Article From:

The Actuary

January 1987 – Volume No. 21, Issue No. 1

CALENDAR DAYS — WITHOUT A CALENDAR

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This article gives a concise, easily remembered way to determine the exact number of days between two specified dates, without reference to the number of days in each of the intervening months.

The following procedure will assign to any date in the 20th or 21st centuries (from March 1, 1900) a "day number" (DN).

(1) Write each date as a year-month-day triplet — y,m,d. (Use only the last two digits of the calendar year for 20th century dates, but after the turn of the century add 100). Examples: 87-1-25 and 102-7-15.

(2) Convert to a year beginning on March 1 instead of January 1 — y',m',d by subtracting 2 from m if m is 3 or greater, or adding 10 to m (and subtracting 1 from y) if m is 1 or 2. Examples above: 86-11-25 and 102-5-15.

(3) Calculate DN = [365.25 y'] + [30.59 m'] + d, where [ ] means "greatest integer in". Continuing same examples:

DN January 25, 1987 = 31411 + 336 + 25 = 31772

DN July 15, 2002 = 37255 + 152 + 15 = 37422

(4) The difference between the DN's is indicative of the number of days between. In our example July 15, 2002 falls 5650 days after January 25, 1987.

The third term of the algorithm for day number is self-evident. The first term will be understandable if one recognizes that a year has 365 days, except one extra day in every year divisible by 4. The integral part of the 30.59 is the 30 days that are part of every month (except February); but the decimal .59 needs further explanation. It is needed to correct for the 31 days in 7 of the 12 months, and will be explained in reference to the following table.

	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB
m'	1	2	3	4	5	6	7	8	9	10	11	12
a	1	0	1	0	1	1	0	1	0	1	1	
b	0	1	1	2	2	3	4	4	5	5	6	7
c	.59	1.18	1.77	2.36	2.95	3.54	4.13	4.72	5.31	5.90	6.49	7.08

Line a shows the extra day for each of the 31-day months. The length of February is omitted, since it is recognized implicitly as the balancing period at the end of the year.

Line b accumulates the extra days of line a for previous months.

Line c is .59m'. When the decimal fractions are dropped from line c, it reproduces line b.

If the coefficient .59 is reduced below 7/12, the 7.08 on line c becomes less than 7, and the day numbers for February become too low: if the .59 is increased to 3/5, the 2.95 and 5.90 on line c are increased to 3 and 6, and the day numbers for July and December become too high. The .59 is therefore not unique. It can be replaced by any amount greater than or equal to 7/12 but less than 3/5.

It is also possible to apply a similar algorithm in reverse. Here the DN is the given and the date is to be determined. The DN of the date 1000 days after January 25, 1987 must be 32772, but what date has a 32772 DN? Here the algorithm is as follows:

$$\frac{DN(y', m', d) - 30.1}{365.25} = y'+f1 \quad , \quad 0<f1<1$$

$$\frac{[365.25\ f1 + 31]}{30.59} = m'+f2 \quad , \quad 0<f2<1$$

$$[30.59\ f2 + 1] = d$$

This means that, for DN = 32772,

y' = 89 and f1 = .6424

m' = 8 and f2 = .6630

d = 21

m = 10

The desired date, 1000 days after January 25, 1987, turns out to be October 21, 1989.

Here one must remember to convert from years beginning in March to the traditional year beginning in January.

I leave to readers how the 30.1 and 31 figures were derived, giving as clues only that the former could just as well have been any value greater than 30 and less than or equal to 30.25, and that March 1, 1900 (when y',m', and d are 0,1,1) is day 31.

The DN algorithm presented here works fine without further correction for dates in the 20th and 21st centuries because the Gregorian correction does not affect the leap-year 2000. The omissions of the 1900 and 2100 leap-years (years divisible by 100 but not by 400) require a small correction if the range is to be extended beyond these two centuries. Even this matter can be accounted for if we add a fourth term to the procedure for DN. The additional term is

$$-[.75(1 + [.01\ y'])]$$

I am indebted to a colleague, Beda Chan, for the observation that the DN also indicates the day of the week. Divide the DN by 7, and write down any remainder. If the DN is divisible by 7 the corresponding date is a Monday; but each unit of remainder advances the weekday by one. Our earlier examples, January 25, 1987 and July 15, 2002, fall respectively on a Sunday and on a Monday.

I have also become aware, but only after I had worked this out for myself, that others have worked along simila. lines. Texas Instruments has a method that handles leap-years and the Gregorian correction in much the same way as mine, but deals with differing month lengths quite differently; and their formulae are much more difficult to remember. □