BAYESIAN GRADUATION OF FHA/HUD SINGLE FAMILY HOME MORTGAGE INSURANCE CONTRACTS -- SECTION 221, 222, 223(e), and 235

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Mortgage insurance is used to guarantee the mortgage investor (mortgagee) against financial loss due to the inability of the home buyer (mortgagor) to repay the loan in accordance with the terms of the mortgage. In this paper we will describe a procedure used to construct predicted survivorship tables for single-family home mortgage insurance contracts written under four sections of the National Housing Act. These sections are 221, 222, 223(e), and 235. All of the mortgages we will be concerned with have 30-year terms.

In an earlier paper (Herzog [1981]), we constructed a survivorship table for Section 203 single-family home mortgage insurance contracts using a procedure known as "Bayesian graduation." In our earlier paper, the prior distributions for claim and non-claim terminations were based solely on Section 203 data. Here we will show how we used the results of the Section 203 Bayesian graduation procedure together with the observed data for the particular section under consideration to construct the appropriate prior distribution. The completion of the Bayesian graduation process is then largely mechanical. The Section 203 survivorship table constitutes Table 1; the two sets of predicted Section 203 central decrement rates are exhibited in Table 2.

To simplify the discussion, we will be concerned primarily with the data of Section 222. After we have described the procedure for the construction of the predicted survivorship table for Section 222, we will indicate how that procedure can be modified to construct survivorship tables for Sections 221, 223(e), and 235.

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The paper consists of seven sections. In the first section, we define the two types of terminations considered. Next, we present a brief overview of the Bayesian graduation process and then discuss the tabulation and adjustment procedures. In the fourth and fifth sections we specify the prior vectors and correlation matrices needed to produce the posterior (i.e., predicted) claim and non-claim termination rates, respectively. Next, we show how these predicted rates can be used to produce the desired survivorship table for Section 222. Finally, we discuss how the procedure can be modified to produce survivorship tables for Sections 221, 223(e), and 235.

In our construction process, we only considered those Section 222 policies endorsed between January 1, 1963 and December 31, 1980. Thus, we have made the implicit, but fundamental, assumption that the data (i.e, the policies) under consideration could be used (without adjustment) to predict future experience. The observed data of Section 222 are exhibited in Table 3.

1. CLAIM AND NON-CLAIM TERMINATIONS

For the purpose of constructing a survivorship table, there are essentially two ways in which a Section 222 mortgage can cease generating premiums:

(1)   as the result of a claim termination where
        the mortgagor defaults on the loan and the
        mortgagee submits an insurance claim to HUD,

        or

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(2)   a <u>non-claim termination</u> or withdrawal--usually
      where the mortgagor prepays the loan in full.

In the first case, claim termination can occur through a
foreclosure by the lender (or deed in lieu of foreclosure where
HUD acquires title to the property) or an assignment of the loan
to HUD. As indicated above, in the second case, the majority of
non-claim terminations are due to the full prepayment of the loan
by the mortgagor. However, there are a number of other ways this
can occur. These include:

(1)   voluntary termination of the insurance whereby
      the mortgagor and mortgagee agree to terminate
      the insurance without termination of the loan,

(2)   default termination where the value of the
      security (house) is greater than the insurance
      benefits and the mortgagee does not apply for
      insurance benefits, or

(3)   cancellation due to fraud or other illegal action.

Non-claim terminations other than full prepayment
historically constitute less than 3 percent of all non-claim
terminations.

Sometimes these loans are partially prepaid. Since the FHA
insurance premiums are based on the scheduled outstanding balance
at the time of endorsement, we have not considered partial
prepayments in our construction process.

For the purpose of constructing a survivorship table, we are concerned with predicting two sets of probabilities. One is a schedule of annual claim termination probabilities, q_x^c ; the second is a schedule of annual non-claim termination probabilities, q_x^n , where the time frame, x , runs from the origination of the loan to the end of the term of the loan. The unit of exposure is taken to be the individual insurance policy; i.e., each policy is given equal weight in the construction process.

2. AN OVERVIEW OF THE CONSTRUCTION PROCESS

We have used a general procedure known as Bayesian graduation (see Kilmeldorf and Jones [1967] and Hickman and Miller [1977]) to construct the Section 222 survivorship table. Bayesian graduation enables us to incorporate our prior subjective notions (about the salient aspects of our data) in a unified and formal framework, and results in a posterior distribution of the predicted central decrement rates.

Before proceeding, we will review some of the notation of the earlier papers:

$\underline{u}$ is the (random) vector of observed central mortality rates.

