Credibility Theory Practices

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Introduction

Study Overview

In the Fall of 2008, the Society of Actuaries (SOA) initiated a research project on the application and adoption of credibility theory within the life insurance and annuity industry. A Request for Proposal was drafted that resulted in the authors of this report performing three distinct tasks:

1) Develop an analytical framework to provide practitioners with an extensive actuarial analysis of applied credibility theory from the standpoint of example using intercompany lapse and mortality.

2) Administer a survey to gauge the application of credibility theory in various actuarial tasks associated with managing life and annuity risks.

3) Create an annotated bibliography for individuals who wish to learn more about applying credibility theory.

Report Structure

For the purposes of presenting the three distinct areas addressed in this report, and to give each area an internal completeness, the report is organized as three independent but related sections, each with its own summary and background sections. We have elected to use a S.n page notation to further clarify and simplify navigation within this document, where S={I,II,III} corresponding to the following reports, respectively:

I. Application of Credibility to Company Lapse and Mortality Experience Data Report,
II. Credibility Survey Report, and
III. Annotated Credibility Bibliography

Each section is numbered separately. Appendices are included within each section as appropriate.

Additional materials containing a numerical example relating to the Application of Credibility to Company Lapse and Mortality Experience Data Report are available by accessing four Excel spreadsheets, available on the SOA Website (www.soa.org), or by contacting MIB at infoline@mib.com:

METHOD	MORTALITY	LAPSE
Bühlmann Empirical Bayesian Method	*Buhlmann_Mortality.xls*	*Buhlmann_Lapse.xls*
Limited Fluctuation Method	*Limited_Fluctuation_Mortality.xls*	*Limited_Fluctuation_Lapse.xls*

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The use of the survey administered by MIB required that actuaries, who received the invitation to participate, devote time to considering the questions included in the survey and supplying meaningful responses. The authors are grateful to each survey participant who took the time to submit a considered response to the survey.

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Application of Credibility to Company Lapse and Mortality Experience Data

Using the Limited Fluctuation Method and the Bühlmann Empirical Bayesian Method

Executive Summary

In recent years, life and annuity actuaries have begun to face issues of using company experience data for both the organization's overall risk assessment process and preparing for principle-based reserving. In the accompanying credibility survey, actuarial judgment based on experience studies is the predominant credibility method currently used by companies for their life and annuity business. Statistical credibility's rigor can validate or improve actuarial judgment applied to company experience data.

Statistical credibility is a mathematical method for adjusting experience-based estimates. Like any statistical method, it is only as good as the degree to which the model and its underlying assumptions hold. Actuaries would generally apply statistical credibility methods for experience assumptions by:

1. Selecting a base estimate using actuarial judgment
2. Obtaining their company's policy by policy experience study results
3. Obtaining an estimate from the statistical credibility model using the base estimate and experience study results
4. Selecting an experience estimate based upon actuarial judgment and estimates from the statistical credibility model

Selecting a base estimate using actuarial judgment – The selection of the base estimate affects the estimate from a statistical credibility approach. For valuation mortality, many actuaries would choose a valuation mortality table. For base policyholder behavior estimates other than lapse, an actuary could use their best opinion until industry policyholder behavior studies are compiled. For base estimates for pricing, actuaries could use their company's pricing assumptions. Regardless of the method used to select the table, it is important to note that while selection of the base estimate is outside the scope of this project, that choice can have a significant effect on the results.

Obtaining their company's policy by policy experience study results – Individual company mortality and lapse experience varies substantially from industry studies. This provides both the experience-based results needed for weighting with the base estimate, but also provides information about variances. This information is necessary for statistical credibility methods; hence merely having experience results, such as a mortality ratio, is not sufficient.

Obtaining an estimate from the statistical credibility methods using the base estimate and experience study results – This paper concentrates on developing formulas for the two well-

established statistical credibility methods: Limited Fluctuation and Bühlmann Empirical Bayesian. The formulas are then demonstrated with applications to actual experience study data.

Credibility formulas use a credibility factor 'Z' which varies from 0 to 1. This is used in the standard form of credibility formula:

$$Z \times (\text{Company A/E Ratio}) + (1\text{-}Z) \times (\text{Prior A/E Ratio})$$

This paper develops the formulas for 'Z' for each method.

The Limited Fluctuation method uses only the policy by policy experience study results of a single company. Consequently, each company can calculate their own estimate using the Limited Fluctuation method.

The Bühlmann Empirical Bayesian method uses both the variance of observations within each company and the variance from one company to another. In order to make calculations using the Bühlmann Empirical Bayesian method, the policy details of each company are needed. Since other company's data is confidential, a company would have to use a statistical agent to access the other company data needed for the Bühlmann Empirical Bayesian method.

Selecting an experience estimate based upon actuarial judgment and estimates from the statistical credibility model – The methods presented in this paper have a common assumption. It is that the true mortality or lapse rates are a constant multiple of the standard table. That is, the same A/E ratio applies at all ages, durations, or any other factor that is part of the experience study. To the extent that this assumption may not be true, judgment will be needed to adjust the credibility results.

The remainder of this paper will concentrate on the third of the four items above: Obtaining an estimate from the statistical credibility model using the base estimate and experience study results.

The paper will provide formulas and demonstrate the results of two statistically-based credibility methods using lapse and mortality experience data from ten companies. For each of the ten companies, the two credibility methods produce predicted A/E ratios by company using the individual company's results and the overall results of the ten companies.

For the demonstrative purposes of this paper, the selection of the base estimates is not critical. Practitioners using these statistical credibility methods in different circumstances will select different base estimates appropriate to their goals. In this paper, the expected bases for the A/E ratios are the mortality rates from the 2001 VBT and the lapse rates by plan from the most recent LIMRA study of individual life lapse experience covering observation years 2002-2004.

The overall experience is from ten companies that participated in the SOA 2004-05 experience study. These companies were selected in order to give a mixture of large, medium and small size companies. In order to maintain strict confidentiality of individual company information, only a portion of each company's mortality and lapse data was used.

The results of this paper use different portions of the data for mortality and lapse. Nonsmoker data was studied because it has the most significant exposure. Also, because most of the companies studied sell universal life, this product was used for purposes of illustrating the application of statistical credibility methods for lapse. The Excel files and appendices include complete lapse results for universal life, term, permanent, and variable universal life plans that have distinctly different lapse experience.

The next section of the paper concentrates on the Limited Fluctuation method. The development of the Limited Fluctuation formulas is shown. Then, the by policy and by amount results for universal life lapses and nonsmoker mortality are provided.

The third section of the paper concentrates on the by policy application of the Bühlmann Empirical Bayesian method. The development of the Bühlmann Empirical Bayesian formulas is shown. Then, the results by policy for universal life lapses and nonsmoker mortality are provided.

In addition, this section of the paper concentrates on the by amount application of the Bühlmann Empirical Bayesian method. The development of the Bühlmann Empirical Bayesian formulas is shown. Then, the results by amount for universal life lapses and nonsmoker mortality are provided.

The fourth section of the paper is a summary of results. First, variability of A/E ratios by company from overall A/E ratios are addressed. Then, the overall variability and 'Z' credibility factor is discussed. Finally, there is a discussion of results by policy and by amount.

The final section of the paper contains conclusions.

The development of all results referred to in the paper is detailed in the Excel files 'Limited Fluctuation Lapse', 'Limited Fluctuation Mortality', 'Buhlmann_Lapse', and 'Buhlmann_Mortality'. In addition to the results for universal life lapses and nonsmoker mortality, these files contain lapse results for term, permanent life and variable universal life plans.

The appendices present the information contained in the Excel files in a different format.

The Limited Fluctuation method is covered in Appendix A and Appendix B. Appendix A contains the Limited Fluctuation method predicted A/E ratios for nonsmoker mortality and lapse by universal life, term, permanent life, and variable universal life plans. Appendix B contains the development of the Limited Fluctuation results.

The by policy results of the Bühlmann Empirical Bayesian method is covered in Appendix C and Appendix D. Appendix C contains the Bühlmann Empirical Bayesian method predicted A/E ratios by policy for nonsmoker mortality and lapse by universal life, term, permanent life, and variable universal life plans. Appendix D contains the development of the Bühlmann Empirical Bayesian results by policy.

The by amount results of the Bühlmann Empirical Bayesian method is covered in Appendix E and Appendix F. Appendix E contains the Bühlmann Empirical Bayesian method predicted A/E ratios by amount for nonsmoker mortality and lapse by universal life, term, permanent life, and variable universal life plans. Appendix F contains the development of the Bühlmann Empirical Bayesian results by amount.

2. Limited Fluctuation Method

The Limited Fluctuation method is based on confidence intervals and only uses data from the particular company being studied to determine the credibility factor. In contrast to Bayesian approaches, the Limited Fluctuation method does not consider variation from one company's A/E ratio to another. As shown in the summary of results, the Limited Fluctuation method is as effective in predicting the A/E ratio by count as the Bühlmann Empirical Bayesian method.

The Limited Fluctuation method will be presented through development of Limited Fluctuation formulas and showing policy and amount A/E ratio results for universal life lapses and nonsmoker mortality. Appendix A contains complete tables of Limited Fluctuation method results for nonsmokers for overall mortality and for overall lapse for term, permanent life, universal life and variable universal life policies. Appendix B contains a discussion of the spreadsheet producing the policy and amount A/E ratio results.

2a. Development of Limited Fluctuation Formulas

For this approach to calculating the credibility factor Z for the Limited Fluctuation method, data is only needed for the company being studied. The data required is:

1. There are n observed lives with index i.
2. For the ith life there are three measurements:
   a. f_i is the fraction of the year for which the life was observed.
   b. b_i is the amount insured.
   c. d_i is equal to 0 if the life did not die and is equal to 1 if it did die.
3. Also, for life i, there is a true mortality rate q_i and a standard table mortality rate q_i^s.

The following six quantities can be calculated:

$$A_c = \sum_{i=1}^{n} d_i$$
$$E_c = \sum_{i=1}^{n} f_i q_i^s$$
$$\hat{m}_c = A_c / E_c$$
$$A_d = \sum_{i=1}^{n} b_i d_i$$
$$E_d = \sum_{i=1}^{n} b_i f_i q_i^s$$
$$\hat{m}_d = A_d / E_d$$

where the subscript c represents counts, the subscript d represents dollars, A is the actual, E is the expected, and $\hat{m}$ is the estimated mortality ratio. When doing lapse studies, simply substitute lapse for death.

In what follows, we assume the expected totals are not random.

