# AN ACTUARIAL CURIOSITY

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Here is an actuarial curiosity which arose as part of a larger study. For simplicity, we show it for a two year term insurance with artificial parameters, but the phenomenon appeared for whole life policies with realistic parameters.

Consider a two year term policy on a standard risk with net single premium A given by

$$A = v \cdot q_1 + v^2 \cdot (1-q_1) \cdot q_2$$

where $q_1$ = probability of dying in year 1
$q_2$ = probability of dying in year 2
$v = 1/(1+i)$
$i$ = annual interest rate

We define a two year term policy on a substandard risk with net single premium A' to be

$$A' = v \cdot (k \cdot q_1) + v^2 \cdot (1-k \cdot q_1) \cdot (k \cdot q_2)$$

where k = extra mortality factor (k>1)

Then the quantity A'-A is simply the extra premium charged the substandard risk. The curiosity is that A'-A can achieve a maximum for an interior value of the interest rate i.

For example, if we set $q_1$=0.80, $q_2$=0.55, k=1.2, then A'-A achieves a maximum at i=0.045. The table shows the results.

| INTEREST RATE (i) | SUBSTANDARD PREMIUM (A') | STANDARD PREMIUM (A) | DIFFERENCE (A'-A) |
|---|---|---|---|
| 0.015 | 0.97144 | 0.89495 | 0.07649 |
| 0.020 | 0.96655 | 0.89004 | 0.07651 |
| 0.025 | 0.96171 | 0.88519 | 0.07653 |
| 0.030 | 0.95692 | 0.88038 | 0.07654 |
| 0.035 | 0.95218 | 0.87563 | 0.07655 |
| 0.040 | 0.94749 | 0.87093 | 0.07655 |
| 0.045 | 0.94284 | 0.86628 | 0.07655 |
| 0.050 | 0.93823 | 0.86168 | 0.07655 |
| 0.055 | 0.93367 | 0.85712 | 0.07655 |
| 0.060 | 0.92916 | 0.85262 | 0.07654 |
| 0.065 | 0.92468 | 0.84816 | 0.07653 |
| 0.070 | 0.92026 | 0.84374 | 0.07651 |
| 0.075 | 0.91587 | 0.83937 | 0.07650 |
| 0.080 | 0.91152 | 0.83505 | 0.07647 |
| 0.085 | 0.90722 | 0.83077 | 0.07645 |

In terms of the original symbols, the maximum value of A'-A occurs when

$$i = (2q_1 \cdot q_2 \cdot (k+1) - 2q_2 - q_1) \;/\; q_1$$

Now there are two curiosities in this result. The first is that the maximum in the extra premium for the substandard risk occurs at an interior value of the interest rate. The second curiosity is a corollary of the first. It is that the extra premium takes on identical values for distinct interest rates, e.g. approximately (0.03,0.06), (0.025,0.065), etc. (there are obviously an infinite number of such pairs). These results follow directly from elementary calculus and algebra, but an explanation based on "general reasoning" eludes me. Can any ARCH readers supply an answer?