B   is the covariance matrix of $\underline{u}$.

$\underline{W}$ is the (random) vector of variables (parameters in the traditional or frequentist sense) for which a

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smooth estimate is sought.

A   is the covariance matrix of $\underline{W}$.

$\underline{m}$ is the (prior) mean vector of $\underline{W}$.

$t(x) = \arcsin\sqrt{x}$.

The procedure is based on the following assumptions:

1. The transformed random vector, $t(\underline{u})$, of observed
    mortality rates, $\underline{u}$, has a multinormal distribution
    with covariance matrix $t(B)$.

2. The transformed prior distribution of $\underline{W}$ is multi-
    normal with mean $t(\underline{m})$ and covariance matrix $t(A)$.

As a result of these assumptions, it can be shown that the
posterior distribution of $t(\underline{W})$ is multinormal with mean

(*) $(t(B)^{-1} + t(A)^{-1})^{-1}(t(B)^{-1}t(\underline{u}) + t(A)^{-1}t(\underline{m}))$

and covariance matrix

$$(t(B)^{-1} + t(A)^{-1})^{-1}.$$

The inverse of the function t (i.e., the square of the sine) of
expression (*) is used as the set of predicted values.

We have employed the arcsin transformation here in order to
"stabilize the variance" in the sense discussed on page 12 of
Hickman and Miler [1977]. (This should become clearer later in
the discussion.) In addition, the arsin transformation
frequently results in a distribution that is closer to the normal
distribution than is that of the pre-transformed variable.

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In our application, we will perform two Bayesian graduations--one for the claim terminations and one for the non-claim terminations.

Unfortunately, unlike life insurance, mortgage insurance must be considered to be a fundamental 1/ (or economic) risk rather than a particular (or personal) risk. This is because the ability of individual homeowners to continue to meet their monthly mortgage obligations is highly dependent on the economy of the United States as well as its effect on local economic conditions. Hence, a group of mortgage insurance policies must be treated as being <u>interdependent</u> rather than independent. As a result, one of the basic assumptions underlying the Bayesian graduation model--that the policies may be assumed to be stochastically independent--is violated. Consequently, Bayesian graduation does not produce here a good model of the posterior distribution of either the claim or non-claim terminations. It is, however, not at all clear how to model the interdependent structure since this largely depends on U. S. economic conditions. We have, therefore, reduced the scope of our problem to that of estimating the average central decrement rates. For this purpose, Bayesian graduation does provide a useful framework and we have used the means of the posterior distribution as our sought-after average central rates of decrement.

1/ The terminology employed here appears in Kulp and Hall [1968, chapter 1]. Also, there is a brief discussion of mortgage insurance on page 69 of Bickelhaupt [1974].

3. TABULATING AND ADJUSTING THE OBSERVED VALUES

Before specifying the mean vectors and covariance arrays just discussed, we need to discuss the observed data. We begin with the following notation:

E_x^j = the number of policies written in year j which survive to age x.

C_x^j = the number of policies written in year j which result in a claim termination between age x and x + 1.

N_x^j = the number of policies written in year j which result in a non-claim termination between age x and x + 1.

Since not all policies endorsed during calendar year 1980 were reported to and/or processed by HUD on or before December 31, 1980, we had to adjust the value of E_0^{1980}.

Because an analysis of recent reporting lags indicated that E_0^{1980} was about 7 percent too low, we decided to replace E_0^{1980} by $(1.07 \times E_0^{1980})$.

Because, on the average, those policies still in force on December 31, 1980, have only completed about half of their most recent policy year and there is frequently a substantial delay (of up to two years in some cases) in the reporting and/or processing of terminations, we decided (1) to replace C_x^{1980-x} and N_x^{1980-x} for x = 1,...,13 by $(3.0 \times C_x^{1980-x})$ and $(3.0 \times N_x^{1980-x})$, respectively, and (2) to replace C_x^{1979-x} and N_x^{1979-x} for x = 1,...,12 by

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$(1.2 \times C_x^{1979-x})$ and $(1.2 \times N_x^{1979-x})$, respectively. Again, these are just ratio adjustments based on reporting lags observed in the recent past.

Having made these adjustments, we may now define more encompassing classes of exposure, claim terminations and non-claim terminations by combining the data across endorsement years as follows:

$$E_x = \sum_{j=1963}^{1980-x} E_x^j$$

$$C_x = \sum_{j=1963}^{1980-x} C_x^j$$

$$N_x = \sum_{j=1963}^{1980-x} N_x^j$$

where $x = 0,1,\ldots,13$.

