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A FINANCIAL MODEL FOR

RETIREMENT INCOME PLANNING

Christoph Haehling von Lanzenauer
The University of Western Ontario
London, Canada

ABSTRACT

The subject of this paper is a financial model for pension planning purposes. The focus of the model is from an individual's point of view with the idea to assist him/her in planning for a financially secure retirement income. The model is dynamic and stochastic, operates in nominal and real terms, and goes beyond the age of retirement (whatever it may be). The model is based on the situation as it currently exists in Canada. Besides some methodological aspects, the use of the model and its implications for decision making in pension planning will be illustrated. Furthermore, the model can be used as a basis for policy making.

I. Introduction

Most analysis of retirement income programs is done either from the plan sponsors or a regulatory's point of view [11,12]. This paper is concerned with retirement income planning taking an individual's perspective. This focus has different roots. Since pension plans are intended to provide retirement income to the individual, its perspective appears to deserve some attention. Furthermore, the issue became more urgent when in 1970, the author's pension plan was changed from a defined benefit to a money purchase plan. Once a year each plan member is not only informed about his/her current status but also given a projection of future retirement income. A few years ago, a substantial raise was projected for retirement! Careful analysis of this situation led to the research effort into pension planning from an individual's point of view. The purpose of this paper is to highlight key features, some methodological considerations and a few results of a financial model for retirement income planning. Although the specifics of the situation are within the Canadian setting, the approach and methodological aspects are general and most findings or at least their nature are transferable.

II. A Framework for Retirement Income Plan

1. Overview

Retirement income planning as any planning is a two step process:

- determining where one wants to be, and
- deciding on how to get there.

Because of its long term nature, it is necessary to include a projection component into the planning framework to determine periodically whether or not one is on target. This, of course, implies that retirement income planning must be viewed as an ongoing rather than once-a-lifetime process.

EXHIBIT I

Exhibit 1 illustrates the framework and its components. Although the framework of retirement income planning is conceptually simple, there appears to be a substantial gap between what ought to be and what is being done. A piece of promotion material (Exhibit II) - designed to assist an individual in determining how much to put aside if a certain retirement income goal is to be achieved - illustrates the point.

EXHIBIT II

For example, a retirement income goal of $10,000 per year requires from a 35-year-old person an annual contribution of $1222 for 30 years if retirement is planned at age 65. The fine print informs the reader that funds are assumed to grow at 6%. Unfortunately, this advice is totally inadequate as a basis for retirement income planning:

- If everything goes according to plan, an annuity of $10,000 will be forthcoming; however its purchasing power is due to inflation reduced to about $3,000.[1]

- Continued inflation after age 65 will erode the purchasing power of the annuity even further, which at age 80 will have declined to about $1700 or 17% of what the individual had hoped for. Planning beyond

1. For a 6% nominal rate of return it is reasonable to assume that the rate of inflation is somewhere close to 4%.

retirement is critical in light of the substantial life expectancy of a 65-year-old person.

- Furthermore, the 6% growth rate will vary over time producing corresponding variability in the resulting retirement income. Assuming a variability as displayed in corporate bonds during the last 30 years leads to significant variability in the retirement income at age 65 (i.e. \$1730(10th percentile) and \$4200 (90th percentile)).

Exhibit II and its implications indicate the urgency of taking an individual's perspective; specifically it highlights the need to deal in real rather than nominal terms, to look beyond the age of retirement and to treat the planning process stochastically rather than deterministically.

2. Goal Setting

Determining the level of retirement income an individual wants to realize must be a personalistic matter. Aspects such as health, family status, continued financial commitments, aspired lifestyle will be of critical importance. It appears, however, that many individuals find it difficult to translate the various aspects into a single retirement income goal. One of the difficulties, of course, is the long planning horizon inherent in retirement income planning.

It is frequently argued [3, 5], that about 70% of the final or final average income represents a reasonable target. Although the use of a percentage figure avoids the need to express one's goal in dollar terms, for planning purposes - that is for designing and executing the pension program necessary to reach the goal - the desired level of retirement income should be expressed in dollars.

Furthermore, it is useful to think in dollars since most 35-year-old persons reject the fact that they would earn at age 65 $1,173,500 annually if the current income of $30,000 grows at 13% due to inflation, merit, productivity and promotion.

