ALLOCATING ACTUARIAL RESOURCES AMONG BRANCH OFFICES

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ABSTRACT

This article shows how one could determine how the workload should be divided between the branch offices of a consulting firm in such a way as to minimize costs.

Introduction

Many uses were recognized for the Simplex Method very soon after it was developed. One such important application was to the solution of a general supply-and-demand transportation problem which was formulated by Dantzig and Schell.[1]

The transportation problem assumes that there are supply centers and demand centers which are in fixed geographic locations. There are various transportation costs associated with supplying a given demand center from a given supply center. The objective of the transportation problem is to satisfy the total demand with the least total cost.

Definitions of Keywords

Supply center. A supply center provides something which is needed at another location. Each supply center is assumed to be in a fixed geographic location, to have a maximum production capacity, and to at least partially meet the needs of some demand center.

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[1]Dantzig (1951, p. 359).

Supply sufficiency. For a feasible solution to exist, the supply centers must be capable of supplying the total quantity needed by demand centers.

Demand center. A demand center requires something which is produced or distributed at another location. Each demand center is assumed to be in a fixed geographic location and to have a known demand.

Decision variable. A decision variable represents the quantity transported to a specific demand center from a specific supply center.

Direct unit cost matrix. The transportation costs per unit of supplying a given demand center from a given supply center are known. Assume that each of these costs is a fixed rate per unit being transported. If there are m supply centers and n demand centers, then these transportation costs per unit may form a matrix of m rows by n columns, known as a direct unit cost matrix.[2] Table 3 is an example.

Objective function. In the transportation problem, the objective is to minimize the total cost of transportation. The direct unit costs are the required coefficients for the decision variables in the objective function.

Constraints. The constraints in the transportation problem are the maximum production capacity at each supply center and the quantity needed at each demand center.

Feasible solution. A feasible solution to the transportation

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[2]Dantzig (1951, p. 367).

reached. This method of finding an arbitrary basic solution is called the northwest-corner rule.

Simplex method for the transportation problem. Given a transportation problem with m supply centers, n demand centers, the sufficiency of the supply to meet the total quantity demanded, the direct unit cost matrix, and an arbitrary basic solution, the simplex method is able to determine an optimal solution in a finite number of iterations. The following procedure is used:

1) If the current feasible solution cannot be improved by the addition of a currently unused direct route, stop. The current feasible solution is an optimal solution.
2) Select one of the currently unused direct routes which could improve the current feasible solution to enter as a nonzero variable in the next solution.
3) Discover which currently used route is replaced by the route which was selected in step two.
4) Repeat steps one through three until an optimal solution is found.

Review the output. It is important to review the final solution given by a computer program to see that the solution is feasible. Also, the number of nonzero variables should be counted. If there are m supply centers and n demand centers, the optimal solution should not contain more than m + n - 1 routes with positive transported amounts.

Summary

The solution to the transportation problem may be summarized as follows:

1) Determine the supply centers and demand centers. Separate any combination supply and demand center by substitution.
2) Determine the quantity needed at each demand center.
3) Determine the maximum production capacity of each supply center.
4) Unless the total production capacity is sufficient, stop. No feasible solution exists.
5) Determine the direct unit cost matrix. Use the big-M method, if necessary, to prevent infeasible solutions.
6) If the total production capacity exceeds the total quantity needed by demand centers, enter a surplus demand center.
7) Use the northwest corner rule to obtain an arbitrary basic solution.
8) Use a computer program adopted for the transportation problem which uses the simplex method.
9) Check the results.

Applications

The following example shows how to use the simplex method to solve problems in a pension consulting firm which are similar to the transportation problem.

The problem. A certain pension consulting firm has three main offices and nine branch offices. The firm's business consists of selling and servicing pension plans.

The main offices have the necessary computer facilities and actuarial support and a company policy is to centralize the difficult duties. The firm's offices are located in: Hartford, Connecticut; Phoenix, Arizona; Springfield, Illinois. Table 1 shows an estimate of how much actuarial time is available next year at each main office.

TABLE 1

Available Actuarial Time

Supply-Center Number	Main Office	Actuary-Hours
1.	Hartford	9,270
2.	Phoenix	9,550
3.	Springfield	6,730

The branch offices obtain new pension plans for the firm through consultants. Staff located at each branch office perform much of the clerical work, such as checking employee census data, for their pension plans. The firm's branch offices are located in: Atlanta, Georgia; Chicago, Illinois; Dallas, Texas; Denver, Colorado; Los Angeles, California; Philadelphia, Pennsylvania; Phoenix, Arizona; San Francisco, California; Seattle, Washington. Table 2 shows an estimate of how much actuarial time will be needed for each branch office's pension plans.

TABLE 2

Needed Actuarial Time

Demand-Center Number	Branch Office	Actuary-Hours
1.	Atlanta	1,760
2.	Chicago	1,590
3.	Dallas	3,670
4.	Denver	3,280
5.	Los Angeles	2,950
6.	Philadelphia	3,870
7.	Phoenix	1,550
8.	San Francisco	3,630
9.	Seattle	2,200

The total available actuarial time at all three main offices is 25,550 hours. The total amount of time needed by all of the pension plans is 24,500 hours. Therefore, the number of hours available is sufficient.

Table 3 gives the direct unit cost matrix. The entries represent expected telephone or traveling expenses per hour of actuarial time.

TABLE 3

Direct Unit Cost Matrix

Supply Center	Demand Center 1	2	3	4	5	6	7	8	9
1	2.00	2.00	2.50	2.50	3.00	1.25	2.75	3.00	3.00
2	2.50	2.50	2.00	1.75	1.50	2.75	0.75	1.75	2.25
3	1.75	1.00	1.75	2.00	2.50	2.00	2.25	2.50	2.50

Assume that the cost per hour of actuarial time is the same for all main offices. What is the optimal distribution of services?

The solution. Because the total hours available exceeds the total number of hours needed, a surplus demand center must be

entered. There are no telephone or traveling expenses related with the surplus demand center.

Next, the northwest-corner rule is used to obtain the arbitrary basic solution shown in Table 4. Note that entries are expressed in units of tens of hours. The Hartford main office spends 1,760 actuarial hours with pension plans from the Atlanta branch office, and there is an excess capacity of 1,050 actuarial hours at the Springfield main office. The total of telephone or traveling expenses is $53,995 for the arbitrary basic solution.

TABLE 4

Arbitrary Basic Solution

(unit = 10 hours)

Supply Center	Demand Center 1	2	3	4	5	6	7	8	9	Surplus
1	176	159	367	225	0	0	0	0	0	0
2	0	0	0	103	295	387	155	15	0	0
3	0	0	0	0	0	0	0	348	220	105

Table 5 contains an optimal solution.* The total of telephone or traveling expenses is $41,310, which is a savings of $12,685 from the arbitrary basic solution.

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*This solution was obtained with a computer program which was written to solve the transportation problem with the simplex method.

TABLE 5

Optimal Solution

(unit = 10 hours)

Supply Center	Demand Center 1	2	3	4	5	6	7	8	9	Surplus
1	176	0	0	39	0	387	0	0	220	105
2	0	0	0	142	295	0	155	363	0	0
3	0	159	367	147	0	0	0	0	0	0

Note that the firm's excess capacity of 1,050 actuarial hours has been reassigned to the Hartford main office.

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