Thus, as defined above, E_0, for example, represents the total number of policies written between January 1, 1963 and December 31, 1980. Unfortunately, because of a computer system problem, there were no data available on the last four policy years of mortgages written during the period 1957-1969.

4. CLAIM TERMINATIONS

4.1 Observed Central Rates

Assuming that terminations are distributed uniformly throughout each policy year, we may approximate the first 14 observed central rates of claim termination by

$$u_x = C_x/(E_x - .5(C_x + N_x)) \qquad \text{for } x = 0,1,\ldots,13.$$

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We define $t(B)^{-1}$, the inverse of the arcsin transformation of the corresponding covariance matrix, to be the 30-by-30 diagonal matrix in which the i-th diagonal is $4E_i$ for i = 0,1,...,13 and 0 for i = 14,...,29. Since the weight assigned to each of the last sixteen policy years is zero, the choice of u_x for $x \geq 14$ is irrelevant. Because the components of $\underline{u}$ are assumed to be mutually independent, given $\underline{W} = \underline{w}$, the off-diagonal elements of B (and hence $t(B)^{-1}$) are all set equal to zero.

4.2 Prior Distribution

Since we want the graduated claim termination rates, m_x, to equal the observed rates for the first seven policy years, we define

$$m_x = u_x^{\cdot} \qquad \text{for } x = 0,1,...,6.$$

The remaining prior central claim termination rates were obtained by multiplying the means of the posterior distribution of the central claim termination rates (u_x^{*}) of Section 203 by the ratio of

$$\frac{\displaystyle\sum_{x=7}^{13} u_x}{\displaystyle\sum_{x=7}^{13} u_x^{*}}.$$

This is in the spirit of maximum likelihood estimation and ensures that the average prior central rate of claim termination (for Section 222) for policy years 8-14 will be equal to that observed. Moreover, the mean vector of the prior distribution

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(of Section 222 claim terminations) from policy year 8 and beyond is assumed to have the shape of the mean vector of the posterior claim distribution of Section 203.

The arcsin transformation, $t(A)$, of the corresponding correlation matrix is $t(A)^*/4N$ where $t(A)^* = [p^{|i-j|}]$ and where $[p^{|i-j|}]$ is a 30-by-30 matrix in which the element in the i-th row and j-th column is equal to $p^{|i-j|} = (.99)^{|i-j|}$. The matrix $t(A)$ is constructed in the spirit of Hickman and Miller [1977]. The value of p was chosen to be .99 to equal the observed value of the serial correlation of the central claim termination rates for policy years 2-19 of Section 203 loans. N is chosen to be the median of the last seven non-zero exposure values, i.e., $N = E_{10}$.

The vector of posterior claim termination values, $\underline{vc}$, is the square of the sine of

$$(t(B)^{-1} + t(A)^{-1})^{-1} (t(B)^{-1}t(\underline{u}) + t(A)^{-1}t(\underline{m}))$$

where $\underline{u} = (u_0, u_1, \ldots, u_{29})$ and $\underline{m} = (m_0, m_1, \ldots, m_{29})$. The observed, prior, and posterior claim termination rates are exhibited in Table 4.

5.   NON-CLAIM TERMINATIONS

5.1   Observed Central Rates

We approximate the central rate of non-claim termination by

$$u_x = N_x/(E_x - .5(C_x + N_x)) \qquad \text{for } x = 0,1,\ldots,13.$$

The matrix $t(B)^{-1}$ is defined as in Section 4.

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5.2 Prior Non-Claim Termination Rates

Since we want each graduated non-claim termination
rate, m_x, to equal the corresponding observed rate for the first
five policy years, we set

$$m_x = u_x \qquad\qquad \text{for } x = 0,1,\ldots,4.$$

The remaining prior central non-claim termination rates were
obtained by multiplying the mean vector of the posterior
distribution of the central non-claim termination rates (u_x^*) of
Section 203 by the ratio of

$$\frac{\displaystyle\sum_{x=7}^{13} u_x}{\displaystyle\sum_{x=7}^{13} u_x^*}.$$

The reasons for this adjustment are those given in the previous
section.

5.3 Correlation Matrix

The arcsin transformation, $t(A)$, of the corresponding
correlation matrix is $t(A)*/4N$ where

$$t(A)* = \begin{pmatrix} I & & & O_{5,25} \\ & & & \\ O_{25,5} & & & p^{|i-j|} \end{pmatrix}$$

and where I is the 5 by 5 identity matrix, $O_{5,25}$ and $O_{25,5}$ are,
respectively, 5-by-25 and 25-by-5 matrices composed entirely of

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zeros, and $p^{|i-j|}$ is a 25-by-25 matrix in which the element in the i-th row and j-th column (of the matrix $t(A)^*$) is equal to $p^{|i-j|} = (.9)^{|i-j|}$.