The process of deriving at a retirement income goal can be assisted by a fairly detailed budgeting process where the budget items are expressed in real (i.e. inflation adjusted) dollars. Translation of these figures into nominal terms at age 65 or thereafter is then an easy matter. The idea is exemplified in Exhibit III.[2]

EXHIBIT III

Although it is reasonable to suggest that the real retirement income goal remains fairly constant throughout the period of retirement, it is also possible that the goal is a function of age. Thus different retirement income goals expressed in today's purchasing power must be specified for different ages.

3. Program Design and Projection

Following the framework set out in Exhibit I, a pension program must be developed and implemented that will realize the selected retirement income goal. A pension program can be defined as the set of interrelated and sequential decisions that

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2. The details justifying the individual items are omitted for brevity.

contribute to the retirement income goal. These decisions involve:

- Determining the degree and the timing of participation in the various pension plans that are available;
- The selection of the age of retirement;
- The choice of the post retirement/post maturity options.

Although the implementation of these decisions is sequential, they are highly interrelated and the current decisions must be made in light of these interdependencies. For example, the participation decisions must be made assuming an age of retirement and specified post maturity options, say, a joint-and-last survivor annuity or a fixed income - Registered Retirement Income Fund (RRIF). [3]

The degree of freedom that exists in making these decisions depends on the particular circumstances prevailing at a given time and jurisdiction. In the Canadian setting, participation is currently available in public pension plans, in private (employer-sponsored) plans including profit-sharing plans and in personal plans (Registered Retirement Savings Plans (RRSP)).[3] Since participation in the public and/or private plans may be mandatory, the greatest flexibility exists in the personal area. Here an individual has to choose among different plans, determine the level of contribution and its timing and specify the investment direction. These decisions must be periodically reviewed and - if circumstances dictate - adjusted. Clearly, these decisions may be constrained by various factors such as disposable income and/or other financial obligations and involve a trade-off between current and future consumption.

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3. Details of an RRIF and RRSP are beyond the scope of this paper.

Needless to say that the evaluation of these decisions (i.e. their effectiveness in realizing the retirement income goal) is not only complex but must also involve the uncertainties and risks that arise in such long-term planning situations. The closely interrelated task of evaluating decisions, making projections and examining assumptions is greatly assisted by a model which accounts for the existing complexities and interdependencies as well as measures the inherent uncertainties.

III. THE MODEL

The model developed to assist an individual in retirement income planning is a mathematical and statistical model consisting of various modules which reflect the various options and aspects of the overall problem. The model is evaluative in nature and uses simulation as well as various analytical concepts for analysis. Normative guidelines can be obtained by using the model iteratively for the discrete options to be evaluated. Exhibit IV illustrates the overall structure of the model.[4] As can be observed the effectiveness of any pension program will also be a function of the uncontrollable variables as indicated in Exhibit IV. For planning

EXHIBIT IV

purposes it is therefore essential to make assumptions about their future behaviour.

Any set of consistent assumptions about the future behaviour of these variables is called a scenario. The use of scenarios in planning is well established and much has been written about the art and science of developing scenarios [4]. Although expert advice should be used, the ultimate choice of a scenario for retirement

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4. A detailed description of the model is beyond the scope of this paper.

income planning purposes must be left to the individual. It is helpful to plan assuming various scenarios, as this allows one to observe the effect of alternate assumptions on the resulting retirement income. Contrary to much practice, the scenarios developed and used in this context are stochastic rather than deterministic.

It appears that one of the key elements in the planning process must be inflation as inflation among others effects the rate of investment return, changes in purchasing power, annuity prices and the level of income which in turn influences contributions. Thus inflation can be considered as one of the driving wheels in the model. A few words regarding the modeling of inflation seem therefore appropriate.

An approach suggested for the development of an inflation scenario is the use of time series analysis [1]. The idea of time series analysis is to select a "meaningful" data base, to extract the relevant characteristics and their changes, and to develop a model based on these characteristics. The key feature of a time series model is the fact that no understanding of the underlying process generating inflation is necessary. The time series model to be developed will describe and reflect the underlying process. If, for example, an appropriate time series model has been developed on the basis of the data from 1950-1975, and is used for forecasting purposes beyond 1975, the model will not reproduce the 1950-1975 period. The model will utilize the conditions existing in 1975 and produce forecasts which display similar characteristics (e.g. variance, autocorrelation, etc.). This is an important distinction. The resulting time series model not only produces a scenario of

future inflation in probabilistic terms but also allows us to follow the path from now to the point in time to be forecasted.