In order to use Limited Fluctuation credibility we need the moments of the estimators:

$$\mu_c = E(\hat{m}_c) = \frac{\sum_{i=1}^{n} E(d_i)}{E_c} = \frac{\sum_{i=1}^{n} f_i q_i}{E_c}$$

$$\sigma_c^2 = Var(\hat{m}_c) = \frac{\sum_{i=1}^{n} Var(d_i)}{E_c^2} = \frac{\sum_{i=1}^{n} f_i q_i (1 - f_i q_i)}{E_c^2}$$

$$\mu_d = E(\hat{m}_d) = \frac{\sum_{i=1}^{n} b_i E(d_i)}{E_d} = \frac{\sum_{i=1}^{n} b_i f_i q_i}{E_d}$$

$$\sigma_d^2 = Var(\hat{m}_d) = \frac{\sum_{i=1}^{n} b_i^2 Var(d_i)}{E_d^2} = \frac{\sum_{i=1}^{n} b_i^2 f_i q_i (1 - f_i q_i)}{E_d^2}.$$

In all the developments in this paper, we assume that the true mortality is a multiple of the standard table. Thus, we have $q_i = m q_i^s$ where *m* is the true ratio. Note that the same value of *m* applies to all values. It is possible to create models where the multiplier depends in some way on the category (such as age or duration). However, in order to develop credibility formulas, the relationship must be explicit and the formulas become much more complex. Modifications for other patterns will not be developed. The formulas become:

$$\mu_c = \frac{\sum_{i=1}^{n} f_i m_c q_i^s}{E_c} = m_c$$

$$\sigma_c^2 = \frac{\sum_{i=1}^{n} f_i m_c q_i^s (1 - f_i m_c q_i^s)}{E_c^2} \approx \frac{\sum_{i=1}^{n} f_i m_c q_i^s}{E_c^2} = \frac{m_c}{E_c} = \frac{A_c}{E_c^2}$$

$$\mu_d = \frac{\sum_{i=1}^{n} b_i f_i m_d q_i^s}{E_d} = m_d$$

$$\sigma_d^2 = \frac{\sum_{i=1}^{n} b_i^2 f_i m_d q_i^s (1 - f_i m_d q_i^s)}{E_d^2}.$$

Where values of the true ratio are needed, the estimated *m* can be used in place of the true value.

The Limited Fluctuation method begins by assuming that answer is of the form $Z\hat{m} + (1 - Z)a$ where *Z* and *a* are to be determined. The method does not specify how to select *a*. The first step is to state that *Z* = 1 (full credibility) if $\Pr(|\hat{m} - m| \le rm) \ge p$. That is, the relative error in the estimate is small with a high probability. The method also does not specify how *r* and *p* are to be selected. If this condition is not met,

then *Z* is selected to reduce the variance of the credibility estimate to the point where it has the desired accuracy. A normal distribution approximation is usually employed in order to evaluate the probability.

Following the development in all standard credibility texts (see the bibliography attached to this report), the credibility factor is then

$$Z = \min\left\{1, \frac{r\hat{m}}{z\hat{\sigma}}\right\}$$

where *z* is the appropriate quantile from the normal distribution, based on the selected value of *p* (for example, *p* = 90% confidence leads to *z* = 1.645). The value of *a* is normally taken as the ratio that would be used if there were no data (that is, assigning zero credibility to any data). This could be 1, implying that with no data the standard table is to be used, or it could be an average mortality ratio over all companies. The choice of *a* can have considerable impact on the final result. In this paper we take the latter approach and have also selected an *r* of .05 and a *z* of 1.96.

If the approximation for claim counts is used, the formula for the credibility factor simplifies to

$$Z = \min\left\{1, \frac{r(A_c / E_c)}{z(\sqrt{A_c} / E_c)}\right\} = \min\left\{1, \frac{r\sqrt{A_c}}{z}\right\}$$

matching the formula presented in the Canadian Institute of Actuaries educational note.

Due to high values of q_i^s for either lapse rates or older age mortality, the simplifying assumption that $(1 - f_i m_c q_i^s)$ is close to 1 is not true. Therefore when using counts, we do not use the simplifying assumption for σ^2 made above.

2b. By policy and by amount results for UL lapse and nonsmoker mortality

Out of the 10 companies, only 8 companies have Universal Life policies.

UL Plan Lapse Results by Policy for Limited Fluctuation					
Overall A/E Ratio by Policy	Company	Company A/E Ratio by Policy	Z	Number of Lapses	Limited Fluctuation A/E Ratio by Policy
120.0%	A	131.0%	1.000	13,472	131.0%
120.0%	B	96.0%	1.000	1,079	96.0%
120.0%	C	117.2%	1.000	3,995	117.2%
120.0%	D	92.7%	1.000	1,879	92.7%
120.0%	F	96.4%	1.000	1,815	96.4%
120.0%	G	106.8%	0.051	3	119.3%
120.0%	H	123.5%	1.000	19,993	123.5%
120.0%	I	87.8%	1.000	949	87.8%

UL Plan Lapse Results by Amount for Limited Fluctuation					
Overall A/E Ratio by Amount	Company	Company A/E Ratio by Amount	Z	Number of Lapses	Limited Fluctuation A/E Ratio by Amount
129.4%	A	135.8%	1.000	13,472	135.8%
129.4%	B	60.4%	0.576	1,079	89.7%
129.4%	C	142.3%	0.843	3,995	140.3%
129.4%	D	89.2%	0.967	1,879	90.5%
129.4%	F	98.3%	0.699	1,815	107.7%
129.4%	G	208.0%	0.022	3	131.1%
129.4%	H	142.1%	0.950	19,993	141.5%
129.4%	I	83.8%	0.577	949	103.1%

Mortality Results by Policy for Limited Fluctuation					
Overall A/E Ratio by Policy	Company	Company A/E Ratio by Policy	Z	Number of Deaths	Limited Fluctuation A/E Ratio by Policy
83.8%	A	115.8%	0.972	1,430	114.9%
83.8%	B	125.6%	0.830	1,038	118.5%
83.8%	C	74.4%	0.664	668	77.6%
83.8%	D	87.6%	0.387	228	85.3%
83.8%	E	75.1%	1.000	13,409	75.1%
83.8%	F	88.7%	1.000	1,988	88.7%
83.8%	G	51.6%	0.044	3	82.4%
83.8%	H	85.9%	1.000	9,978	85.9%
83.8%	I	91.4%	1.000	3,609	91.4%
83.8%	J	101.6%	0.952	1,349	100.7%

Mortality Results by Amount for Limited Fluctuation					
Overall A/E Ratio by Amount	Company	Company A/E Ratio by Amount	Z	Number of Deaths	Limited Fluctuation A/E Ratio by Amount
77.0%	A	106.0%	0.708	1,430	97.5%
77.0%	B	118.5%	0.285	1,038	88.8%
77.0%	C	63.5%	0.254	668	73.6%
77.0%	D	89.2%	0.219	228	79.6%
77.0%	E	61.4%	1.000	13,409	61.4%
77.0%	F	71.6%	0.236	1,988	75.7%
77.0%	G	36.8%	0.020	3	76.2%
77.0%	H	81.2%	0.409	9,978	78.7%
77.0%	I	82.8%	0.833	3,609	81.8%
77.0%	J	97.9%	0.453	1,349	86.5%

Appendix A contains the Limited Fluctuation method predicted A/E ratios for nonsmoker mortality and lapse for universal life, term, permanent life, and variable universal Life plans. Appendix B contains the development of the Limited Fluctuation A/E ratio results.

3. Bühlmann Empirical Bayesian Method

The Limited Fluctuation method has the advantage of being simple and also of requiring only data from the company whose experience is being evaluated. However, with simplicity comes some drawbacks. First there is the arbitrary nature of the parameters *a*, *r*, and *p*. Second, there is no explicit model used or quantity optimized. Third, the accuracy of *a* is not factored into the calculation of *Z*.

The Bayesian approach rectifies many of these issues by beginning with a model for the probability distribution of observations. In the Bayesian model an initial or prior distribution is based on past data, professional experience, and/or opinion. Observed results are then used to formulate a predictive or posterior distribution.

The Bühlmann Empirical Bayesian method is based on a linear Bayesian model that relies only on the first two moments of the distribution. This method is empirical because the prior distribution's moments are based on past data. In the application used in this paper, the A/E ratios used are the nonsmoker 2001 VBT modified by the experience of ten companies from the 2004-2005 study. The observed data of both the company under study and all other companies are used in formulating the predictive A/E ratios for the company under study.

The Bühlmann Empirical Bayesian method will be presented though development of predicted A/E ratios by policy results for universal life lapses and nonsmoker mortality, and predicted A/E ratios by amount results for universal life lapses and nonsmoker mortality. Appendices C and E contain respectively predicted A/E ratios by policy and predicted A/E ratios by amount for nonsmokers for overall mortality and by lapse for term, permanent life, universal life and variable universal life policies. Appendices D and F describe the spreadsheet used to develop A/E ratios by policy and the A/E ratios by amount.

3a. Development of Bühlmann Empirical Bayesian Formulas

This form of credibility differs in that it starts with a model and then proceeds to a statistical estimation process for the unknown items in the model. To make this work, we need information from more than one organization. The data are now:

1. There are *r* companies, with index *h*.
2. For company *h* there are n_h observed lives with index *hi*.
3. For the *i*th life from company *h* there are three measurements:
   a. f_{hi} is the fraction of the year for which the life was observed.
   b. b_{hi} is the amount insured.
   c. d_{hi} is equal to 0 if the life did not die and is equal to 1 if it did die.
4. Also, for life *hi*, there is a true mortality rate q_{hi} and a standard table mortality rate q^s_{hi}.

For this development, the distinction between counts and amounts will not be made. The formulas will be developed for amounts. However, by setting all the *b* values to 1, the formulas for counts will result.

For each company, the following three quantities can be calculated:

$$A_h = \sum_{i=1}^{n_h} b_{hi} d_{hi}$$
$$E_h = \sum_{i=1}^{n_h} b_{hi} f_{hi} q_{hi}^s$$
$$\hat{m}_h = A_h / E_h$$

where *A* is the actual, *E* is the expected, and *m* is the mortality ratio.

The model is in two stages:

Stage 1: For a given company there is a true mortality ratio, m_h. Given this value, each d_{hi} is a random outcome where it takes on the value 1 with probability $m_h f_{hi} q_{hi}^s$ and the value 0 otherwise.

Stage 2: The mortality ratios are distributed among companies according a probability distribution. The distribution is not specified. However, $E(m_h) = \mu$ and $Var(m_h) = \sigma^2$.

Development of the Credibility Factor *Z*

The Bühlmann approach to credibility features two key components. The first is that the solution must take on a specific form. The estimator must be $Z\hat{m}_h + W$. The second component is that the two coefficients are selected to minimize the expected squared error with regard to the true ratio,

$$E[(m_h - Z\hat{m}_h - W)^2].$$

The solution will now be developed.

