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PENSION MATHEMATICS FOR ACTUARIES

Arthur W. Anderson
Author and Publisher
Reviewed by Deborah Poppel

Arthur Anderson has written a book on the subject of Pension Mathematics which can be easily followed by a reader who understands life contingencies, elementary calculus, and algebra. He has used an informal style of writing ("We" are talking to "you"). The definitions and concepts are surrounded by conversation, which makes them more meaningful than the usual concise statements one would expect in such a book. He has taken pains to organize the subject in an order which makes the principles easily grasped.

The book contains seven chapters, each of which is divided into sections of three to seven pages. Each section is followed by exercises which are well-designed to keep the reader involved in the evolution of the subject. Some of the exercises prove mathematical relationships which were glossed over in the preceding section. Others lead to development of new but related material. Hints are provided to lead the reader through new topics.

Occasionally, one might wish for more hints than are given. A criticism of the text might be that there are not enough questions of the reinforcement type. Formulas can be interpreted as instructions for processing data and actuarial functions. Does the reader really know how to execute these instructions? Some problems testing this ability would be useful. Anderson does not, and did not intend to, provide exercises which develop problem solving skills needed to pass professional or licensing examinations.

The first chapter introduces the nature of the subject and the purpose of the book. It significantly discusses the difference between the philosophy and application of actuarial mathematics in the pension field versus the life insurance field.

The next two chapters develop, from reasonable premises, the formulas for normal cost and accrued liablity for the following methods: unit credit, entry age normal, individual level premium, frozen initial liability and the aggregate methods. Included is a clear insight into the formula for actuarial gains which enables the reader to observe an analysis of these gains from various sources. Both contributory and non-contributory plans are discussed.

The formulas are expressed in terms of commutation symbols. Other writers have used summations of probabilities (Winklevoss), annuity symbols (Trowbridge and Farr), or words (McGill). A mixture of commutation symbols, probabilities, and the spreadsheet concept was used by another author (Berin). There is some feeling in actuarial circles that computers have eliminated the need for commutation symbols. However, commutation symbols can be used for conceptual model building as illustrated by Anderson. The ideas could have been handled with probabilities just as easily as with commutation symbols. Anderson chose commutation symbols. It should make no difference to an actuary whether ideas are expressed in commutation symbols or as summation of probabilities. Actual computation can be handled by any state-of-the-art method.

The fourth chapter gives formulas for ancillary benefits: vesting, early retirement, late retirement. In the exercises the reader is led to produce formulas for the Social-Security-Adjustment option, joint and survivor option, etc.

Chapter 5 discusses the valuation of assets. A smoothed market value is rationalized for stocks. The new-money approach to valuing bonds is explained clearly. Some interesting problems arise in the valuation of assets for plans funded by group annuity contracts or individual insurance policies.

The chapter on actuarial assumptions contains some surprises (at least to this reader). Anderson develops the concept that, under certain conditions (small plans or small probabilities of decrement), more accurate results are obtained by ignoring a decrement than by using a precise measure of it. Anderson is concerned with portraying the nature and significance of the various assumptions rather than the actual choice of them.

The final chapter begins by dealing with the question as to who should be included in a pension valuation. Following this, Anderson gives variations of both the unit credit method (projected unit credit, etc.) and the individual level premium method (individual-aggregate, etc.) Some of these methods may not be permitted for plans subject to ERISA.

Anderson has provided an exposition of pension mathematics in a manner which gives the reader a very complete grasp of the subject. His rationalizations are extremely thorough. This book will become one of the classics of actuarial literature.

References

1. Berin, Barnet N. (1978) *The Fundamentals of Pension Mathematics*, William M. Mercer, Inc.
2. McGill, Dan M. (1984) *Fundamentals of Private Pensions*, Richard D. Irwin, Inc.
3. Trowbridge, C.L. and Farr, C.E. (1976) *The Theory and Practice of Pension Funding*, Richard D. Irwin, Inc.
4. Winklevoss, Howard E. (1977) *Pension Mathematics*. Richard D. Irwin, Inc.

Editor's Note: The Anderson book, pp. 175, is available from Windsor Press, Wellesley Hills, MA 02181, $49.

Some Magic Numbers *(Continued from page 5)*

(8) What is the monthly payment for a 240-month mortgage with a 1% monthly interest rate?

Ans: $MN_{(1+k)/k} = 240$. $MN_{10} = 72 + 167 = 239$. Hence $(1+k)/k \simeq 10$, $k \simeq 1/9$, and $(1+k)\cdot i \simeq 1.11\%$.

Conclusion

Those who like to solve problems involving exponential growth at a constant rate, without use of interest tables, logarithms, or calculators, will find much of value in the three magic numbers 72, 114, and 167. Learn to use them, and amaze your friends! □

COMPETITION RESULTS

by Charles G. Groeschell, Competition Editor

Esther Portnoy and Robert Hohertz continue to solve all Actucrossword puzzles with apparent ease. They remain our co-champions with ten 100%

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