Thus, $t(A)^*$ is constructed so that (1) each of the first five graduated values is set equal to the corresponding prior (i.e, observed) value, and (2) each of the graduated values for policy years 6-29 is influenced to some extent by each of the prior values for policy years 6-29. In the next section, we will show how we determined the non-claim termination rate for the final policy year. Since the observed serial correlation among the initial 19 Section 203 central non-claim termination rates is 0.90, we set $p = 0.90$; N is as defined in Section 4.

The vector of graduated non-claim termination values, vn, is the square of the sine of

$$(t(B)^{-1} + t(A)^{-1})^{-1} \quad (t(B)^{-1}t(\underline{u}) + t(A)^{-1}t(\underline{m}))$$

where

$$\underline{u} = (u_0, u_1, \ldots, u_{29}) \quad \text{and} \quad \underline{m} = (m_0, m_1, \ldots m_{29}).$$

The observed, prior, and posterior (predicted) non-claim termination rates of Section 222 are exhibited in Table 5.

6. CONSTRUCTING THE SURVIVORSHIP SCHEDULE

Using the posterior means (i.e., the predicted values), vc and vn, determined above, we are able to approximate the corresponding vectors of termination probabilities, $\underline{q}^c$ and $\underline{q}^n$, by

$$q_x^c = vc_x / (1 + (.5)vt_x)$$

and   for $x=0,1,\ldots,29$

$$q_x^n = vn_x / (1 + (.5)vt_x)$$

where $vt_x = vc_x + vn_x$. The equations above are derived on pages 273-274 of Jordan [1967].

Before proceeding, we need to introduce the following notation: S_x = the number of survivors at the beginning of policy year x.

We first set the radix at 100,000-i.e., $S_0 = 100,000$. We next define

$$C_x = S_x q_x^c$$
$$N_x = S_x q_x^n$$
$$S_{x+1} = S_x - (N_x + C_x)$$

for $x=0,\ldots,28$.

Note that the last three equations represent a recursive procedure in which S_x must be computed before C_x and N_x and in which S_{x+1} is computed after C_x and N_x.

Finally, we define

$$C_{29} = S_{29} q_{29}^c \quad \text{and} \quad N_{29} = S_{29} (1-q_{29}^c)$$

so that $S_{30} = 0$.

The resulting survivorship schedule is shown in Table 6.

7. SECTIONS 221, 223(e), and 235

Because HUD began collecting data for actuarial purposes on
both 221 and 222 mortgage insurance contracts in 1963, the
procedure applied to the Section 221 data is precisely that
applied to the 222 data. Since 1969 was the first year in which
substantial amounts of Section 223(e) and 235 mortgage insurance
contracts were written, we have only considered those 223(e) and
235 mortgages written on or after January 1, 1969. Hence, we
must make two minor changes in the procedure applied to those
sections. First, for the claim terminations only, we set the
first five (rather than seven) prior central rates equal to those
observed. Second, since we only had eleven exposure values, we
set $N = (E_8 + E_9)/2$ --i.e., the median of the last six exposure
values.

The observed data for these sections as well as the
resulting survivorship tables are shown in Tables 7-18.

TABLE 1

*SURVIVORSHIP AND DECREMENT TABLE AS OF DECEMBER 31, 1980
BASED ON AGGREGATE INSURANCE AND TERMINATION EXPERIENCE
FOR HOME MORTGAGES INSURED SINCE 1957*

*SECTION 203
30 YEAR TERM
U.S. TOTALS*

POLICY YEAR	SURVIVORS BEGINNING OF POLICY YEAR	TERMINATIONS DURING POLICY YEAR	
		CLAIM	NON-CLAIM
1	100000.0	413.1	706.9
2	98880.1	1445.2	2262.8
3	95172.1	1352.3	3729.6
4	90090.2	1042.3	4082.8
5	84965.0	776.6	4349.5
6	79838.9	567.9	4595.4
7	74675.5	415.4	4475.2
8	69785.0	282.4	4311.2
9	65191.3	183.0	3908.4
10	61099.9	119.3	3494.6
11	57486.0	73.7	3414.6
12	53997.7	44.9	3204.7
13	50748.1	26.8	3006.5
14	47714.7	15.7	2819.5
15	44879.5	9.1	2644.2
16	42226.2	5.2	2681.9
17	39539.1	2.9	2695.1
18	36841.2	1.6	2684.8
19	34154.8	.8	2651.1
20	31502.8	.5	2593.8
21	28908.5	.3	2515.9
22	26392.3	.2	2420.3
23	23971.9	.1	2309.9
24	21661.9	.1	2187.7
25	19474.2	.0	2056.5
26	17417.6	.0	1919.3
27	15498.2	.0	1778.7
28	13719.5	.0	1637.0
29	12082.5	.0	2092.7
30	9989.8	.0	9989.8