Taking the derivative with respect to *W* produces

$$0 = -2E[(m_h - Z\hat{m}_h - W)]$$
$$0 = E(m_h) - ZE(\hat{m}_h) - W$$
$$E(m_h) = \mu$$
$$E(\hat{m}_h) = \frac{1}{E_h} E\left[E\left(\sum_{i=1}^{n_h} b_{hi} d_{hi} \mid m_h \right) \right] = \frac{1}{E_h} E\left(\sum_{i=1}^{n_h} b_{hi} f_{hi} m_h q_{hi}^s \right) = \frac{1}{E_h} \mu E_h = \mu$$
$$0 = \mu - Z\mu - W$$
$$W = (1 - Z)\mu.$$

Now take the derivative with respect to *Z*.

$$0 = -2E\left[(m_h - Z\hat{m}_h - W)\hat{m}_h\right]$$

$$0 = E(m_h\hat{m}_h) - ZE(\hat{m}_h^2) - W\mu$$

$$E(m_h\hat{m}_h) = \frac{1}{E_h}E\left[m_h E\left(\sum_{i=1}^{n_h} b_{hi}d_{hi} \mid m_h\right)\right] = \frac{1}{E_h}E\left[m_h\sum_{i=1}^{n_h} b_{hi}f_{hi}m_h q_{hi}^s\right]$$

$$= \frac{1}{E_h}E(E_h m_h^2) = \sigma^2 + \mu^2$$

$$E(\hat{m}_h^2) = \frac{1}{E_h^2}E\left[E\left(\sum_{i=1}^{n_h}\sum_{j=1}^{n_h} b_{hi}b_{hj}d_{hi}d_{hj} \mid m_h\right)\right]$$

$$= \frac{1}{E_h^2}E\left[\sum_{i\neq j} b_{hi}b_{hj}f_{hi}m_h q_{hi}^s f_{hj}m_h q_{hj}^s + \sum_{i=1}^{n_h} b_{hi}^2 f_{hi}m_h q_{hi}^s\right]$$

$$= \frac{1}{E_h^2}\left[\sum_{i\neq j} b_{hi}b_{hj}f_{hi}q_{hi}^s f_{hj}q_{hj}^s(\sigma^2+\mu^2) + \sum_{i=1}^{n_h} b_{hi}^2 f_{hi}q_{hi}^s\mu\right]$$

$$= \frac{1}{E_h^2}\left[\sum_{i=1}^{n_h}\sum_{j=1}^{n_h} b_{hi}b_{hj}f_{hi}q_{hi}^s f_{hj}q_{hj}^s(\sigma^2+\mu^2) + \sum_{i=1}^{n_h}\left(b_{hi}^2 f_{hi}q_{hi}^s\mu - b_{hi}^2 f_{hi}^2\left\{q_{hi}^s\right\}^2(\mu^2+\sigma^2)\right)\right]$$

$$= \frac{1}{E_h^2}(\sigma^2+\mu^2)E_h^2 + \frac{\mu}{E_h^2}\sum_{i=1}^{n_h} b_{hi}^2 f_{hi}q_{hi}^s - \frac{\mu^2+\sigma^2}{E_h^2}\sum_{i=1}^{n_h} b_{hi}^2 f_{hi}^2\left(q_{hi}^s\right)^2.$$

For convenience, define

$$B_h = \sum_{i=1}^{n_h} b_{hi}^2 f_{hi}q_{hi}^s \text{ and } C_h = \sum_{i=1}^{n_h} b_{hi}^2 f_{hi}^2\left(q_{hi}^s\right)^2.$$

Then,

$$0 = \sigma^2 + \mu^2 - Z\left(\sigma^2 + \mu^2 + \frac{\mu}{E_h^2}B_h - \frac{\mu^2+\sigma^2}{E_h^2}C_h\right) - (1-Z)\mu^2$$

$$= \sigma^2 - Z\left(\sigma^2 + \frac{\mu}{E_h^2}B_h - \frac{\mu^2+\sigma^2}{E_h^2}C_h\right)$$

$$Z = \frac{\sigma^2}{\sigma^2 + \frac{\mu}{E_h^2}B_h - \frac{\mu^2+\sigma^2}{E_h^2}C_h} = \frac{E_h}{E_h + \frac{\mu}{\sigma^2 E_h}B_h - \frac{\mu^2+\sigma^2}{\sigma^2 E_h}C_h}.$$

Note that if counts are being used, then $B_h = E_h$ and $C_h = \sum_{i=1}^{n_h} f_{hi}^2\left(q_{hi}^s\right)^2$.

Development of Estimators of the Mean and Variance

The mean and variance must be estimated using data from more than one company.

Development of the estimator for the mean

$$\hat{\mu} = \frac{\sum_{h=1}^{r}\sum_{i=1}^{n_h} b_{hi} d_{hi}}{\sum_{h=1}^{r} E_h} = \frac{\sum_{h=1}^{r} A_h}{\sum_{h=1}^{r} E_h} = \frac{A}{T}$$

(where $T = \sum_{h=1}^{r} E_h$) which just the ratio of actual to expected, each totaled over all contributors.

Note that the estimator is unbiased:

$$\begin{aligned} E(\hat{\mu}) &= \frac{1}{T} E\left[E\left(\sum_{h=1}^{r}\sum_{i=1}^{n_h} b_{hi} d_{hi} \mid m_h \right) \right] \\ &= \frac{1}{T} E\left[\sum_{h=1}^{r}\sum_{i=1}^{n_h} b_{hi} f_{hi} q_{hi}^{s} m_h \right] \\ &= \frac{1}{T} \sum_{h=1}^{r}\sum_{i=1}^{n_h} b_{hi} f_{hi} q_{hi}^{s} \mu = \mu. \end{aligned}$$

Development of the estimator for the variance

Developing an estimator for the variance is more complicated. Rather than announcing the estimator, begin with something reasonable:

$$\begin{aligned} \sum_{h=1}^{r} E_h (\hat{m}_h - \hat{\mu})^2 &= \sum_{h=1}^{r} E_h \hat{m}_h^2 - 2\hat{\mu}\sum_{h=1}^{r} E_h \hat{m}_h + \sum_{h=1}^{r} E_h \hat{\mu}^2 \\ &= \sum_{h=1}^{r} E_h \hat{m}_h^2 - 2\hat{\mu} A + E\hat{\mu}^2 = \sum_{h=1}^{r} E_h \hat{m}_h^2 - T\hat{\mu}^2. \end{aligned}$$

The expected value of the first component is

$$\begin{aligned} E\left(\sum_{h=1}^{r} E_h \hat{m}_h^2 \right) &= \sum_{h=1}^{r} E_h \left[(\sigma^2 + \mu^2) + \frac{\mu}{E_h^2} B_h - \frac{\mu^2 + \sigma^2}{E_h^2} C_h \right] \\ &= T(\sigma^2 + \mu^2) + \mu \sum_{h=1}^{r} \frac{B_h}{E_h} - (\mu^2 + \sigma^2) \sum_{h=1}^{r} \frac{C_h}{E_h} \end{aligned}$$

And the second component is

$$\begin{aligned}
E(T\hat{\mu}^2) &= \frac{1}{T}E\left[\left(\sum_{h=1}^{r}E_h\hat{m}_h\right)^2\right] = \frac{1}{T}E\left(\sum_{g=1}^{r}\sum_{h=1}^{r}E_gE_h\hat{m}_g\hat{m}_h\right)\\
&= \frac{1}{T}E\left(\sum_{g\neq h}E_gE_h\hat{m}_g\hat{m}_h + \sum_{h=1}^{r}E_h^2\hat{m}_h^2\right)\\
&= \frac{1}{T}\left[\sum_{g\neq h}E_gE_h\mu^2 + \sum_{h=1}^{r}(\sigma^2+\mu^2)E_h^2 + \mu B_h - (\mu^2+\sigma^2)C_h\right]\\
&= \frac{1}{T}\left[T^2\mu^2 + \sum_{h=1}^{r}\left\{\sigma^2E_h^2 + \mu B_h - (\mu^2+\sigma^2)C_h\right\}\right].
\end{aligned}$$

Putting these together gives the expected value of the variance estimator:

$$\begin{aligned}
&T(\sigma^2+\mu^2) + \mu\sum_{h=1}^{r}\frac{B_h}{E_h} - (\mu^2+\sigma^2)\sum_{h=1}^{r}\frac{C_h}{E_h} - T\mu^2 - \frac{1}{T}\sum_{h=1}^{r}\left\{\sigma^2E_h^2 + \mu B_h - (\mu^2+\sigma^2)C_h\right\}\\
&= \sigma^2\left(T - \frac{1}{T}\sum_{h=1}^{r}E_h^2 - \sum_{h=1}^{r}\frac{C_h}{E_h} + \frac{1}{T}\sum_{h=1}^{r}C_h\right)\\
&+ \mu\left(\sum_{h=1}^{r}\frac{B_h}{E_h} - \frac{1}{T}\sum_{h=1}^{r}B_h\right) - \mu^2\left(\sum_{h=1}^{r}\frac{C_h}{E_h} - \frac{1}{T}\sum_{h=1}^{r}C_h\right).
\end{aligned}$$

Solving for the variance and substituting the estimator for the mean, gives a reasonable (though biased) estimator.

$$\hat{\sigma}^2 = \frac{\sum_{h=1}^{r}E_h(\hat{m}_h-\hat{\mu})^2 - \hat{\mu}\left(\sum_{h=1}^{r}\frac{B_h}{E_h} - \frac{1}{T}\sum_{h=1}^{r}B_h\right) + \hat{\mu}^2\left(\sum_{h=1}^{r}\frac{C_h}{E_h} - \frac{1}{T}\sum_{h=1}^{r}C_h\right)}{T - \frac{1}{T}\sum_{h=1}^{r}E_h^2 - \sum_{h=1}^{r}\frac{C_h}{E_h} + \frac{1}{T}\sum_{h=1}^{r}C_h}.$$

3b. By policy results for UL lapse and nonsmoker mortality

Out of the 10 companies, only 8 companies have UL policies.

UL Lapse by Policy - Bühlmann Empirical Bayesian Method					
Company	Overall A/E Ratio by Policy	Z	Company A/E Ratio by Policy	Number Of Lapses	Bühlmann estimates A/E Ratio by Policy
A	120.0%	0.995	131.0%	13,472	131.0%
B	120.0%	0.957	96.0%	1,079	97.1%
C	120.0%	0.985	117.2%	3,995	117.3%
D	120.0%	0.976	92.7%	1,879	93.4%
F	120.0%	0.974	96.4%	1,815	97.0%
G	120.0%	0.052	106.8%	3	119.3%
H	120.0%	0.997	123.5%	19,993	123.4%
I	120.0%	0.955	87.8%	949	89.2%

Mortality by Policy - Bühlmann Empirical Bayesian Method					
Company	Overall A/E Ratio by Policy	Z	Company A/E Ratio by Policy	Number Of Deaths	Bühlmann estimates A/E Ratio by Policy
A	83.8%	0.962	115.8%	1,430	114.6%
B	83.8%	0.945	125.6%	1,038	123.3%
C	83.8%	0.949	74.4%	668	74.9%
D	83.8%	0.843	87.6%	228	87.0%
E	83.8%	0.997	75.1%	13,409	75.1%
F	83.8%	0.979	88.7%	1,988	88.6%
G	83.8%	0.106	51.6%	3	80.4%
H	83.8%	0.996	85.9%	9,978	85.9%
I	83.8%	0.988	91.4%	3,609	91.3%
J	83.8%	0.965	101.6%	1,349	101.0%

Appendix C contains Bühlmann Empirical Bayesian method predicted A/E ratios by policy for nonsmoker mortality and lapse for universal life, term, permanent life, and variable universal life plans. Appendix D contains the development of the Bühlmann Empirical Bayesian predicted A/E ratios by policy results.

