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1986 AERF GRANTS COMPETITION

The Actuarial Education and Research Fund (AERF) is sponsoring a 1986 grants competition. One or more grants, totalling up to $10,000, will be available from AERF to support actuarial education or research projects. In addition, worthy applications will be brought to the attention of the AERF constituent organizations. Funds may be used to compensate grant recipients or to cover expenses for services utilized in carrying out the project.

The goal of the competition is the production of publications which will advance actuarial science, especially in regard to practical applications. For this purpose, proposals are invited from members of the actuarial organizations supporting AERF, from faculty members of a U.S. or Canadian university or college who have teaching and research responsibilities in the actuarial or related fields, and others qualified by knowledge and experience to contribute to the goal.

To begin application, a letter should be submitted to the AERF Research Director outlining the scope of the proposed project. The Research Director will then supply further information on applying for a project grant, and application forms.

Completed applications will be due by Feb. 1, 1986 and will proceed to an Awards Committee drawn from the actuarial and academic professions. Awards will be announced in April 1986.

Correspondence should be addressed to Cecil J. Nesbitt, AERF Research Director, Department of Mathematics, University of Michigan, Ann Arbor, MI 48109-1003. □

WRIGHT. . . OR WRONG?

He was controversial enough to be interesting. He was a pioneer of our profession whose work has influenced our balance sheet practices even today. Elizur Wright's death in November 1885 will be commemorated at a short and lively session at New Orleans on Monday, Oct. 14. Chief Commemorator will be former editor E.J.M. See the final program booklet for time and place.

SOME MAGIC NUMBERS — 72, 114, AND 167

By C. L. Trowbridge

Long well-known to compound interest practitioners (actuaries clearly included) is a simple but very useful rule: The number of years for money to double (at annually compounded interest) is approximately 72 divided by the interest rate. This simple relationship tells us that for $1 to become $2 it takes about:

36 years at i = 2%
18 years at i = 4%
12 years at i = 6%
9 years at i = 8%
6 years at i = 12%
4 years at i = 18%

Some readers may have seen this "rule of 72" referred to in recent *Wall Street Journal* columns.

The derivation of this rule is simple enough:

$$\text{If } (1+i)^n = 2$$
$$n \cdot \log(1+i) = \log 2$$

$$n = \log 2/\log(1+i) = .693/\delta = .693/i \cdot i/\delta$$

But i/δ is very close to $1 + i/2$, and at 8% is approximately 1.04.

$$\text{Then } n \approx .693\,(1.04)\,/\,i \approx 72/100i.$$

Hence the rule gives almost exact results at $i = .08$. For greater accuracy the 72 should be increased (decreased) by 1% for each 2% that the rate of interest exceeds (is less than) 8%.

Note that 72 is easily divisible by 2, 3, 4, 6, 8, 9, and 12, and hence the rule gives quick and easy answers for many of the common interest rates. Except for its limited applicability, the rule of 72 leaves little to be desired.

A natural question is whether this rule can be extended to make it more widely applicable; and if so, can its simplicity be preserved? This article is an investigation of these interesting questions.

A First Extension

A similar derivation, but replacing the factor 2 by first 3 and then 5, quickly leads to the "rule of 114" (for tripling) and the "rule of 167" (for a five-fold increase). If we let the "magic number" for an increase by a factor of f be represented by MN_f, and be calculated by 104 log f, we can apply the usual rules for manipulating logarithms and come up with the following:

f	MN_f	
2	72	
3	114	
4	144	= 72 + 72
5	167	
6	186	= 114 + 72
8	216	= 72 + 72 + 72
9	228	= 114 + 114
10	239	= 72 + 167
12	258	= 114 + 144

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Some Magic Numbers

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Note that once we know that 72, 114, and 167 are the magic numbers for $f = 2, 3$, and 5, we have little further use for a log table, since the values of MN_f for all other integers up to 12 (the prime numbers 7 and 11 being exceptions) can be obtained by combination. The MN_f for some fractional values of f are also easily found; e.g., $MN_{2.5} = MN_5 - MN_2 = 95$. Still other values of MN_f can be obtained by interpolation. (Linear interpolation in a log table is reasonably accurate if the log is not too close to zero).

Once one has memorized the numbers 72, 114, and 167, and has mastered the technique for finding other MN_f values, his range of application is extended. Given any two of i, n, and f, the third can be approximated, with recourse to mental arithmetic only. Here are some examples.