Judicious use of different periods for the data base of the time series model makes it possible to develop alternative scenarios and to add judgmental factors and hunches into the forecasting process. In fact, the use of different models for different time spans to be forecasted not only makes it possible to introduce dynamic elements but also facilitates coping with discontinuities. It is the combination of human intuition and judgment with powerful statistical tools which contributes to the generation of meaningful scenarios as a basis for planning. For more details see [6].

As has been pointed out above the rate of inflation impacts, at times with a lag, other random variables. It is important to reflect the underlying dependencies in the modeling process. This problem is well recognized in the field [10]. A new procedure has been developed by the author and a doctoral student which seems to overcome some of the shortcomings of previous approaches in coping with this problem. The basic idea is as follows: Let x be the independent variable with density function f(x) and CDF F(X). Correspondingly, y is the dependent variable with g(y) and G(Y). Let r_{xy} describe the observed and ρ_{xy} the desired correlation between x and y. Sampling from f(x) and g(y) must assure that the following conditions are satisfied:

(1) $$g(y) = \int_x t(y|x)\, f(x)dx \qquad \text{(consistency)}$$

(2) $$r_{xy} = \rho_{xy} \qquad \text{(reproductivity).}$$

Procedurely, both conditions will be satisfied if for a given value of x the value of y is selected from a restricted range of y, $L \leq y \leq U$,

with L such that $G(L) = \max \{ [G(Y)-R], 0 \}$

and U such that $G(U) = \min \{ [G(Y)+R], 1 \}$

where R is determined iteratively such that $r_{xy} = \rho xy$. Thus, we have

$$t(y\ x) = \begin{cases} \frac{g(y)}{2R} & L \leq y \leq U \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

if $[G(Y)-R] \geq 0$ and $[G(Y)+R] \leq 1$. Further details and results for other conditions are given in [7].

As pointed out earlier in planning for a secure retirement income, one must account for the loss in purchasing power due to inflation. The accepted way to deal with this problem is to work in real rather than in nominal terms. The real rate of growth (return), R, is given by

$$R = \frac{N-I}{1+I} \tag{4}$$

with N being the nominal rate of growth (return) and I the rate of inflation; frequently, however, R is incorrectly defined simply as

$$R = N - I. \tag{5}$$

While the error introduced of using (5) instead of (4) is small for shorter planning periods, it becomes significant for long planning horizons such as in retirement income planning,

specifically with substantial rates of inflation. Exhibit V illustrates the magnitude of the error for three different investment cases. Case (a) is characterized by a single initial investment. In case (b) the investment process consists of

EXHIBIT V

periodic investments. The periodic investments are constant in nominal terms but due to inflation decline in real terms. Case (c) represents a situation of periodic investments in which investments remain constant in real terms but increase in nominal terms. For further details see [9].

IV. RESULTS

Results obtained from the financial planning model will now be presented to illustrate its use.

Consider Mr. X who is 35 years old, and has an income of $28,000. He is a member of his employers' defined benefit plan which promises a retirement income of 1.5 times years of service of the final average salary. The plan is contributory at 5% of salary. Mr. X contributes annually also the maximum tax-sheltered amount to an RRSP which has currently assets of $7,500. Mr. X expects his salary to grow by inflation + 2% and his retirement income goal is $30,000, or about 60% of final salary expressed in today's purchasing power. Of course, Mr. X is also part of the public plans currently offered (CPP and OAS). Assuming retirement at age 65 and a basic annuity as a post maturity option for the RRSP assets, his pension program will produce the retirement income as described in Exhibit VI.

EXHIBIT VI

As can be observed, the expected retirement income is above the goal at age 65 for all three scenarios but reduces with time.[5] The dramatic decline in scenario A results from continued high inflation during retirement. An indication of the variability present due to inflation and investment returns is provided by the 10th and 90th percentile. The variability can, of course, be influenced by the investment direction decisions in the pension planning process.