ULTIMATE CLAIM TERMINATION RATE   6.78 *PERCENT*
ULTIMATE NON-CLAIM TERMINATION RATE   93.22 *PERCENT*
ESTIMATED LIFE EXPECTANCY   13.98 *YEARS*

TABLE 2

PREDICTED CENTRAL TERMINATION RATES FOR SECTION 203 (MULTIPLIED BY 100.000)

POLICY YEAR	CLAIMS	NONCLAIMS
1	415	711
2	1490	2332
3	1460	4026
4	1191	4665
5	942	5278
6	735	5948
7	575	6196
8	419	6388
9	290	6189
10	201	5894
11	132	6126
12	86	6119
13	54	6107
14	34	6090
15	21	6071
16	13	6560
17	8	7057
18	4	7563
19	3	8076
20	2	8587
21	1	9099
22	1	9611
23	0	10124
24	0	10636
25	0	11149
26	0	11662
27	0	12175
28	0	12689
29	0	18962
30	0	6067

TABLE 3

DATA USED TO CONSTUCT SECTION 222 SURVIVORSHIP SCHEDULE
(ROUNDED TO NEAREST INTEGER)

POLICY ANNIVERSARY	E_X	C_X	N_X
1	151790	664	964
2	149468	2375	4028
3	140556	2232	6187
4	129136	1559	6264
5	119155	928	6136
6	109892	472	6116
7	101240	280	6151
8	93367	168	6118
9	80289	92	5292
10	63978	64	3942
11	48176	27	3221
12	32936	17	2246
13	18837	6	1304
14	7801	3	692

TABLE 4

CLAIM TERMINATION RATES FOR SECTION 222
(MULTIPLIED BY 100,000)

POLICY YEAR	OBSERVED	PRIOR	PREDICTED
1	440	440	438
2	1623	1623	1620
3	1637	1637	1633
4	1245	1245	1241
5	803	803	799
6	442	442	439
7	286	286	283
8	186	204	201
9	119	141	139
10	104	98	96
11	58	65	63
12	53	42	41
13	33	27	26
14	40	17	16
15		10	10
16		6	6
17		4	4
18		2	2
19		1	1
20		1	1
21		0	0
22		0	0
23		0	0
24		0	0
25		0	0
26		0	0
27		0	0
28		0	0
29		0	0
30		0	0

TABLE 5

NON-CLAIM TERMINATION RATES FOR SECTION 222
(MULTIPLIED BY 100,000)

POLICY YEAR	OBSERVED	PRIOR	PREDICTED
1	638	638	638
2	2754	2754	2754
3	4538	4538	4538
4	5002	5002	5002
5	5307	5307	5307
6	5737	6743	6112
7	6275	7023	6451
8	6781	7241	6787
9	6820	7016	6691
10	6360	6681	6447
11	6919	6944	6807
12	7062	6936	6883
13	7172	6923	6933
14	9284	6903	6963
15		6882	6936
16		7436	7486
17		7999	8046
18		8573	8617
19		9154	9194
20		9734	9771
21		10314	10348
22		10895	10926
23		11476	11505
24		12057	12083
25		12638	12662
26		13220	13242
27		13802	13822
28		14384	14402
29		21495	21515
30		6878	6889

TABLE 6

SURVIVORSHIP AND DECREMENT TABLE AS OF DECEMBER 31, 1980
BASED ON AGGREGATE INSURANCE AND TERMINATION EXPERIENCE
FOR HOME MORTGAGES INSURED SINCE 1963

