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## EXPONENTIAL DECAY MODEL FOR WITHDRAWAL RATES

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The procedures used in this paper were developed to solve a practical problem. Crude withdrawal data was available by calendar duration, but was distorted by reporting lags. In order to develop graduated policy year withdrawal rates, the following formula was utilized:

$$\ell_t = e^{R(t)} \quad , \quad R(t) = \sum_{j=1}^{p} \beta_j \cdot t^{s \cdot j}$$

$t$ = policy year duration

$\ell_0$ = 1

$\ell_t$ = portion persisting after t policy years, assuming the only decrement to be voluntary withdrawal.

$\beta_j$ = parameters to be determined in curve fitting process

$p$ = number of $\beta_j$ parameters to be used in curve fitting

$s$ = another parameter to be determined in curve fitting process

The choice of this formula evolved from the initial assumption that withdrawal rates follow an exponential decay pattern. The above formula is an extension of the concepts inherent in the application of the simple formula

$$\ell_t = 1/e^{\beta \cdot t^{1/n}}$$

Using a fractional power in the denominator allows for rapid early decay and slow ultimate decay, typical of withdrawal rate patterns. For this reason this formula does not work well for graduating $\ell_t$ where $\ell_t$ is a function of both withdrawal and mortality decrements. Whereas many graduation methods familiar to actuaries were developed with mortality studies in mind, this formula was developed solely for graduation of withdrawal rates. This formula handles the end of the curve (early durations) without special handling. This is obviously an important criteria when evaluating graduation methods for withdrawal data. Many graduation techniques work poorly at curve ends.

To use this formula, one must have a means by which to fit it to the observed data. The method used is least squares. In order to have a linear model, we must take logarithms, giving

$$\ln(\ell_t) = \sum_{j=1}^{p} \beta_j \cdot t^{s \cdot j}$$

This still does not yield a linear form due to the positioning of the parameter s. Thus, one must use trial and error to find the s producing the smallest sum of squares. Usually s will be in the range of .02 to.80, most frequently toward the low end of this range.

For a given s the following technique yields the $\beta_j$ for $j = 1$ to p which minimize the sum of squares:

1. Define the matrix X such that $X_{ij} = t_i^{s \cdot j}$ where $t_i$ corresponds to the duration of the $i^{th}$ observed value for $\ell_t$. Then X will be of size n × p where n is the number of observed values.

2. Define the p × 1 matrix $\beta$ where $\beta_{j1} = \beta_j$

3. Define the n × 1 matrix Y of observed values where $Y_{i1}$ the logarithm of the $i^{th}$ observed value for $\ell_t$.

4. Then one has the linear model $Y = X\beta$

5. To solve for $\beta$ one performs the following matrix manipulation:

$$\beta = (X^T X)^{-1} X^T Y$$

where $X^T$ is the transpose of X. Development of this matrix solution appears in Graybill[1].

If one has observed withdrawals on a calendar year basis, then $\ell_t$ will be available only at the end of each calendar year. Thus one will have data available for $\ell_0$, $\ell_A$, $\ell_{1+A}$, $\ell_{2+A}$ .... where A is the average policy year duration at the end of the calendar year 1. If withdrawals were reported immediately, then A would be 0 for annual premium policies and ½ for continuous premium policies. If one has a reporting lag of about 2½ months on a block of monthly premium mode business, then A is about .275.

Often one will have a first year policy lapse rate and calendar lapse rates for all durations. In such a case one merely needs to try different A's until one is found which closely reproduces the first year policy lapse rate. Fortunately, the formula produces about the same graduated policy year lapse rates for any logical choice of A. The graduated policy year withdrawal rates usually are not greatly affected by the number of terms used in the exponent.

In addition to calendar year lapses, monthly lapses for the first policy year can be used in the graduation process. The first year monthly lapse information is not particularly important, but it can suppress graduation anomalies occurring in the first one or two policy months.

Up to this point, withdrawal rates as a function of duration have been discussed. Typically one wishes to reflect issue age as well. In building an issue age/duration withdrawal table, one usually desires that the rates progress smoothly from issue age to issue age and from duration to duration. Thus, before applying the formula to each issue age's withdrawal rates, one may wish to smooth the crude withdrawal data so that for each given duration (or duration group) the rates are smooth as one goes from issue age to issue age. One approach that works well starts by calculating the average withdrawal rate as a function of duration only. Then one calculates an actual to expected ratio for each issue age group where the expected are the duration withdrawal rates.

Next one builds an expected table by applying the issue age actual to expected ratios to the duration withdrawal rates. Using this expected table, one can obtain a matrix of actual to expected ratios. Then for each duration one can graduate the actual to expected ratios. Weighted polynomial least-squares work well for graduating the actual to expected ratios.

Examples and more information about this method can be obtained from the author at his *Yearbook* address.

1 Graybill, Franklin, A. *Theory and Application of the Linear Model*. North Scituate, Mass.: Duxbury Press, 1976. □