3c. By amount results for UL lapse and nonsmoker mortality

Out of the 10 companies, only 8 companies have Universal Life policies.

UL Lapse by Amount - Bühlmann Empirical Bayesian Method					
Company	Overall A/E Ratio by Amount	Z	Company A/E Ratio by Amount	Number Of Lapses	Bühlmann estimates A/E Ratio by Amount
A	129.4%	0.997	135.8%	13,472	135.8%
B	129.4%	0.919	60.4%	1,079	66.0%
C	129.4%	0.982	142.3%	3,995	142.1%
D	129.4%	0.979	89.2%	1,879	90.0%
F	129.4%	0.964	98.3%	1,815	99.4%
G	129.4%	0.051	208.0%	3	133.4%
H	129.4%	0.986	142.1%	19,993	141.9%
I	129.4%	0.940	83.8%	949	86.5%

Mortality by Amount - Bühlmann Empirical Bayesian Method					
Company	Overall A/E Ratio by Amount	Z	Company A/E Ratio by Amount	Number Of Deaths	Bühlmann estimates A/E Ratio by Amount
A	77.0%	0.935	106.0%	1,430	104.1%
B	77.0%	0.678	118.5%	1,038	105.1%
C	77.0%	0.757	63.5%	668	66.8%
D	77.0%	0.623	89.2%	228	84.6%
E	77.0%	0.986	61.4%	13,409	61.6%
F	77.0%	0.704	71.6%	1,988	73.2%
G	77.0%	0.033	36.8%	3	75.6%
H	77.0%	0.863	81.2%	9,978	80.6%
I	77.0%	0.963	82.8%	3,609	82.6%
J	77.0%	0.865	97.9%	1,349	95.1%

Appendix E contains Bühlmann Empirical Bayesian method predicted A/E ratios by amount for nonsmoker mortality and lapse for universal life, term, permanent life, and variable universal life plans. Appendix F contains the development of the Bühlmann Empirical Bayesian predicted A/E ratios by amount results.

4. Summary of Results

4a. Variation of A/E ratios by company

In SOA experience studies, the results of individual companies are not presented to protect company confidentiality. As noted earlier, this project uses a portion of the experience of ten companies that contributed mortality and lapse experience to the SOA 2004-05 experience study. Only a portion of each company's data was used in order to prevent identifying any company. The ten companies were selected to give a mixture of larger, medium and smaller size companies. For this paper, the base estimates are applied to the experience of these ten companies to produce the overall A/E ratios as well as the company A/E ratios.

For this study of ten companies' experience, the nonsmoker A/E ratios for each company and overall are:

Company	NS Mortality A/E Ratio by Amount	Number Of Deaths
A	**106.0%**	**1,430**
B	**118.5%**	**1,038**
C	**63.5%**	**668**
D	**89.2%**	**228**
E	**61.4%**	**13,409**
F	**71.6%**	**1,988**
G	**36.8%**	**3**
H	**81.2%**	**9,978**
I	**82.8%**	**3,609**
J	**97.9%**	**1,349**
Overall	**77.0%**	**33,700**

Some actuaries may not be familiar with the dispersion of individual company A/E ratios around the overall A/E ratio. Actuaries working with individual company contributions to studies commonly find that the highest company A/E ratio is about twice the lowest company A/E ratio. Ignoring the small company with only three deaths, the results for the ten companies are consistent with this finding. The authors of this paper consider this study to be a reasonable experience basis on which to evaluate the Limited Fluctuation method and the Bühlmann Empirical Bayesian method.

When reviewing the results of mortality experience studies, actuaries customarily look at the actual to expected (A/E) ratio and look at the number of deaths as an indicator of reliability of the A/E ratio. For the ten company A/E ratio, the overall nonsmoker 2001 VBT is 77.0%. Each company has a corresponding nonsmoker A/E ratio. With Company E, the 13,409 deaths would give actuaries credence in the corresponding 61.4% A/E ratio. With Company G, the 3 deaths would lead actuaries to discount the 36.8% A/E ratio. Although the use of the number of deaths to give credence to A/E ratios is useful as an overall indicator of comfort with a specific company result, the credibility methods help to quantify the degree of confidence in specific company results versus the industry as a whole.

4b. Overall Variability and *Z* factors

In both the Limited Fluctuation and the Bühlmann Empirical Bayesian methods, the results are calculated with respect to a mean (A/E ratio) and incorporate a variance. The methods differ in the treatment of the components of the variance (σ^2). The total variance of the observations is the sum over all companies of two different sources of variation, which are:

1. For each company, the variation of a company's observations about that company's mean.
2. The variation between each company's mean and the overall mean.

Limited Fluctuation credibility uses only the first source while the Bühlmann Empirical Bayesian method uses both. Thus, Limited Fluctuation credibility requires only data from the company being studied. For the Bühlmann Empirical Bayesian approach, data is needed for all companies under study.

In either case, a credibility factor Z is determined. The credibility factor determines the result using the following formula:

Estimated A/E Ratio = Z × (Company A/E Ratio) + (1-Z) × (Overall A/E Ratio)

Both approaches assume that the mean (overall A/E ratio) is constant over time. Many actuaries would accept this assumption for both mortality and lapse over a one year period. For a five year period, many actuaries would view the results of a mortality study to be representative for the middle of the five year period. Since lapse rates are known to vary with factors such as economic conditions as well as industry events including introduction of new products or changes in the regulatory environment, many actuaries would limit the assumption of lapse rates constant over time for shorter periods of time than five years.

4c. Comparison of results

The results are a function of the data and the method. The mortality and lapse data of policy years ending in 2005 was from ten companies selected to give a mixture of large, medium and small size companies. Although useful in presenting results, we cannot say that this data is totally representative of mortality and lapse data. Therefore, we cannot produce a systemic relationship between the two methods based on this data. However, we can observe how the Limited Fluctuation and the Bühlmann Empirical Bayesian method are similar or different.

i. By Policy

For all the 10 companies, the differences in the A/E ratios by policy of the Limited Fluctuation and the Bühlmann Empirical Bayesian methods differ by less than 5%.

Nonsmoker A/E Ratios by Policy		
Company	Limited Fluctuation	Bühlmann
A	**114.9%**	**114.6%**
B	**118.5%**	**123.3%**
C	**77.6%**	**74.9%**
D	**85.3%**	**87.0%**
E	**75.1%**	**75.1%**
F	**88.7%**	**88.6%**
G	**82.4%**	**80.4%**
H	**85.9%**	**85.9%**
I	**91.4%**	**91.3%**
J	**100.7%**	**101.0%**

ii. By Amount

For 5 of the 10 companies, the differences in the A/E ratios by amount of the Limited Fluctuation and the Bühlmann Empirical Bayesian methods differ by at least 5%.

Nonsmoker A/E Ratios by Amount		
Company	Limited Fluctuation	Bühlmann
A	**97.5%**	**104.1%**
B	**88.8%**	**105.1%**
C	**73.6%**	**66.8%**
D	**79.6%**	**84.6%**
E	**61.4%**	**61.6%**
F	**75.7%**	**73.2%**
G	**76.2%**	**75.6%**
H	**78.7%**	**80.6%**
I	**81.8%**	**82.6%**
J	**86.5%**	**95.1%**

5. Conclusion

This paper evaluated the Limited Fluctuation method and the Bühlmann Empirical Bayesian method by applying them to a portion of the experience of 10 companies that contributed mortality and lapse experience to the SOA 2004-05 experience study. The ten companies were selected to give a mixture of large, medium and small size companies.

For both the Limited Fluctuation and the Bühlmann Empirical Bayesian methods, the paper showed the development of the formulas and the appendices show how the calculations are applied. The paper and appendices show results of both methods for nonsmoker mortality and lapse for universal life, term, permanent life, and variable universal life plans.

The results of the data applied to the methods were compared. For all the 10 companies, the differences in the A/E ratios by policy for the Limited Fluctuation and the Bühlmann Empirical Bayesian methods differ by less than 5%. For 5 of the 10 companies, the differences in the A/E ratios by amount of the Limited Fluctuation and the Bühlmann Empirical Bayesian methods differ by at least 5%.

A difference in the use of total variance between the methods affects the data required. The total variance of the observations is the sum over all companies of two different sources of variation, which are:

1. For each company, the variation of a company's observations about that company's mean.
2. The variation between each company's mean and the overall mean.

Appendix A:

Limited Fluctuation Method Results for Lapse and Nonsmoker Mortality

Lapse - Universal Life

Out of the 10 companies, only 8 companies have Universal Life policies.

UL Plan Lapse Results by Policy for Limited Fluctuation					
Overall A/E Ratio by Policy	Company	Company A/E Ratio by Policy	Z	Number of Lapses	Limited Fluctuation A/E Ratio by Policy
120.0%	A	131.0%	1.000	13,472	131.0%
120.0%	B	96.0%	1.000	1,079	96.0%
120.0%	C	117.2%	1.000	3,995	117.2%
120.0%	D	92.7%	1.000	1,879	92.7%
120.0%	F	96.4%	1.000	1,815	96.4%
120.0%	G	106.8%	0.051	3	119.3%
120.0%	H	123.5%	1.000	19,993	123.5%
120.0%	I	87.8%	1.000	949	87.8%

Appendix A (Continued):

UL Plan Lapse Results by Amount for Limited Fluctuation					
Overall A/E Ratio by Amount	Company	Company A/E Ratio by Amount	Z	Number of Lapses	Limited Fluctuation A/E Ratio by Amount
129.4%	A	135.8%	1.000	13,472	135.8%
129.4%	B	60.4%	0.576	1,079	89.7%
129.4%	C	142.3%	0.843	3,995	140.3%
129.4%	D	89.2%	0.967	1,879	90.5%
129.4%	F	98.3%	0.699	1,815	107.7%
129.4%	G	208.0%	0.022	3	131.1%
129.4%	H	142.1%	0.950	19,993	141.5%
129.4%	I	83.8%	0.577	949	103.1%

Appendix A (Continued):

Lapse - Term

Out of the 10 companies, only 9 companies have Term policies. Note that Term includes both YRT and level premium term that have different overall lapse experience.

Term Plan Lapse Results by Policy for Limited Fluctuation					
Overall A/E Ratio by Policy	Company	Company A/E Ratio by Policy	Z	Number of Lapses	Limited Fluctuation A/E Ratio by Policy
106.3%	B	86.2%	1.000	12,245	86.2%
106.3%	C	120.9%	1.000	3,712	120.9%
106.3%	D	95.1%	1.000	4,670	95.1%
106.3%	E	83.8%	1.000	11,592	83.8%
106.3%	F	93.8%	1.000	6,194	93.8%
106.3%	G	99.1%	0.457	269	103.0%
106.3%	H	143.1%	1.000	33,507	143.1%
106.3%	I	111.7%	1.000	23,738	111.7%
106.3%	J	57.4%	1.000	3,357	57.4%

Appendix A (Continued):

Term Plan Lapse Results by Amount for Limited Fluctuation					
Overall A/E Ratio by Amount	Company	Company A/E Ratio by Amount	Z	Number of Lapses	Limited Fluctuation A/E Ratio by Amount
114.0%	B	121.6%	1.000	12,245	121.6%
114.0%	C	117.6%	0.935	3,712	117.3%
114.0%	D	84.1%	1.000	4,670	84.1%
114.0%	E	89.9%	1.000	11,592	89.9%
114.0%	F	93.1%	1.000	6,194	93.1%
114.0%	G	75.6%	0.320	269	101.7%
114.0%	H	139.4%	1.000	33,507	139.4%
114.0%	I	113.4%	1.000	23,738	113.4%
114.0%	J	76.5%	1.000	3,357	76.5%

Appendix A (Continued):

Lapse – Permanent Life

Out of the 10 companies, only 6 companies have Permanent Life policies.