SECTION 222
30 YEAR TERM
U.S.TOTALS

POLICY YEAR	SURVIVORS BEGINNING OF POLICY YEAR	TERMINATIONS DURING POLICY YEAR	
		CLAIM	NON-CLAIM
1	100000.0	436.0	635.0
2	98929.0	1568.6	2666.1
3	94694.4	1500.2	4168.5
4	89025.7	1071.2	4318.3
5	83636.2	648.4	4307.0
6	78680.8	334.5	4656.2
7	73690.0	201.4	4598.8
8	68889.8	133.7	4517.6
9	64238.5	86.1	4156.2
10	59996.2	56.0	3745.3
11	56194.9	34.4	3698.3
12	52462.2	20.9	3490.1
13	48951.2	12.4	3279.6
14	45659.2	7.2	3071.9
15	42580.1	4.1	2854.1
16	39721.9	2.3	2866.2
17	36853.4	1.3	2850.4
18	34001.7	.7	2808.7
19	31192.3	.4	2741.8
20	28450.1	.2	2650.3
21	25799.6	.1	2538.5
22	23261.0	.1	2409.9
23	20851.1	.0	2268.3
24	18582.7	.0	2117.5
25	16465.2	.0	1960.8
26	14504.4	.0	1801.4
27	12703.0	.0	1642.3
28	11060.7	.0	1486.0
29	9574.7	.0	1859.9
30	7714.8	.0	7714.8

ULTIMATE CLAIM TERMINATION RATE   6.12 PERCENT
ULTIMATE NON-CLAIM TERMINATION RATE   93.88 PERCENT
ESTIMATED LIFE EXPECTANCY   13.38 YEARS

TABLE 7

DATA USED TO CONSTUCT SECTION 221 SURVIVORSHIP SCHEDULE
(ROUNDED TO NEAREST INTEGER)

POLICY ANNIVERSARY	E_X	C_X	N_X
1	740140	7878	2825
2	718083	31233	11918
3	653868	24689	21290
4	584763	15658	25073
5	521396	10392	26044
6	469123	7321	26893
7	420537	5453	25361
8	373672	4282	22500
9	287392	2791	16232
10	206883	1667	10024
11	131803	688	7139
12	73778	193	4353
13	39633	93	2476
14	14889	23	1067

TABLE 8

CLAIM TERMINATION RATES FOR SECTION 221
(MULTIPLIED BY 100,000)

POLICY YEAR	OBSERVED	PRIOR	PREDICTED
1	1072	1072	1068
2	4484	4484	4475
3	3913	3913	3903
4	2774	2774	2763
5	2065	2065	2053
6	1620	1620	1605
7	1346	1346	1329
8	1188	1457	1433
9	1005	1009	997
10	829	700	697
11	538	461	459
12	270	298	298
13	243	190	189
14	160	118	118
15		73	73
16		44	44
17		26	26
18		15	15
19		9	9
20		5	5
21		3	3
22		2	2
23		1	1
24		1	1
25		1	1
26		1	1
27		1	1
28		0	0
29		0	0
30		0	0

TABLE 9

NON-CLAIM TERMINATION RATES FOR SECTION 221
(MULTIPLIED BY 100,000)

POLICY YEAR	OBSERVED	PRIOR	PREDICTED
1	384	384	384
2	1711	1711	1711
3	3375	3375	3375
4	4442	4442	4442
5	5176	5176	5176
6	5950	5926	5899
7	6260	6172	6117
8	6245	6364	6217
9	5841	6166	5921
10	4986	5871	5555
11	5582	6102	5823
12	6088	6096	5890
13	6457	6084	5934
14	7439	6067	5952
15		6048	5945
16		6535	6438
17		7030	6940
18		7535	7451
19		8045	7967
20		8554	8483
21		9064	8998
22		9575	9513
23		10085	10028
24		10596	10544
25		11107	11059
26		11618	11574
27		12129	12089
28		12641	12604
29		18890	18851
30		6044	6023

TABLE 10

SURVIVORSHIP AND DECREMENT TABLE AS OF DECEMBER 31, 1980
BASED ON AGGREGATE INSURANCE AND TERMINATION EXPERIENCE
FOR HOME MORTGAGES INSURED SINCE 1963

SECTION 221
30 YEAR TERM
U.S.TOTALS

| POLICY YEAR | SURVIVORS BEGINNING OF POLICY YEAR | TERMINATIONS DURING POLICY YEAR | |
		CLAIM	NON-CLAIM
1	100000.0	1060.5	381.7
2	98557.8	4278.5	1635.9
3	92643.4	3489.2	3016.7
4	86137.6	2297.6	3693.5
5	80146.5	1587.9	4003.7
6	74554.9	1153.4	4238.7
7	69162.8	886.0	4078.7
8	64198.1	886.3	3844.3
9	59467.5	573.2	3403.3
10	55491.1	374.8	2988.9
11	52127.4	232.1	2942.9
12	48952.4	141.3	2796.6
13	46014.6	84.5	2649.4
14	43280.6	49.7	2500.0
15	40731.0	28.7	2350.6
16	38351.7	16.3	2391.7
17	35943.7	9.1	2410.5
18	33524.1	5.0	2407.9
19	31111.2	2.7	2383.6
20	28724.9	1.5	2337.4
21	26386.1	.8	2271.9
22	24113.3	.5	2189.8
23	21923.1	.3	2093.6
24	19829.2	.2	1986.0
25	17843.0	.1	1869.8
26	15973.0	.1	1747.6
27	14225.4	.1	1621.7
28	12603.6	.1	1494.4
29	11109.2	.0	1913.8
30	9195.4	.0	9195.3