(1) How long does it take for a dollar at 10% interest to increase by a factor of 10?
Ans: (72 + 167) / 10 = 24 years approximately.

(2) At what interest rate does $1 become $8 after 36 years?
Ans: (72 + 72 + 72) / 36 = 6%.

(3) By what factor will a dollar invested at 9½% for 15 years have increased?
Ans: $MN_f = 15 \cdot 9.5 = 142.5$. $MN_4 = 144$. The required factor is approximately 4.

Although the examples so far given come from the theory of interest, our magic numbers are equally useful in calculations involving other forms of exponential growth.

(4) How long will it take for a population growing at a 1½% annually rate to triple?
Ans: 114/1½ = 76 years, approximately. Here we can improve the approximation by dividing by 1.0325, to recognize that the 1½% growth rate is rather far from 8%. 76/1.0325 = 73.6.

(5) If a corporation's sales have increased 10-fold in 20 years, what is the average annual rate of sales growth?
Ans: (167 + 72) / 20 = 12%. Here we know that our first approximation is a bit low, because the MN_fs at 12% should be 2% higher. 12 (1.02) = 12¼%.

A Second Extension

Having learned to approximate $(1+i)^n$ by using 72, 114, and 167, we can now approximate more complicated functions involving $(1+i)^n$, or its reciprocal, v^n. Consider this relationship. Let the annual payment for an n year mortgage of unit initial debt be $(1+k) \cdot i$, where i is the interest rate (and k will ordinarily be > 0).

Then: $(1+k) \cdot i = 1/a_{\overline{n|}} = i / (1-v^n)$

$$(1+i)^n = (1+k)/k$$

$$n = \frac{\log\ [(1+k)/k]}{\delta} \approx \frac{MN_{(1+k)/k}}{100\,i}$$

(6) What is the mortgage term if the annual payment is 10% and the interest rate 8%?
Ans: k = .25. (1 + k) / 5 = 5. MN_5 = 167· n = 167/8 = 21 years, approximately.

(7) What is the annual payment for a 6%, 30-year mortgage?
Ans: $MN_{(1+k)/k} = 30 \cdot 6 = 180$. By interpolation, $(1+k)/k$ must be about 5.7, leading to k = .213 and an annual payment of 1.213 · 6 = 7.3% of the initial debt.

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ACTUARIAL BIRTHPLACE

The Actuary is indebted to John Angle for noting an advertisement of the Nathaniel Bowditch Guest House, behind the Salem Witch Museum, Salem, Mass. The ad reads in part:

Birthplace of
NATHANIEL BOWDITCH
Born in 1773
Author of New Practical Navigator

Bowditch was the leading American mathematician of the early 19th century, best known for his work in celestial navigation and celestial mechanics. The volume of tables referred to above is still in use today (in much revised form) and is familiarly known as "Bowditch's".

But Bowditch was a mathematician by avocation. He earned his living in the insurance business. He was both actuary and chief executive of the Massachusetts Hospital Life Insurance Company for the 15 years ending with his death in 1838.

E.J. Moorhead's *Transactions* paper entitled "Sketches of Early North American Actuaries" (still only in preprint form) contains a very brief account of Bowditch's actuarial career. A much more extended account appears in an article by Dwight K. Bartlett, III in the June 1979 issue of this newsletter.

John Angle wonders whether we should enlighten the guest house owners as to Bowditch's work as an actuary; and whether the presence of the nearby Witch Museum has any significance. ☐

10,000 Readers

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members and non-member subscribers who reside outside of North America.

The Actuary accepts no advertising, and has no need of circulation claims. For what these members may be worth, however, *The Actuary* now thinks of itself as having a circulation of 10,000, with 2% not-yet-member students, and 5% overseas.

The Society pays roughly $30,000 for printing the 10 monthly issues, and an equal amount for postage and other mailing costs. The cost per copy works out to about 60¢. ☐

PENSION MATHEMATICS FOR ACTUARIES

Arthur W. Anderson
Author and Publisher
Reviewed by Deborah Poppel

Arthur Anderson has written a book on the subject of Pension Mathematics which can be easily followed by a reader who understands life contingencies, elementary calculus, and algebra. He has used an informal style of writing ("We" are talking to "you"). The definitions and concepts are surrounded by conversation, which makes them more meaningful than the usual concise statements one would expect in such a book. He has taken pains to organize the subject in an order which makes the principles easily grasped.