Permanent Life Plan Lapse Results by Policy for Limited Fluctuation					
Overall A/E Ratio by Policy	Company	Company A/E Ratio by Policy	Z	Number of Lapses	Limited Fluctuation A/E Ratio by Policy
84.7%	C	103.5%	0.294	193	90.2%
84.7%	E	67.8%	1.000	19,292	67.8%
84.7%	F	83.6%	1.000	2,654	83.6%
84.7%	H	124.1%	1.000	9,335	124.1%
84.7%	I	115.1%	1.000	9,179	115.1%
84.7%	J	76.4%	1.000	5,318	76.4%

Appendix A (Continued):

Permanent Life Plan Lapse Results by Amount for Limited Fluctuation					
Overall A/E Ratio by Amount	Company	Company A/E Ratio by Amount	Z	Number of Lapses	Limited Fluctuation A/E Ratio by Amount
103.1%	C	86.3%	0.105	193	101.3%
103.1%	E	74.6%	1.000	19,292	74.6%
103.1%	F	100.0%	0.498	2,654	101.6%
103.1%	H	165.0%	0.585	9,335	139.3%
103.1%	I	118.6%	1.000	9,179	118.6%
103.1%	J	91.6%	1.000	5,318	91.6%

Appendix A (Continued):

Lapse – Variable Universal Life

Out of the 10 companies, only 5 companies have Variable Universal Life policies.

VUL Plan Lapse Results by Policy for Limited Fluctuation					
Overall A/E Ratio by Policy	Company	Company A/E Ratio by Policy	Z	Number of Lapses	Limited Fluctuation A/E Ratio by Policy
95.6%	B	109.7%	0.378	273	100.9%
95.6%	C	88.9%	1.000	4,732	88.9%
95.6%	F	89.9%	0.809	853	91.0%
95.6%	H	101.1%	1.000	6,831	101.1%
95.6%	I	66.4%	0.064	3	93.7%

VUL Plan Lapse Results by Amount for Limited Fluctuation					
Overall A/E Ratio by Amount	Company	Company A/E Ratio by Amount	Z	Number of Lapses	Limited Fluctuation A/E Ratio by Amount
94.0%	B	95.7%	0.290	273	94.5%
94.0%	C	90.9%	0.936	4,732	91.1%
94.0%	F	80.2%	0.472	853	87.5%
94.0%	H	98.3%	0.706	6,831	97.0%
94.0%	I	110.2%	0.025	3	94.4%

Appendix A (Continued):

Nonsmoker Mortality Results

Mortality Results by Policy for Limited Fluctuation					
Overall A/E Ratio by Policy	Company	Company A/E Ratio by Policy	Z	Number of Deaths	Limited Fluctuation A/E Ratio by Policy
83.8%	A	115.8%	0.972	1,430	114.9%
83.8%	B	125.6%	0.830	1,038	118.5%
83.8%	C	74.4%	0.664	668	77.6%
83.8%	D	87.6%	0.387	228	85.3%
83.8%	E	75.1%	1.000	13,409	75.1%
83.8%	F	88.7%	1.000	1,988	88.7%
83.8%	G	51.6%	0.044	3	82.4%
83.8%	H	85.9%	1.000	9,978	85.9%
83.8%	I	91.4%	1.000	3,609	91.4%
83.8%	J	101.6%	0.952	1,349	100.7%

Appendix A (Continued):

Mortality Results by Amount for Limited Fluctuation					
Overall A/E Ratio by Amount	Company	Company A/E Ratio by Amount	Z	Number of Deaths	Limited Fluctuation A/E Ratio by Amount
77.0%	A	106.0%	0.708	1,430	97.5%
77.0%	B	118.5%	0.285	1,038	88.8%
77.0%	C	63.5%	0.254	668	73.6%
77.0%	D	89.2%	0.219	228	79.6%
77.0%	E	61.4%	1.000	13,409	61.4%
77.0%	F	71.6%	0.236	1,988	75.7%
77.0%	G	36.8%	0.020	3	76.2%
77.0%	H	81.2%	0.409	9,978	78.7%
77.0%	I	82.8%	0.833	3,609	81.8%
77.0%	J	97.9%	0.453	1,349	86.5%

Appendix B:

Development of Limited Fluctuation Method Results

The sheet 'UL - Ltd Fluctuation' in the file 'Limited Fluctuation Lapse.xls' contains the development of lapse results for the Limited Fluctuation method. The values in the columns come from Access tables.

In the first section of the sheet 'UL - Ltd Fluctuation' contains:

h	bh	Awhc	Awhd	Wfh	Ewhc	Ewhd	wmhc	wmhd

In each row, the column is:

- 'h' is a company code of an individual company
- 'bh' or $\sum_{i=1}^{n} b_{hi}$ is the sum of the amount insured for company h ,
- 'Awhc' is the sum of the number of contracts lapsed for company h,
- 'Awhd' is the sum of the amount of contracts lapsed for company h,
- 'Wfh' is the sum of policies exposed to lapse for company h,
- 'Ewhc' or E_{whc} is the expected contracts lapsed for company h,
- 'Ewhd' or E_{whd} is the expected amount of policies lapsed for company h,
- 'wmhc' or m_{whc} is the company A/E lapse ratio by count, and
- 'wmhd' or m_{whd} is the company A/E lapse ratio by amount

Appendix B (Continued):

The second section of the sheet 'UL - Ltd Fluctuation' contains:

			Ltd Fluc Variance by Policy			Ltd Fluc Variance by Amount
H	LFvarwqcnum	Lfvarwqcden		LFvarwqdnum	Lfvarwqdden	

In each row, the column is:

- 'h' is a company code of an individual company
- 'LFvarwqcnum' is the numerator of the variance by policy, that is $\sum_{i=1}^{n} m_{whc} wf_{hi} wq_{hi}$.
- 'LFvarwqcden' is the denominator of the variance by policy which is Ewhc squared or E_{whc}^2
- 'Ltd Fluc Variance by Policy' is 'LFvarwqcnum' /'LFvarwqcden' which equals

$$\frac{\sum_{i=1}^{n} m_{whc} wf_{hi} wq_{hi}}{E_{whc}^2}$$

This corresponds to the formula in the section on development of limited fluctuation formulas:

$$\sigma_c^2 = \frac{\sum_{i=1}^{n} f_i m_c q_i^s (1 - f_i m_c q_i^s)}{E_c^2}$$

- 'LFvarwqdnum' is the numerator of the variance by amount that is $\sum_{i=1}^{n} m_{wfd} b_{hi}^2 wf_{hi} wq_{hi}^s (1 - m_{whd} wf_{hi} wq_{hi}^s)$
- 'LFvarwqcden' is the denominator of the variance by amount which is 'Ewhd' squared or E_{whd}^2
- 'Ltd Fluc Variance by Policy' is 'LFvarwqdnum'/'LFvarwqcden' which equals

Appendix B (Continued):

$$\frac{\sum_{i=1}^{n} m_{wfd} b_{hi}^2 wf_{hi} wq_{hi}^s (1 - m_{whd} wf_{hi} wq_{hi}^s)}{E_{whd}^2}$$

This corresponds to the formula in the section on development of limited fluctuation formulas:

$$\sigma_d^2 = \frac{\sum_{i=1}^{n} b_i^2 f_i m_d q_i^s (1 - f_i m_d q_i^s)}{E_d^2}$$

The third section of the sheet ‘Lapse – Limited Fluctuation’ contains Z for policies lapsed

$$Z = \min\left\{1, \frac{rm_{whc}}{z\sigma_{hc}}\right\}$$

The fourth section of the sheet ‘Lapse – Limited Fluctuation’ contains Z for amount lapsed

$$Z = \min\left\{1, \frac{rm_{whd}}{z\sigma_{hd}}\right\}$$

These Z formulas are consistent with the formula in the section on development of limited fluctuation formulas:

$$Z = \min\left\{1, \frac{r\hat{m}}{z\hat{\sigma}}\right\}$$

The spreadsheet uses r = .05 and the corresponding z of 1.96.

The development on the sheet s ‘Term - Limited Fluctuation’, ‘Perm Plan Ltd Fluctuation ‘, ‘VUL Ltd Fluctuation ‘ of the file ‘Limited Fluctuation Lapse.xls’ uses the same development on the sheet ‘UL - Ltd Fluctuation’. Additionally, the development on the sheet ‘Mortality – Limited Fluctuation’ of the file ‘Limited Fluctuation_Mortality.xls parallels that with the development on the sheet ‘UL - Ltd Fluctuation’

Appendix C:

Bühlmann Empirical Bayesian Method by Policy Results

Lapse - Universal Life

Out of the 10 companies, only 8 companies have Universal Life policies.

UL Lapse by Policy - Bühlmann empirical Bayesian Method					
Company	Overall A/E Ratio by Policy	Z	Company A/E Ratio by Policy	Number Of Lapses	Bühlmann estimates A/E Ratio by Policy
A	120.0%	0.995	131.0%	13,472	131.0%
B	120.0%	0.957	96.0%	1,079	97.1%
C	120.0%	0.985	117.2%	3,995	117.3%
D	120.0%	0.976	92.7%	1,879	93.4%
F	120.0%	0.974	96.4%	1,815	97.0%
G	120.0%	0.052	106.8%	f3	119.3%
H	120.0%	0.997	123.5%	19,993	123.4%
I	120.0%	0.955	87.8%	949	89.2%

Appendix C (Continued):

Lapse - Term

Out of the 10 companies, only 9 companies have Term policies. Note that Term includes both YRT and level premium term that have different overall lapse experience.

Term Lapse by Policy - Bühlmann empirical Bayesian Method					
Company	Overall A/E Ratio by Policy	Z	Company A/E Ratio by Policy	Number Of Lapses	Bühlmann estimates A/E Ratio by Policy
B	106.3%	0.999	86.2%	12,245	86.2%
C	106.3%	0.996	120.9%	3,712	120.8%
D	106.3%	0.997	95.1%	4,670	95.1%
E	106.3%	0.999	83.8%	11,592	83.8%
F	106.3%	0.998	93.8%	6,194	93.8%
G	106.3%	0.955	99.1%	269	99.5%
H	106.3%	0.999	143.1%	33,507	143.0%
I	106.3%	0.999	111.7%	23,738	111.7%
J	106.3%	0.998	57.4%	3,357	57.5%

Appendix C (Continued):

Lapse – Permanent Life

Out of the 10 companies, only 6 companies have Permanent Life policies.

