \frac{\sum_{x=a}^{b} q_x \sum_{j=1}^{m_x} b_{xj}^2}{\sum_{x=a}^{b} q_x \sum_{j=1}^{m_x} b_{xj}}$$

Since the actual mortality rates are unknown, it is reasonable to use $\hat{f}\, q_x^E$ to estimate q_x .

Applying the previously-defined formula for *N* to the data from our example, $\hat{N}$ becomes 1,635 while the actual number of study deaths is only 703. Therefore, the adjusted table in this case would not be fully credible.

Adjusting a Standard Table—Partial Credibility

If there are insufficient data for the adjustment to a standard table described in the previous section to be fully credible, a more appropriate approach would be to assign partial credibility to the results of the mortality study. In this case, q_x would be estimated as:

$Z\, \hat{f}\, q_x^E + (1 - Z) q_x^E$, where $0 < Z < 1$ is the credibility weighting assigned to the experience study.

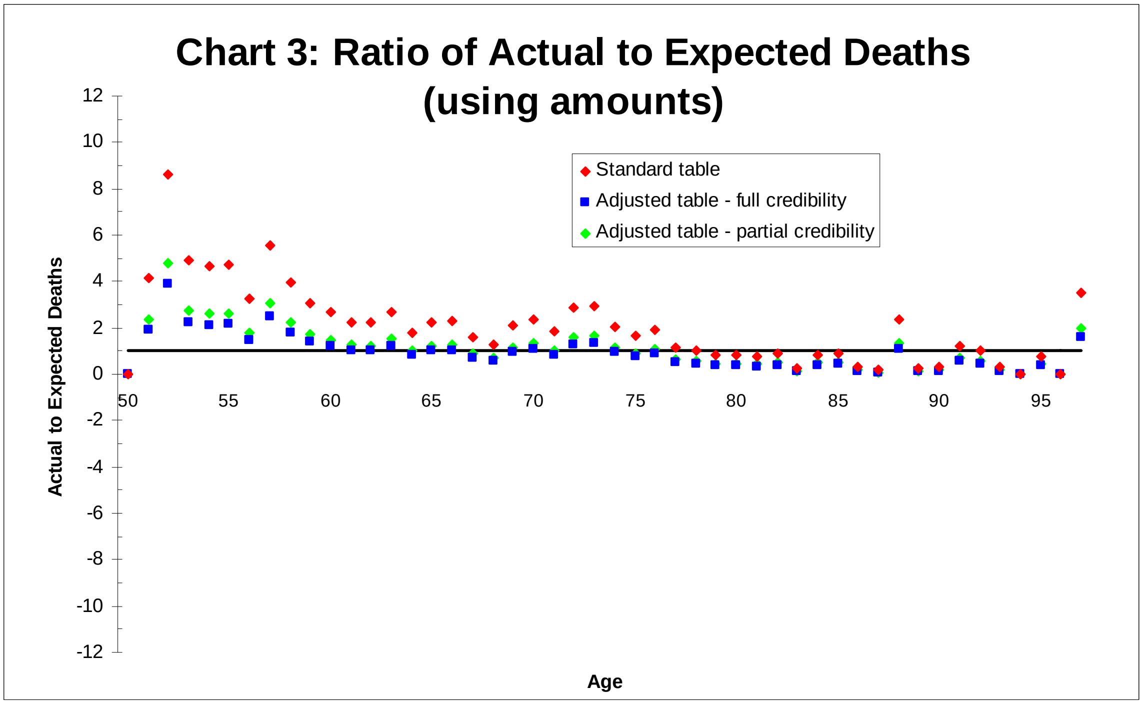
Z can be determined as:

$\sqrt{\dfrac{\sum_{x=a}^{b}\sum_{j=1}^{m_x} d_{xj}}{\hat{N}}}$, or equivalently $\sqrt{\dfrac{\sum_{x=a}^{b}\sum_{j=1}^{m_x} b_{xj} d_{xj}}{\hat{D}}}$.

In our example, Z is equal to $\sqrt{\dfrac{703}{1{,}635}} = 0.656$ and

$\hat{q}_{70}$ would be set equal to $Z\,\hat{f}\,q_{70}^{E} + (1-Z)q_{70}^{E} = 0.656 \text{ x } 2.20 \text{ x } 0.022 + (1-0.656) \text{ x } 0.022 = 0.0393$ which is between the standard table value of 0.022 and the "full credibility" adjusted value of 0.0484 determined above.

Chart 3 shows, in green, the actual to expected deaths based on adjusted mortality rates using the partial credibility approach.



Conclusion

As with all actuarial work, judgment must be used when adjusting mortality assumptions. For example, the credibility approach outlined above assumes that the <u>shape</u> of the standard table is appropriate for the plan being valued, and all that is required is a proportional adjustment (either up or down) to the standard table. If an actuary believes strongly that the underlying shape of the mortality curve for a plan differs significantly from all available standard tables, he or she may choose to build a table from scratch using experience data from the plan, even if a credible amount of experience data is not available. Nevertheless, as long as judgment is used, credibility theory can serve as a powerful tool for an actuary setting the mortality assumption for a pension plan.

Gavin Benjamin, FSA, FCIA, is a principal with Towers Perrin in Toronto, Ontario. He is Chair of the Pension Section Retirement Plans Experience Committee. He can be reached at *gavin.benjamin@towersperrin.com*.