Risk can be defined as the probability of not meeting the retirement income goal at specified points in time. Of more interest is the degree of shortfall with various levels of confidence. Defining shortfall as the difference between goal and realized pension, it is possible to specify that, say, at age 75 under scenario A there exists a 50% chance that the shortfall will not exceed $3,192; correspondingly Mr. X can be 90% sure that the shortfall will not exceed $7,704.

This information is of importance since it forms the basis for determining the additional contributions that are required to avoid such a shortfall with specified levels of confidence. If, for whatever reasons, it is impossible to come up with the added level of participation in pension plans, Mr. X will be aware of the anticipated shortfall long before retirement. In a similar fashion, we can determine the level of retirement income and the degree of confidence if, for example, the pension planning process is postponed for some years in order to channel more income into current rather than future consumption.

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5. Scenario A is characterized by high inflation and a depressed equity market, while scenario C reflects a strong and growing economy with little inflation. Scenario B is a middle-of-the-road scenario.

Consider now Mr. Y who is 60 years old, earns $30,000 and has no dependents. His retirement income goal is $15,000 and he considers taking early retirement at age 60. In addition to the indexed public plans which come on stream at age 65, his employer would provide him now with a pension of $9,375 resulting from 25 years participation in the company's pension plan. Furthermore, Mr. Y has contributed to an RRSP since their inception in 1957 and has accumulated funds totalling $65,632. Conversion of these funds into an annuity produces with the other pension sources a retirement income profile as given in Exhibit VII. As can be observed, the expected retirement income falls below the target

EXHIBIT VII

most of the time implying that early retirement is not recommended. Exhibit VII also shows how significantly the additional five years of employment and participation in the pension plans contribute to the realization of the income goal. Although the retirement income is far in excess of the goal in the early years of retirement, it falls sharply below after about age 77. While this may represent a desirable feature to some, most people would prefer a more balanced income profile.[6] With the current post maturity options available in Canada, such a level profile can be realized by pulsing and mixing. Pulsing implies maturing of RRSP assets at different points in time, however, prior to age 71

EXHIBIT VIII

which is currently the latest possible date. The purpose of pulsing is to increase the profile by bringing new funds on stream

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6. The pattern of the desired retirement income goal is of course a key factor in the planning process.

periodically. Mixing suggests that funds are split between post maturity vehicles having different characteristics such as an annuity and a RRIF. But note that a RRIF stops payments at age 90 while an annuity continues till death. It must also be pointed out that in addition to the inflation risk, the investment risk continues with the RRIF. For more details, see [8].

The financial planning model can also be used to evaluate public policy decisions in pension planning. For example, the model is well suited to evaluate and compare the adequacy and equity of different pension plan features. Furthermore, the model provides a vehicle for setting RRSP contribution limits as well as for pointing out the consequences of not or only partial indexing of those limits, just to name a few.

VI. SUMMARY

The purpose of the paper is to present a financial model for retirement income planning. The model is designed to assist an individual in planning realistically for a secure retirement income. Key features of the model include formulation in real rather than nominal terms, planning beyond the age of retirement and the use of stochastic scenarios.

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Exhibit I

Retirement Income Planning

As An Iterative Process

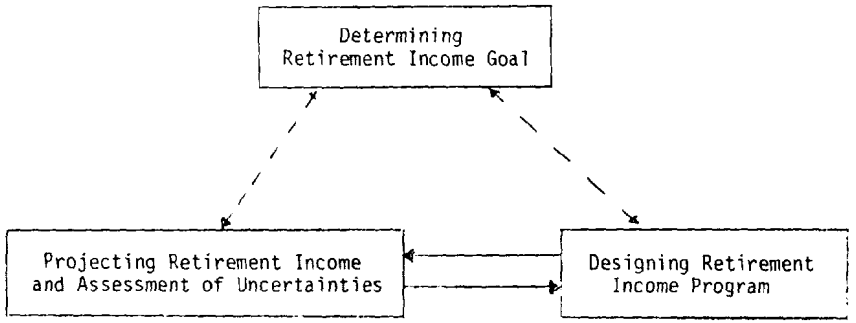


Exhibit II

Retirement Income Planning in Practice

Registered Retirement Savings Plans

You know about the tax savings. We thought you should know about the long term benefits.