Permanent Life Lapse by Policy - Bühlmann empirical Bayesian Method					
Company	Overall A/E Ratio by Policy	Z	Company A/E Ratio by Policy	Number Of Lapses	Bühlmann estimates A/E Ratio by Policy
C	84.7%	0.997	103.5%	193	103.4%
E	84.7%	1.000	67.8%	19,292	67.8%
F	84.7%	1.000	83.6%	2,654	83.6%
H	84.7%	1.000	124.1%	9,335	124.1%
I	84.7%	1.000	115.1%	9,179	115.1%
J	84.7%	1.000	76.4%	5,318	76.4%

Appendix C (Continued)**:**

Lapse – Variable Universal Life

Out of the 10 companies, only 5 companies have Variable Universal Life policies.

VUL Lapse by Policy - Bühlmann empirical Bayesian Method					
Company	Overall A/E Ratio by Policy	Z	Company A/E Ratio by Policy	Number Of Lapses	Bühlmann estimates A/E Ratio by Policy
B	95.6%	0.635	109.7%	273	104.5%
C	95.6%	0.974	88.9%	4,732	89.1%
F	95.6%	0.869	89.9%	853	90.7%
H	95.6%	0.979	101.1%	6,831	101.0%
I	95.6%	0.030	66.4%	3	94.7%

Appendix C (Continued):

Nonsmoker Mortality

Mortality by Policy - Bühlmann empirical Bayesian Method					
Company	Overall A/E Ratio by Policy	Z	Company A/E Ratio by Policy	Number Of Deaths	Bühlmann estimates A/E Ratio by Policy
A	83.8%	0.962	115.8%	1,430	114.6%
B	83.8%	0.945	125.6%	1,038	123.3%
C	83.8%	0.949	74.4%	668	74.9%
D	83.8%	0.843	87.6%	228	87.0%
E	83.8%	0.997	75.1%	13,409	75.1%
F	83.8%	0.979	88.7%	1,988	88.6%
G	83.8%	0.106	51.6%	3	80.4%
H	83.8%	0.996	85.9%	9,978	85.9%
I	83.8%	0.988	91.4%	3,609	91.3%
J	83.8%	0.965	101.6%	1,349	101.0%

Appendix D:

Development of Bühlmann Empirical Bayesian A/E by Policy Results

The sheet 'UL Buhlmann by Policy' in the file 'Buhlmann_Lapse.xls' contains the development of lapse results for the Bühlmann empirical Bayesian method. The values in the columns come from Access tables.

In the first section of the sheet 'UL Buhlmann by Policy' contains:

h	Awhc	Wfh	Ewhc	Ewhc^2	wmhc	Ewhc(wmhc-uwc)^2	Bhc	Chc

In each row, the column is:

- 'h' is a company code of an individual company,
- 'Awhc' is the sum of the number of contracts lapsed for company h,
- 'Wfh' is the sum of policies exposed to lapse for company h,
- 'Ewhc' or E_{whc} is the sum of the expected contracts lapsed for company h,
- 'Ewhc^2' is the square of the sum of expected contracts lapsed for company h,
- 'wmhc' or m_{whc} is the company A/E lapse ratio by policy,
- 'Ewhc(wmhc-uwc)^2' is the product of the expected contracts lapsed for company h and the square of the difference between the company A/E lapse ratio by policy and the overall A/E lapse ratio by policy,
- 'Bhc' is identical to the sum of the expected contracts lapsed for company h, and
- 'Chc' is the sum of the square of the expected contracts lapsed for each policy of company h.

Appendix D (Continued):

In the second section of the sheet ‘UL Buhlmann by Policy’ contains:

h	Bhc/Ewhc	Chc/Ewhc	c_Sigma^2 Num1	c_Sigma^2 Num2	c_Sigma^2 Num3	c_Sigma^2 Den	c_Sigma^2

In each row, the column is:

- ‘h’ is a company code of an individual company,
- ‘Bhc/Ewhc’ is 1 because Bhc and Ewhc are identical,
- ‘Chc/Ewhc’ is $\sum_{i=1}^{n_h} f_{hci}^2 (q_{hci}^s)^2$ divided by the expected amount of policies lapsed for company h,
- ‘c_Sigma^2 Num1’ is $\sum_{h=1}^{8} E_{whc} (wmhc - \mu wc)^2$,
- ‘c_Sigma^2 Num2’ is - $\mu_{wc} \left(\sum_{h=1}^{8} \frac{B_{hc}}{E_{whc}} - \frac{\sum_{h=1}^{8} B_{hc}}{\sum_{h=1}^{8} E_{whc}} \right)$,
- ‘c_Sigma^2 Num3’ is $\mu_{wc}^2 \left(\sum_{h=1}^{8} \frac{C_{hc}}{E_{whc}} - \frac{\sum_{h=1}^{8} C_{hc}}{\sum_{h=1}^{8} E_{whc}} \right)$,
- ‘c_Sigma^2 Den’ is $\sum_{h=1}^{8} E_{whc} - \frac{\sum_{h=1}^{8} E_{whc}^2}{\sum_{h=1}^{8} E_{whc}} - \sum_{h=1}^{8} \frac{C_{hc}}{E_{whc}} + \frac{\sum_{h=1}^{8} C_{hc}}{\sum_{h=1}^{8} E_{whc}}$, and
- ‘c_Sigm^2’ or σ^2 is (‘c_Sigma^1 Num1’ + ‘c_Sigma^2 Num2’ + ‘c_Sigma^2 Num3’)/ ‘c_Sigma^2 Den’.

Appendix D (Continued):

The third section of the sheet ‘UL Buhlmann by Policy’ contains:

‘Z’ which is $E_{whc} / (E_{whc} + \mu_{wc} B_{hc} \ / (\sigma^2 E_{whc}) - (\mu^2 + \sigma^2) C_{hc} / (\sigma^2 E_{whc}))$

This corresponds to the development of the credibility factor Z:

$$Z = \frac{E_h}{E_h + \frac{\mu}{\sigma^2 E_h} B_h - \frac{\mu^2 + \sigma^2}{\sigma^2 E_h} C_h}$$

The development on the sheets ‘Term Buhlmann by Policy’, ‘Perm Life Buhlmann by Policy ‘, ‘VUL Buhlmann by Policy’ of the file ‘Buhlmann_Lapse.xls’ uses the same development on the sheet ‘UL Buhlmann by Policy Additionally, the development on the sheet ‘Buhlmann by Count’ of the file ‘Buhlmann_Mortality.xls’ parallels that with the development on the sheet ‘UL Buhlmann by Policy’.

Appendix E:

Bühlmann Empirical Bayesian Method by Amount Results

Lapse - Universal Life

Out of the 10 companies, only 8 companies have Universal Life policies.

UL Lapse by Amount - Bühlmann empirical Bayesian Method					
Company	Overall A/E Ratio by Amount	Z	Company A/E Ratio by Amount	Number Of Lapses	Bühlmann estimates A/E Ratio by Amount
A	129.4%	0.997	135.8%	13,472	135.8%
B	129.4%	0.919	60.4%	1,079	66.0%
C	129.4%	0.982	142.3%	3,995	142.1%
D	129.4%	0.979	89.2%	1,879	90.0%
F	129.4%	0.964	98.3%	1,815	99.4%
G	129.4%	0.051	208.0%	3	133.4%
H	129.4%	0.986	142.1%	19,993	141.9%
I	129.4%	0.940	83.8%	949	86.5%

Appendix E (Continued):

Lapse - Term

Out of the 10 companies, only 9 companies have Term policies. Note that Term includes both YRT and level premium term that have different overall lapse experience.

Term Lapse by Amount - Bühlmann empirical Bayesian Method					
Company	Overall A/E Ratio by Amount	Z	Company A/E Ratio by Amount	Number Of Lapses	Bühlmann estimates A/E Ratio by Amount
B	114.0%	0.995	121.6%	12,245	121.5%
C	114.0%	0.982	117.6%	3,712	117.5%
D	114.0%	0.987	84.1%	4,670	84.4%
E	114.0%	0.996	89.9%	11,592	90.0%
F	114.0%	0.989	93.1%	6,194	93.3%
G	114.0%	0.806	75.6%	269	83.0%
H	114.0%	0.993	139.4%	33,507	139.2%
I	114.0%	0.998	113.4%	23,738	113.4%
J	114.0%	0.987	76.5%	3,357	77.0%

Appendix E (Continued):

Lapse – Permanent Life

Out of the 10 companies, only 6 companies have Permanent Life policies.

Permanent Life Lapse by Amount - Bühlmann empirical Bayesian Method					
Company	Overall A/E Ratio by Amount	Z	Company A/E Ratio by Amount	Number Of Lapses	Bühlmann estimates A/E Ratio by Amount
C	103.1%	0.494	86.3%	193	94.8%
E	103.1%	0.996	74.6%	19,292	74.7%
F	103.1%	0.962	100.0%	2,654	100.2%
H	103.1%	0.983	165.0%	9,335	163.9%
I	103.1%	0.995	118.6%	9,179	118.5%
J	103.1%	0.995	91.6%	5,318	91.6%

Appendix E (Continued):

Lapse – Variable Universal Life

Out of the 10 companies, only 5 companies have Variable Universal Life policies.

VUL Lapse by Amount - Bühlmann empirical Bayesian Method					
Company	Overall A/E Ratio by Amount	Z	Company A/E Ratio by Amount	Number Of Lapses	Bühlmann estimates A/E Ratio by Amount
B	94.0%	0.272	95.7%	273	94.5%
C	94.0%	0.787	90.9%	4,732	91.5%
F	94.0%	0.454	80.2%	853	87.8%
H	94.0%	0.694	98.3%	6,831	97.0%
I	94.0%	0.003	110.2%	3	94.1%

Appendix E (Continued):

Nonsmoker Mortality

Mortality by Amount - Bühlmann empirical Bayesian Method					
Company	Overall A/E Ratio by Amount	Z	Company A/E Ratio by Amount	Number Of Deaths	Bühlmann estimates A/E Ratio by Amount
A	77.0%	0.935	106.0%	1,430	104.1%
B	77.0%	0.678	118.5%	1,038	105.1%
C	77.0%	0.757	63.5%	668	66.8%
D	77.0%	0.623	89.2%	228	84.6%
E	77.0%	0.986	61.4%	13,409	61.6%
F	77.0%	0.704	71.6%	1,988	73.2%
G	77.0%	0.033	36.8%	3	75.6%
H	77.0%	0.863	81.2%	9,978	80.6%
I	77.0%	0.963	82.8%	3,609	82.6%
J	77.0%	0.865	97.9%	1,349	95.1%

Appendix F:

Development of Bühlmann Empirical Bayesian A/E by Amount Results

The sheet 'UL Buhlmann by Amount' in the file 'Buhlmann_Lapse.xls' contains the development of amount lapse results for the Bühlmann empirical Bayesian method. The values in the columns come from Access tables.