ULTIMATE CLAIM TERMINATION RATE   17.16 PERCENT   .
ULTIMATE NON-CLAIM TERMINATION RATE   82.84 PERCENT
ESTIMATED LIFE EXPECTANCY   13.02 YEARS

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TABLE 11

*DATA USED TO CONSTUCT SECTION 223(E) SURVIVORSHIP SCHEDULE
(ROUNDED TO NEAREST INTEGER)*

POLICY ANNIVERSARY	E X	C X	N X
1	153508	1791	764
2	147411	8887	2153
3	130882	7950	3109
4	114632	5844	2652
5	100292	3976	2392
6	89425	3236	2563
7	80033	2476	2495
8	70242	2056	2647
9	35692	1094	1494
10	25109	749	899
11	13526	465	897

TABLE 12

CLAIM TERMINATION RATES FOR SECTION 223(E)
(MULTIPLIED BY 100,000)

POLICY YEAR	OBSERVED	PRIOR	PREDICTED
1	1176	1176	1078
2	6264	6264	6026
3	6342	6342	6074
4	5295	5295	5011
5	4094	4094	3799
6	3740	6203	5775
7	3193	4852	4480
8	3028	3531	3256
9	3181	2446	2257
10	3083	1697	1565
11	3620	1116	1020
12		723	646
13		459	399
14		287	240
15		176	140
16		107	79
17		63	43
18		37	22
19		22	11
20		13	5
21		8	2
22		5	1
23		3	0
24		2	0
25		2	0
26		2	0
27		1	0
28		1	0
29		1	0
30		1	0

TABLE 13

NON-CLAIM TERMINATION RATES FOR SECTION 223(E)
(MULTIPLIED BY 100,000)

POLICY YEAR	OBSERVED	PRIOR	PREDICTED
1	502	502	502
2	1517	1517	1517
3	2480	2480	2480
4	2402	2402	2402
5	2464	2464	2464
6	2962	4065	3353
7	3217	4234	3577
8	3899	4365	3885
9	4344	4229	3935
10	3701	4027	3858
11	6983	4186	4150
12		4181	4149
13		4173	4144
14		4161	4135
15		4149	4125
16		4483	4461
17		4822	4802
18		5168	5149
19		5518	5501
20		5868	5852
21		6218	6203
22		6568	6554
23		6918	6905
24		7268	7256
25		7619	7608
26		7969	7959
27		8320	8311
28		8671	8662
29		12958	12949
30		4146	4141

TABLE 14

SURVIVORSHIP AND DECREMENT TABLE AS OF DECEMBER 31, 1980
BASED ON AGGREGATE INSURANCE AND TERMINATION EXPERIENCE
FOR HOME MORTGAGES INSURED SINCE 1969

SECTION 223(E)
30 YEAR TERM
U.S.TOTALS

		TERMINATIONS DURING POLICY YEAR	
	SURVIVORS		
POLICY	BEGINNING OF		NON-
YEAR	POLICY YEAR	CLAIM	CLAIM
1	100000.0	1070.0	497.9
2	98432.0	5715.6	1439.0
3	91277.4	5316.9	2170.7
4	83789.8	4048.5	1941.0
5	77800.3	2866.1	1858.5
6	73075.7	4036.2	2343.3
7	66696.2	2872.5	2293.5
8	61530.1	1934.3	2307.9
9	57287.9	1254.3	2186.5
10	53847.0	820.5	2022.6
11	51004.0	506.9	2063.4
12	48433.7	305.6	1962.5
13	46165.6	180.1	1870.7
14	44114.8	103.6	1785.3
15	42225.9	57.9	1705.6
16	40462.4	31.4	1764.9
17	38666.1	16.2	1812.8
18	36837.1	8.0	1849.1
19	34980.1	3.7	1872.6
20	33103.8	1.6	1882.0
21	31220.2	.6	1878.2
22	29341.4	.2	1861.9
23	27479.2	.1	1834.1
24	25645.1	.0	1795.7
25	23849.3	.0	1747.9
26	22101.5	.0	1691.8
27	20409.7	.0	1628.5
28	18781.2	.0	1559.3
29	17221.8	.0	2094.4
30	15127.4	.0	15127.4