Your age	Growth assuming interest at 6% compounded annually. Annual Contributions $2,000	$4,000	$5,500
41	$ 2,120	$ 4,240	$ 5,830
42	4,367	8,734	12,010
43	6,749	13,498	18,560
44	9,274	18,548	25,504
45	**$ 11,995**	**$ 23,901**	**$ 32,864**
46	14,786	29,575	40,666
47	17,795	35,590	48,936
48	20,983	41,965	57,702
49	24,362	48,723	66,994
50	**$ 27,943**	**$ 55,887**	**$ 76,844**
51	31,740	63,480	87,285
52	35,764	71,529	98,352
53	40,030	80,060	110,083
54	44,552	89,104	122,518
55	**$ 49,345**	**$ 98,690**	**$135,699**
56	54,426	108,852	149,671
57	59,811	119,623	164,481
58	65,520	131,040	180,180
59	71,571	143,142	196,821
60	**$ 77,985**	**$155,971**	**$214,460**
61	84,785	169,569	233,158
62	91,992	183,983	252,977
63	99,631	199,262	273,986
64	107,729	215,458	296,255
65	**$116,313**	**$232,626**	**$319,860**

Select the desired retirement income you would like to have at age 65 and determine how much you should save each year to reach this goal.

Desired Annual Retirement Income*	Annual Contribution Required. Your age at commencement of plan. 30	35	40	45	50
$10,000	$ 868	$1,222	$1,763	$2,627	$4,048
15,000	1,302	1,833	2,644	3,940	
20,000	1,736	2,444	3,526	5,254	
25,000	2,170	3,055	4,407		
30,000	2,604	3,666	5,289		
35,000	3,038	4,277			
40,000	3,472	4,888			

Growth assuming interest at 6% compounded annually
*Based on annuity payments for the greater of the annuitant's lifetime or 10 years. Current annuity rates were used for illustration purposes only and are subject to change.

Registered Retirement Savings Plans let you save today on income tax while you're saving for your retirement.

Exhibit III

A Guide to Retirement Income Goal Setting

GUIDE TO ANNUAL RETIREMENT INCOME NEEDS FOR DIFFERENT LIFESTYLES

(TWO PERSON HOUSEHOLD)

	NEEDS * / LIFESTYLE *	FIRST CLASS	COMFORTABLE	MODERATE	DO IT YOURSELF **
1	HOUSING (MAINTENANCE, TAXES, INSURANCE FOR FULLY PAID HOUSE)	4,400	2,800	1,320	
2	UTILITIES (HEAT, ELECTRICITY, WATER, TELEPHONE)	2,200	1,750	1,380	
3	FOOD (INCLUDING GENERAL HOUSEHOLD SUPPLIES)	6,300	5,700	4,600	
4	CLOTHING (INCLUDING CLEANING)	2,000	1,320	660	
5	MEDICAL AND DENTAL EXPENSES (INCLUDING DRUGS, EYEGLASSES, HEARING AIDS, ETC.)	1,300	880	570	
6	TRANSPORATION (CAR, INSURANCE, TAXIS, ETC.)	3,500	2,600	1,050	
7	VACATION EXPENSES AND/OR A SECOND HOME	8,000	4,950	2,750	
8	SOCIAL AND RECREATIONAL ACTIVITIES INCLUDING CLUB MEMBERSHIPS	5,400	3,200	1,320	
9	APPLIANCES & FURNITURE, REPAIRS AND REPLACEMENTS	3,000	1,500	680	
10	DONATIONS AND GIFTS	3,250	1,925	930	
11	MISCELLANEOUS AND UNEXPECTED EXPENSES	1,500	1,200	670	
12	APPROXIMATE INCOME TAXES (BASED ON 1979 RATES AND DEDUCTIONS)	17,650	7,495	1,840	
	REQUIRED PRE-TAX RETIREMENT INCOME	59,300	35,320	18,570	

* - A DESCRIPTION OF SPECIFIED NEEDS FOR THESE LIFESTYLES IS GIVEN ON THE FOLLOWING PAGE.