In the first section of the sheet 'UL Buhlmann by Amount' contains:

h	bh	Awhd	Wfh	Ewhd	Ewhd^2	wmhd	Bhd	Chd

In each row, the column is:

- 'h' is a company code of an individual company,
- 'bh' or $\sum_{i=1}^{n_h} b_{hi}$ is the sum of the amount insured for company h ,
- 'Awhd' is the sum of the amount of contracts lapsed for company h,
- 'Wfh' is the sum of policies exposed to lapse for company h,
- 'Ewhd' or E_{whd} is the expected amount of policies lapsed for company h,
- 'Ewhd^2' is the square of the sum of expected amount of policies lapsed for company h,
- 'wmhd' or m_{whd} is the company A/E lapse ratio by amount for company h,
- 'Bhd' is $\sum_{i=1}^{n_h} b_{hi} E_{whdi}$ for company h, and
- 'Chd' is $\sum_{i=1}^{n_h} b_{hdi}^2 f_{hdi}^2 \left(q_{hdi}^s\right)^2$ for company h.

Appendix F (Continued):

In the second section of the sheet ‘UL Buhlmann by Amount’ contains:

h	Bhd/Ewhd	Chd/Ewhd	d_Sigma^2 Num1	d_Sigma^2 Num2	d_Sigma^2 Num3	d_Sigma^2 Den	d_Sigma^2	Awhc

In each row, the column is:

- ‘h’ is a company code of an individual company,
- ‘Bhd/Ewhd’ is $\sum_{i=1}^{n_h} b_{hi} E_{whdi}$ divided by the expected amount of policies lapsed for company h,
- ‘Chd/Ewhd’ is $\sum_{i=1}^{n_h} b_{hdi}^2 f_{hdi}^2 \left(q_{hdi}^s\right)^2$ divided by the expected amount of policies lapsed for company h,
- ‘d_Sigma^2 Num1’ is $\sum_{h=1}^{8} E_{whd} (wmhd - \mu wd)^2$,
- ‘d_Sigma^2 Num2’ is $-\mu_{wd} \left(\sum_{h=1}^{8} \frac{B_{hd}}{E_{whd}} - \frac{\sum_{h=1}^{8} B_{hd}}{\sum_{h=1}^{8} E_{whd}} \right)$,
- ‘d_Sigma^2 Num3 ’ is $\mu_{wd}^2 \left(\sum_{h=1}^{8} \frac{C_{hd}}{E_{whd}} - \frac{\sum_{h=1}^{8} C_{hd}}{\sum_{h=1}^{8} E_{whd}} \right)$,
- ‘d_Sigma^2 Den ’ is $\sum_{h=1}^{8} E_{whd} - \frac{\sum_{h=1}^{8} E_{whd}^2}{\sum_{h=1}^{8} E_{whd}} - \sum_{h=1}^{8} \frac{C_{hd}}{E_{whd}} + \frac{\sum_{h=1}^{8} C_{hd}}{\sum_{h=1}^{8} E_{whd}}$,
- ‘d_Sigma^2 ’ or σ^2 is (‘d_Sigma^2 Num1’ + ‘d_Sigma^2 Num2’ + ‘d_Sigma^2 Num3’)/ ‘d_Sigma^2 Den,’ and
- ‘Awhc’ is the sum of the number of contracts lapsed for company h.

Appendix F (Continued):

The third section of the sheet ‘UL Buhlmann by Policy’ contains:

‘Z’ which is $E_{whd} / (E_{whd} + \mu_{wd} B_{hd} / (\sigma^2 E_{whd}) - (\mu^2 + \sigma^2) C_{hd} / (\sigma^2 E_{whd}))$

This corresponds to the development of the credibility factor Z:

$$Z = \frac{E_h}{E_h + \frac{\mu}{\sigma^2 E_h} B_h - \frac{\mu^2 + \sigma^2}{\sigma^2 E_h} C_h}$$

The development on the sheets ‘Term Buhlmann by Amount’, ‘Perm Life Buhlmann by Amount‘, ‘VUL Buhlmann by Amount ‘ of the file ‘Buhlmann_Lapse.xls’ uses the same development on the sheet ‘UL Buhlmann by Amount’. Additionally, the development on the sheet ‘Buhlmann by Amount’ of the file ‘Buhlmann_Mortality.xls’ is parallel to the development on the sheet ‘UL Bühlmann by Amount’.

Credibility Theory Survey Report

Introduction

MIB Solutions (www.mibsolutions.com) sought, on behalf of the Society of Actuaries' Credibility Theory Practices Project Oversight Group (www.soa.org), to provide an understanding of the degree to which credibility theory is understood and applied in the life insurance industry. To find out the level of understanding in the industry, actuaries employed by US insurance companies were surveyed to ascertain who uses credibility theory and how credibility theory is applied at responding insurers.

Executive Summary

The major conclusion from this survey of 190 US insurers is that credibility theory is not widely adopted among surveyed actuaries at United States life and annuity carriers to date in managing mortality-, lapse- and expense- related risks. Comments submitted as part of the survey strongly suggest that there is a growing awareness of the value of credibility theory, but operational risk management has yet to adopt the key principles of applied credibility theory. To the extent that those who responded did indicate some level of adoption, the findings show that mortality is the area where credibility theory is most commonly applied, followed by lapse, followed by expense. The life & annuity grouping of responding companies shows a higher percentage of companies using credibility theory than the whole, as do the respondents representing large companies.

One potentially confusing artifact from the study relates to the question regarding the specific methods applied. Even among the group indicating that they use credibility theory in their work, the majority indicate that either they cannot specify the particular method they use, or else they use "actuarial judgment" (See Question 7 in the body of the report and Appendix A)

The 10% proportion of usable survey responses raises questions about whether this issue is seen as relevant in actuarial practice among the surveyed group, or whether the application of credibility theory among the surveyed group is low, or both.

Overview

The body of the report provides information on:

- Survey Design
- Methods
- Analysis and Key Findings
- Limitations
- Recommendations

The appendices include:

- Appendix A – The question-by-question response tabulation (including any comments submitted by survey participants)

- Appendix B – The company roster for those companies invited to participate in the survey
- Appendix C – The survey instrument. Please note this survey was conducted through an internet browser-based method, permitting respondents to participate by clicking a link in an email that directed the participant to a website displaying the survey form.

Survey Design

The Credibility Theory Survey was sent to 190 insurance companies via email on Monday March 2, 2009, and it was available to these companies until Friday March 27, 2009. The survey consists of 20 questions: 11 multiple choice questions and 9 "comment" responses. Multiple follow-up and reminder emails were sent to maximize participation.

Of the 11 multiple choice questions, 3 of them restrict the participant to choose only one option, whereas the other 8 allow the participant to "choose all that applied".

The survey instrument itself was organized to explore the use of credibility theory as it applies to experience analysis for mortality, lapse and expense. The survey results make clear that to the extent credibility theory is currently practiced, its application to mortality experience is relatively better understood than with regard to expenses. Lapse experience falls somewhere between those two levels.

The survey directions defined credibility theory as follows:
"In this survey the application of credibility theory refers to the methods and practices employed in determining the level of reliance that can be placed on company experience and/or the approaches used in blending company data with industry or other data. An in-depth description of Credibility Theory can be found at http://www.actuary.org/pdf/practnotes/life_credibility08.pdf."

The survey instrument itself is included as Appendix C.

Methods

Initializing the Data

At the close of the survey, the raw data was tabulated and graphed in Microsoft Excel. Once entered into Excel, it was determined that 31 of the 190 target companies responded to the survey, with only 13 fully completed survey responses. The 18 partial responses were instances where the participant dropped out of the survey prior to completing all questions. Regardless of completion status, all 31 responses were considered for analysis.

Data Cleaning

Eight multiple choice questions permitted the participant to choose as many responses as applied to a given question. With this option, some participants provided a specific choice, along with a 'not sure' or 'not yet applied'. In these instances, the 'not sure' or 'not yet applied' options were omitted from the data for the given survey response.

"60% Yes Method"
In order to accept a given response into the survey analysis, the response had to be substantial. The methodology developed to determine whether or not a response contained usable data is defined as follows:

A multiple choice question was considered to contain relevant data if the participant **did not select** 'not sure' or 'not yet applied'. With this in mind, after excluding the first question, at least **6 of the remaining 10** questions had to contain relevant data in order to be kept for analysis.

Using this methodology, 19 responses of the 31 were considered to have substantial data, and they were used in the analysis.

Analysis
The primary goal of the survey analysis was to try to answer the question of "Who uses credibility theory?" by segregating the responses again in a 'Yes-No' fashion. Again, a response is considered to be a 'Yes' when the participant **did not select** 'Not sure' or 'Not yet applied'. A percentage accompanies each graph to show its 'Yes' value with its complement labeled with a 'No' value.

THE WHOLE

The analysis of 'The Whole' is an analysis of all of the 19 responses taken together. A complete tabulation and presentation of responses is included in Appendix A. This view illustrates trends and practices within responding companies overall. The analysis consistently shows that mortality has the highest 'yes' rate, followed by lapse, followed by expense. Also, we can see this hierarchy for response rate when it comes to questions that permit the participant to choose more than one choice per area.

Key Findings

- Responses to Question 2, the most fundamental question regarding the use of credibility theory, indicate a strong 'yes' response for mortality (89.5%) while indicating a weak 'yes' response for expense (21.1%).

- Responses to Question 7 show the same trend: mortality (76.9%), lapse (59.1%), and expense (21.1%). One company specifically commented, "We don't use credibility for expenses."

- Responses to Question 10 show the same behavior, and include the same comment (from a different respondent): "We don't use credibility for expenses."

LIFE & ANNUITY VS. MULTI-LINE
In this grouping, life & annuity companies are considered those companies where the focus is centered on individual life and annuity products. A multi-line company is one where the products sold by the company can also include health, group and other types of products. Of the 19 companies, 15 were considered life & annuity, and 4 were considered multi-line.

Key Findings

- The same trend appears in this analysis as it does in the Whole analysis: mortality has the highest rate of 'Yes' responses, followed by lapse, followed by expense.

- For responses to Question 4, the overall 'yes' response rate increases when examining the large company responses alone.

- Responses to Question 6 also demonstrate an increase in the 'yes' response rate.

FINANCIAL SIZE
For this analysis, financial size was determined by AM Best ratings. Of the 19 companies, 13 were considered large companies, and 6 were considered small companies.

This analysis shows the same kind of trend, with the decreasing 'yes' rate as we go from mortality, to lapse, to expense. However, two interesting finds: the first one being that it appears to be that the large companies have nearly 100% 'yes' rate for mortality, and the second one being an extremely low 'yes' rate for expense. The small companies tend to exhibit the same sort of trend seen in other analysis, but with less uniformity. Small companies also exhibit a relatively more common use of credibility theory for expense experience analysis than large company respondents indicated. This may be due to a small number of overall responses skewing the small company findings in this area. Reliability of such findings based on small numbers must always be treated as suspect.

Key Findings

- Responses to Question 2 show respondents have (or will have) a standard for applying credibility theory to mortality experience at a 90 percent level, whereas the same measure for expense weighs in at 10 percent.