ULTIMATE CLAIM TERMINATION RATE   31.15 PERCENT
ULTIMATE NON-CLAIM TERMINATION RATE   68.85 PERCENT
ESTIMATED LIFE EXPECTANCY   13.61 YEARS

TABLE 15

DATA USED TO CONSTUCT SECTION 235 SURVIVORSHIP SCHEDULE
(ROUNDED TO NEAREST INTEGER)

POLICY ANNIVERSARY	E_X	C_X	N_X
1	456598	11329	672
2	444591	25298	2848
3	416415	20843	7370
4	388193	12992	11822
5	363367	7913	17776
6	337604	4932	22566
7	306949	3177	23622
8	272920	2294	20387
9	204334	1274	13210
10	126686	627	6002
11	48813	282	2862

TABLE 16

CLAIM TERMINATION RATES FOR SECTION 235
(MULTIPLIED BY 100,000)

POLICY YEAR	OBSERVED	PRIOR	PREDICTED
1	2514	2514	2491
2	5876	5876	5840
3	5181	5181	5144
4	3457	3457	3423
5	2257	2257	2226
6	1523	1636	1605
7	1082	1280	1251
8	877	931	909
9	646	645	630
10	508	448	437
11	597	294	287
12		191	185
13		121	116
14		76	72
15		46	44
16		28	26
17		17	15
18		10	9
19		6	5
20		3	3
21		2	2
22		1	1
23		1	1
24		1	0
25		0	0
26		0	0
27		0	0
28		0	0
29		0	0
30		0	0

TABLE 17

NON-CLAIM TERMINATION RATES FOR SECTION 235
(MULTIPLIED BY 100,000)

POLICY YEAR	OBSERVED	PRIOR	PREDICTED
1	149	149	149
2	662	662	662
3	1832	1832	1832
4	3146	3146	3146
5	5071	5071	5071
6	6968	6546	6967
7	8047	6819	7296
8	7794	7030	7367
9	6703	6812	6905
10	4865	6486	6384
11	6058	6741	6625
12		6734	6630
13		6721	6627
14		6702	6618
15		6682	6606
16		7219	7149
17		7766	7700
18		8324	8262
19		8887	8831
20		9450	9398
21		10014	9965
22		10577	10532
23		11141	11100
24		11705	11667
25		12270	12235
26		12834	12802
27		13399	13370
28		13964	13938
29		20868	20840
30		6677	6662

TABLE 18

SURVIVORSHIP AND DECREMENT TABLE AS OF DECEMBER 31, 1980
BASED ON AGGREGATE INSURANCE AND TERMINATION EXPERIENCE
FOR HOME MORTGAGES INSURED SINCE 1969

SECTION 235
30 YEAR TERM
U.S.TOTALS

POLICY YEAR	SURVIVORS BEGINNING OF POLICY YEAR	TERMINATIONS DURING POLICY YEAR CLAIM	NON- CLAIM
1	100000.0	2458.6	147.2
2	97394.2	5508.5	624.0
3	91261.7	4535.9	1615.4
4	85110.3	2820.9	2592.4
5	79697.1	1711.7	3899.5
6	74086.0	1140.2	4949.7
7	67996.1	815.7	4757.6
8	62422.8	545.1	4416.1
9	57461.6	348.6	3823.7
10	53289.3	225.1	3289.9
11	49774.2	138.0	3187.6
12	46448.6	83.0	2977.9
13	43387.7	48.9	2781.5
14	40557.4	28.2	2597.1
15	37932.1	16.0	2425.1
16	35491.0	8.9	2449.3
17	33032.9	4.8	2449.2
18	30578.9	2.5	2426.2
19	28150.2	1.3	2380.6
20	25768.2	.7	2312.9
21	23454.7	.3	2226.3
22	21228.0	.2	2124.0
23	19103.8	.1	2009.0
24	17094.7	.1	1884.6
25	15210.1	.0	1753.7
26	13456.4	.0	1619.1
27	11837.3	.0	1483.5
28	10353.8	.0	1349.1
29	9004.7	.0	1699.5
30	7305.2	.0	7305.2

ULTIMATE CLAIM TERMINATION RATE   20.44 *PERCENT*
ULTIMATE NON-CLAIM TERMINATION RATE   79.56 *PERCENT*
ESTIMATED LIFE EXPECTANCY   12.38 *YEARS*

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