** - THE DO IT-YOURSELF COLUMN WILL PERMIT AN INDIVIDUAL TO MODIFY, IF REQUIRED, THE BUDGET SHOWN FOR ANY LIFESTYLE BY SUBSTITUTING FIGURES FOR PARTICULAR NEEDS FROM OTHER LIFESTYLES

Exhibit IV

Modeling the Retirement Income Planning Process

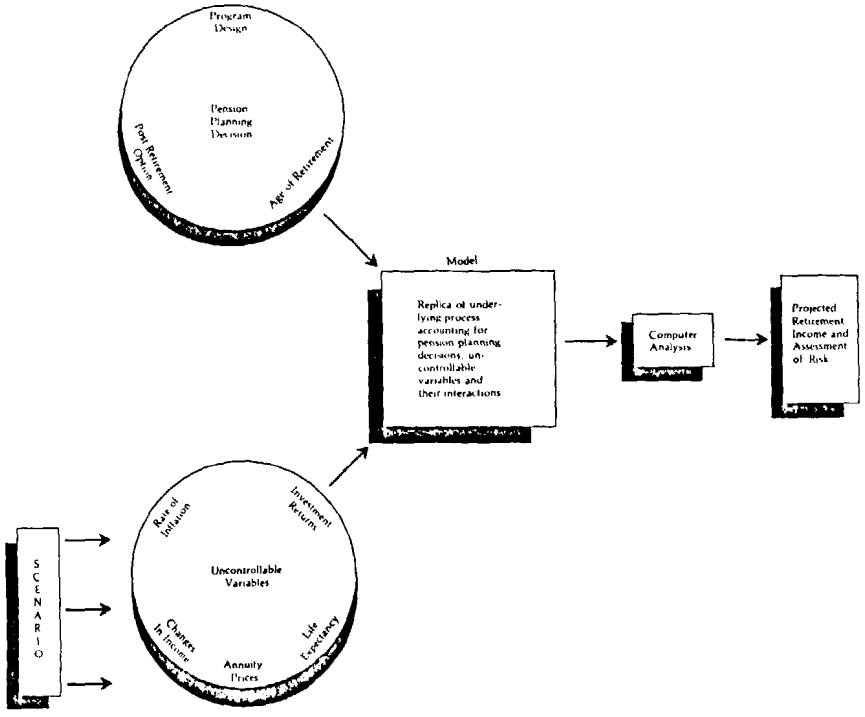
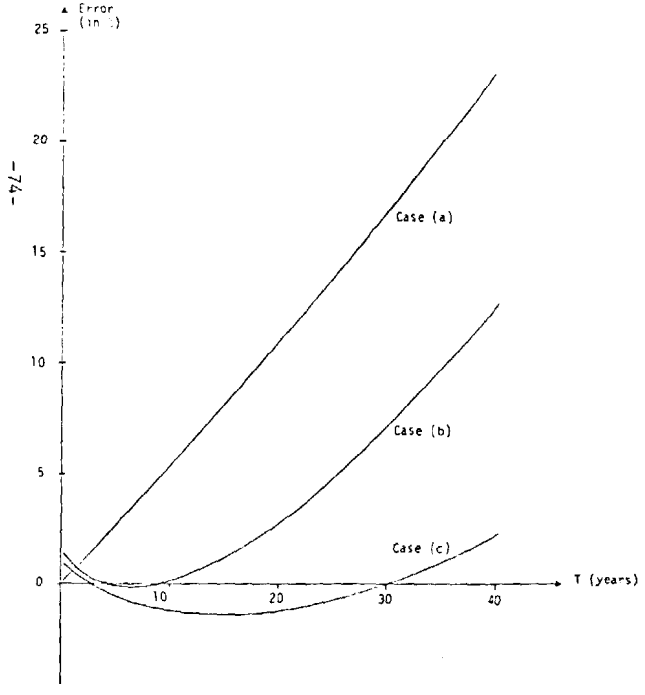


