DIVIDED DIFFERENCES AND DETERMINANTS

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In problem 24, Chapter 5, of Kellison's "Fundamentals of Numerical Analysis" it is indicated how a divided difference can be expressed as a ratio of two determinants. This method works well also in the case where some of the arguments of collocation coincide. Example:

$$\underset{x,x,x,y,y}{\Delta^5} f(z) = \frac{\begin{vmatrix} 1 & x & x^2 & x^3 & x^4 & f(x) \\ 0 & 1 & 2x & 3x^2 & 4x^3 & f'(x) \\ 0 & 0 & 2 & 6x & 12x^2 & f''(x) \\ 1 & y & y^2 & y^3 & y^4 & f(y) \\ 0 & 1 & 2y & 3y^2 & 4y^3 & f'(y) \\ 1 & z & z^2 & z^3 & z^4 & f(z) \end{vmatrix}}{\begin{vmatrix} 1 & x & x^2 & x^3 & x^4 & x^5 \\ 0 & 1 & 2x & 3x^2 & 4x^3 & 5x^4 \\ 0 & 0 & 2 & 6x & 12x^2 & 20x^3 \\ 1 & y & y^2 & y^3 & y^4 & y^5 \\ 0 & 1 & 2y & 3y^2 & 4y^3 & 5y^4 \\ 1 & z & z^2 & z^3 & z^4 & z^5 \end{vmatrix}}$$

Thus the rule is: If an argument occurs k times, the derivatives of the corresponding row up to order $k-1$ should be used in numerator and denominator. Déjà vu?