The book contains seven chapters, each of which is divided into sections of three to seven pages. Each section is followed by exercises which are well-designed to keep the reader involved in the evolution of the subject. Some of the exercises prove mathematical relationships which were glossed over in the preceding section. Others lead to development of new but related material. Hints are provided to lead the reader through new topics.

Occasionally, one might wish for more hints than are given. A criticism of the text might be that there are not enough questions of the reinforcement type. Formulas can be interpreted as instructions for processing data and actuarial functions. Does the reader really know how to execute these instructions? Some problems testing this ability would be useful. Anderson does not, and did not intend to, provide exercises which develop problem solving skills needed to pass professional or licensing examinations.

The first chapter introduces the nature of the subject and the purpose of the book. It significantly discusses the difference between the philosophy and application of actuarial mathematics in the pension field versus the life insurance field.

The next two chapters develop, from reasonable premises, the formulas for normal cost and accrued liablity for the following methods: unit credit, entry age normal, individual level premium, frozen initial liability and the aggregate methods. Included is a clear insight into the formula for actuarial gains which enables the reader to observe an analysis of these gains from various sources. Both contributory and non-contributory plans are discussed.

The formulas are expressed in terms of commutation symbols. Other writers have used summations of probabilities (Winklevoss), annuity symbols (Trowbridge and Farr), or words (McGill). A mixture of commutation symbols, probabilities, and the spreadsheet concept was used by another author (Berin). There is some feeling in actuarial circles that computers have eliminated the need for commutation symbols. However, commutation symbols can be used for conceptual model building as illustrated by Anderson. The ideas could have been handled with probabilities just as easily as with commutation symbols. Anderson chose commutation symbols. It should make no difference to an actuary whether ideas are expressed in commutation symbols or as summation of probabilities. Actual computation can be handled by any state-of-the-art method.

The fourth chapter gives formulas for ancillary benefits: vesting, early retirement, late retirement. In the exercises the reader is led to produce formulas for the Social-Security-Adjustment option, joint and survivor option, etc.

Chapter 5 discusses the valuation of assets. A smoothed market value is rationalized for stocks. The new-money approach to valuing bonds is explained clearly. Some interesting problems arise in the valuation of assets for plans funded by group annuity contracts or individual insurance policies.

The chapter on actuarial assumptions contains some surprises (at least to this reader). Anderson develops the concept that, under certain conditions (small plans or small probabilities of decrement), more accurate results are obtained by ignoring a decrement than by using a precise measure of it. Anderson is concerned with portraying the nature and significance of the various assumptions rather than the actual choice of them.

The final chapter begins by dealing with the question as to who should be included in a pension valuation. Following this, Anderson gives variations of both the unit credit method (projected unit credit, etc.) and the individual level premium method (individual-aggregate, etc.) Some of these methods may not be permitted for plans subject to ERISA.

Anderson has provided an exposition of pension mathematics in a manner which gives the reader a very complete grasp of the subject. His rationalizations are extremely thorough. This book will become one of the classics of actuarial literature.

References

1. Berin, Barnet N. (1978) *The Fundamentals of Pension Mathematics*, William M. Mercer, Inc.
2. McGill, Dan M. (1984) *Fundamentals of Private Pensions*, Richard D. Irwin, Inc.
3. Trowbridge, C.L. and Farr, C.E. (1976) *The Theory and Practice of Pension Funding*, Richard D. Irwin, Inc.
4. Winklevoss, Howard E. (1977) *Pension Mathematics*, Richard D. Irwin, Inc.

Editor's Note: The Anderson book, pp. 175, is available from Windsor Press, Wellesley Hills, MA 02181, $49.

Some Magic Numbers (Continued from page 5)

(8) What is the monthly payment for a 240-month mortgage with a 1% monthly interest rate?

Ans: $MN_{(1+k)/k} = 240$. $MN_{10} = 72 + 167 = 239$. Hence $(1+k)/k \simeq 10$, $k \simeq 1/9$, and $(1+k)\cdot i \simeq 1.11\%$.

Conclusion

Those who like to solve problems involving exponential growth at a constant rate, without use of interest tables, logarithms, or calculators, will find much of value in the three magic numbers 72, 114, and 167. Learn to use them, and amaze your friends! □

COMPETITION RESULTS

by Charles G. Groeschell, Competition Editor

Esther Portnoy and Robert Hohertz continue to solve all Actucrossword puzzles with apparent ease. They remain our co-champions with ten 100%

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