Exhibit V

Errors Resulting from Incorrect Definition of Real Rate of Return

Nominal Rate of Return: 16%
Rate of Inflation: 10%



Nominal Rate of Return: 12%
Rate of Inflation: 10%

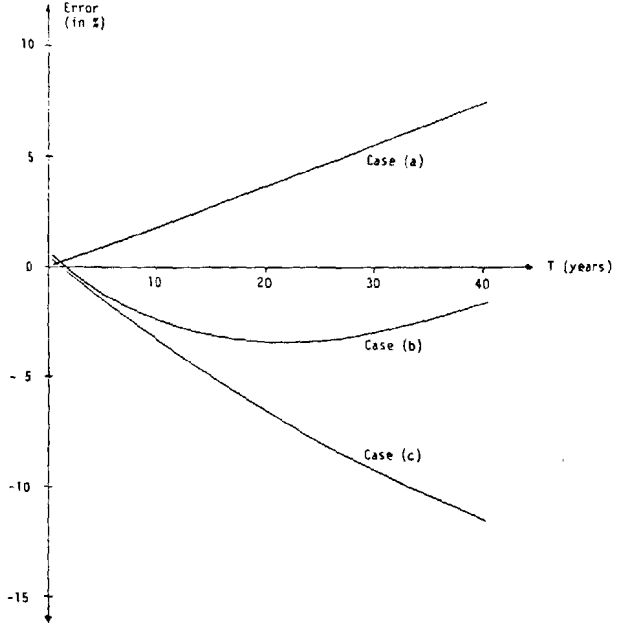


Exhibit VI

Expected Retirement Income for Mr. X

Age		SCENARIO A	B	C
65	Low	$ 42,060	$ 42,288	$ 40,224
	Mean	44,916	45,564	43,392
	High	48,072	48,528	46,728
75	Low	22,296	24,588	25,860
	Mean	26,808	33,012	31,644
	High	31,680	40,368	37,656
85	Low	13,824	16,128	18,480
	Mean	17,280	24,408	24,084
	High	20,940	33,468	29,676

Exhibit VII

Expected Retirement Income Profiles for Mr. Y

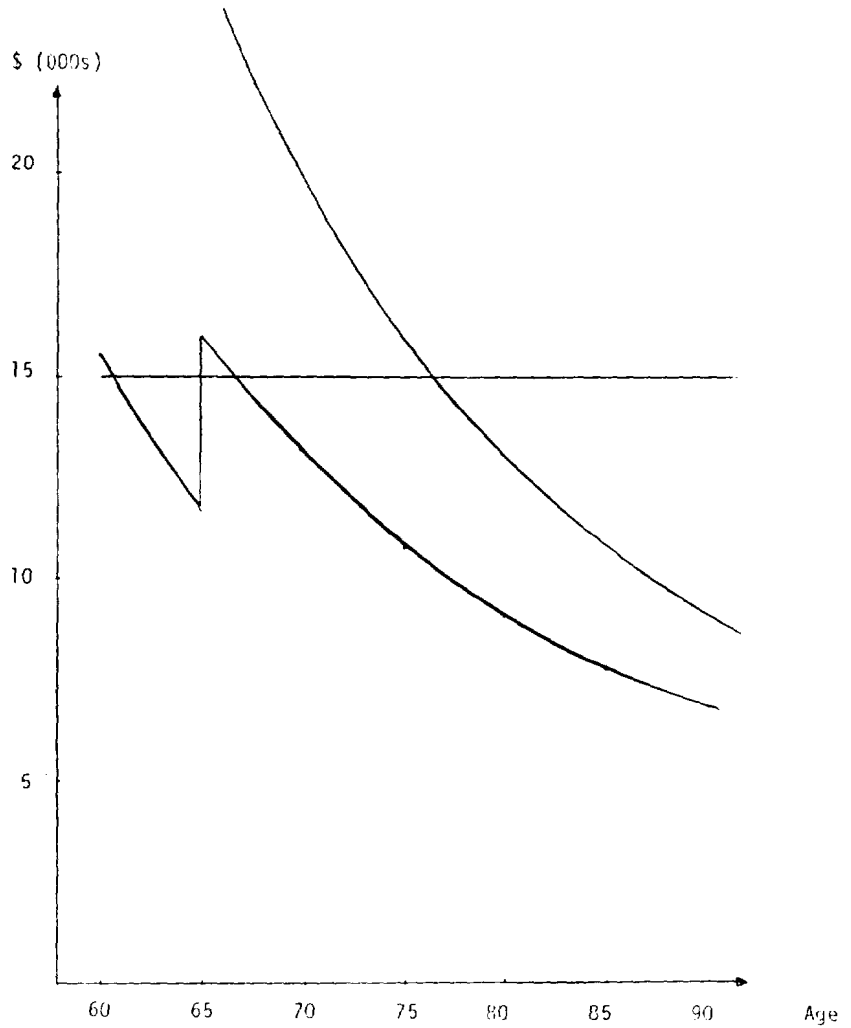


Exhibit VIII

Retirement Income Profiles with Pulsing and Mixing