- Question 6 responses demonstrate a high response rate among large companies for the products to which credibility theory is applied.

- Question 13 responses demonstrate a high response rate among large companies with regard to who determines the methods of credibility analysis used.

Limitations

Any analysis and conclusions are based on limited feedback from the industry, and should not be generalized. The response rate for the companies polled was 31/190, or 16.3 percent. 10.0 percent (19/190) of the polled group supplied the responses that were used in the survey analysis. The practices disclosed by the responding companies 'partial participation' may not accurately represent the prevalence of such practices for the industry. The voluntary, self-selecting nature of the survey responses, however, gives one a basis for the belief that the responses for this sample are reliable.

Based on limited survey responses, we cannot use this survey alone to characterize the entire industry, but the survey responses do provide some insight as to the degree that credibility theory is used and understood. For those that did not participate in the survey, one might conclude that credibility theory is not widely used, and that there is therefore room to improve this understanding.

Recommendations

Based on the findings and feedback associated with the survey, the following recommendations are submitted for consideration:

1) Build More Awareness.

The low survey response rate among insurer-employed actuaries may be interpreted to denote either a low level of awareness in the application of credibility theory, or a low level of overall interest in exploring the applications of credibility theory. Either way, it is incumbent on the leaders of the actuarial profession to promote the value and benefit of applying credibility theory to the everyday problems of estimation faced by actuaries in business situations, and where appropriate. Actuarial work is intrinsically fraught with issues of managing risk by the use of techniques to maximize the value of imperfect and incomplete information in measuring risk and identifying optimal strategies. Credibility theory represents an improvement in such estimation by weighting an overall risk parameter against the risk parameter of a subclass of interest, based on the "credibility" of the subclass's experience. The level of awareness of the techniques of achieving this can be improved among life and annuity actuaries.

2) Provide More Training.

Comments submitted by those who did respond to the survey suggest that actuaries working within life and annuity companies are aware of the potential benefits of applying credibility theory, but have not applied those techniques in order to realize those benefits in their work of analyzing mortality, lapse and expense experience. There is an educational opportunity to "connect the dots" and provide actuaries with the means to apply credibility theory appropriately.

3) Conduct More Research.

Additional research would be beneficial to the actuarial profession in exploring various methods of applying credibility theory. Developing practice notes and writing articles to provide greater understanding of the appropriate use and application of credibility theory are critical success factors in assuring consistent and widespread use of the principles of credibility theory in actuarial practice.

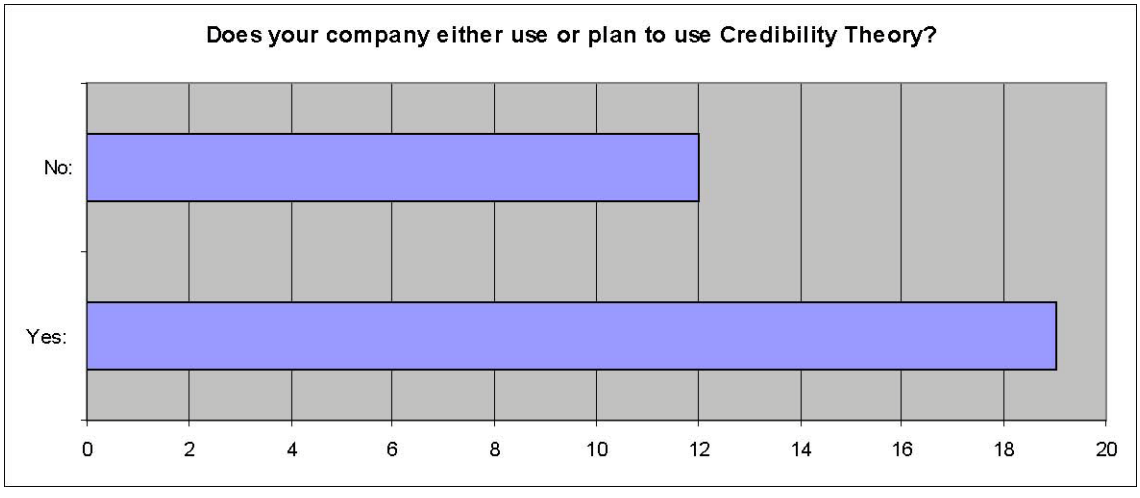
4) Seek More Feedback.

The survey population in this exercise was confined to actuaries employed by US-based life and annuity carriers. A follow-on survey of consulting actuaries would provide important information regarding the awareness and application of credibility theory among members of the actuarial profession who are by definition oriented toward working with a variety of client organizations and are presumably less constrained by the accepted practices of any single insurer. Such a survey would be helpful in providing a more complete picture of how actuaries use credibility theory.

Any additional survey might also include actuaries employed by non-insurance financial institutions (banks, brokerages, rating agencies, etc.), offering insights into the use and understanding of credibility theory by this sort of practicing actuary. Such actuaries are engaged in substantially different types of work than their consulting and carrier-oriented colleagues, and their use of innovative techniques for assessing unique risks would be important to gauge.

Appendix A: Analysis of Survey Responses

Question 1: Does your company either use or plan to use Credibility Theory on your Mortality, Lapse, or Expense data for individual life or annuities?



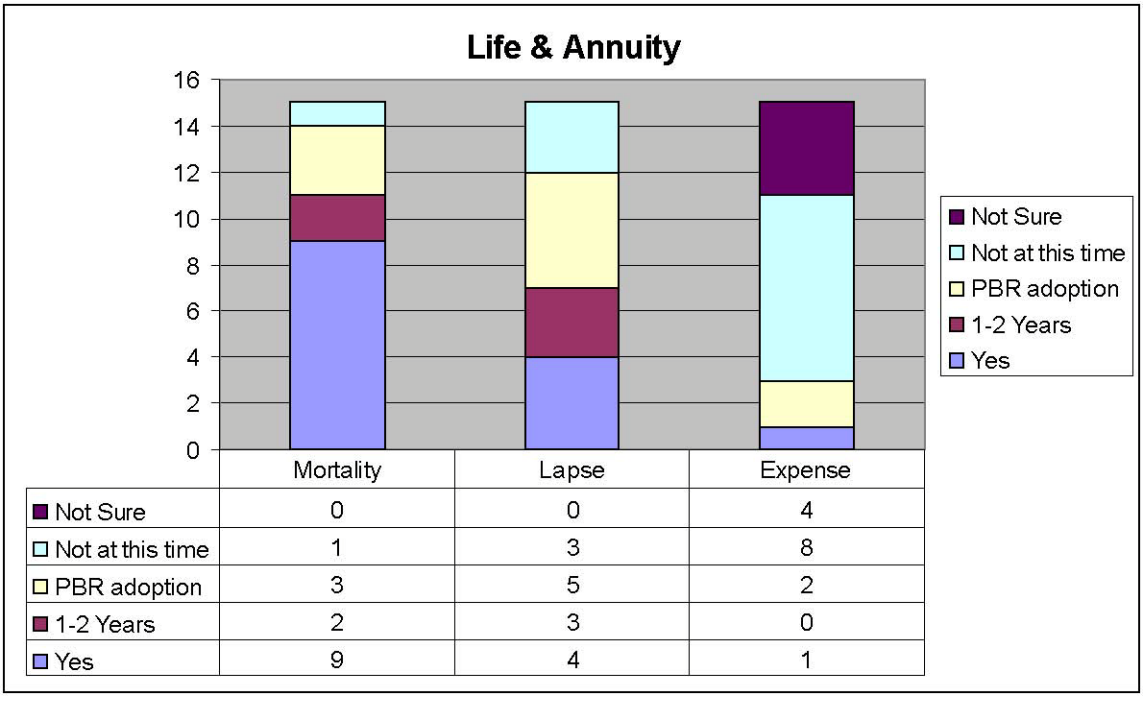
Please note:

- The analysis in this Appendix is confined to the 19 "Yes" responses shown above.
- 14 "Life & Annuity" companies responded; 5 "Multi-Line".
- 13 "Large" companies responded; 6 "Small".
- Comment-questions asked throughout the survey are included with the question the comments pertain to.

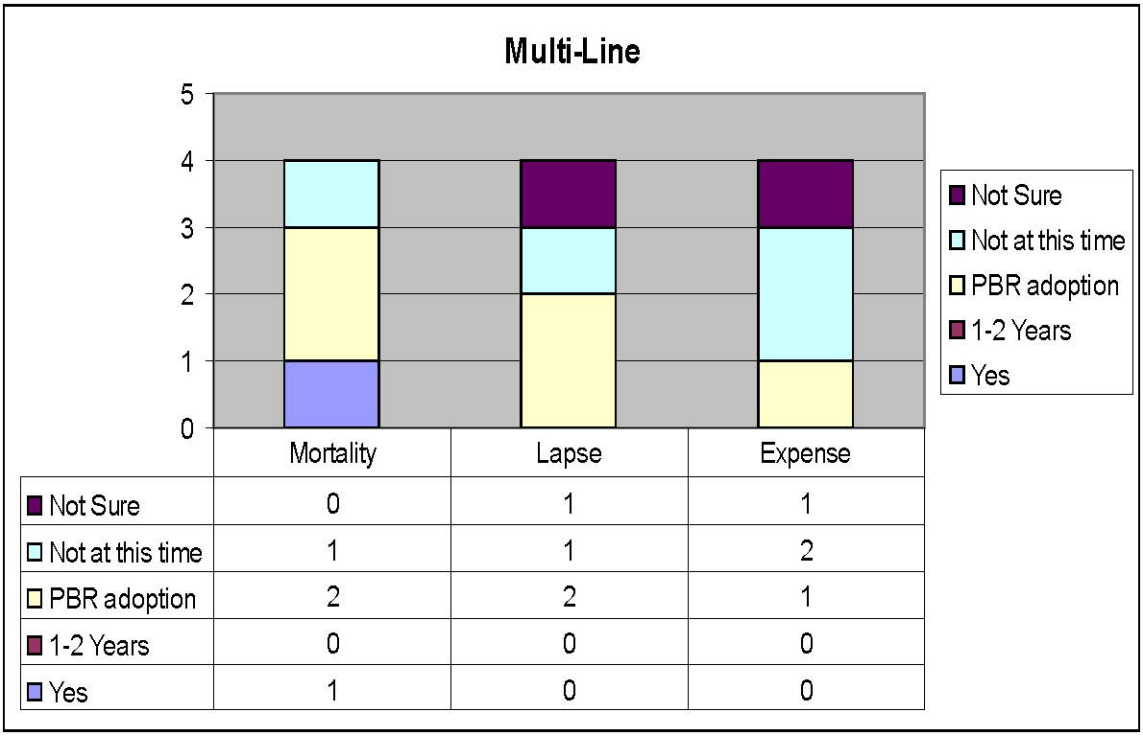
Appendix A: Analysis of Survey Responses

Question 2: Does your company have a standard for applying Credibility Theory?

LIFE & ANNUITY VS. MULTI-LINE



	Mortality	Lapse	Expense
Not Sure	0	0	4
Not at this time	1	3	8
PBR adoption	3	5	2
1-2 Years	2	3	0
Yes	9	4	1

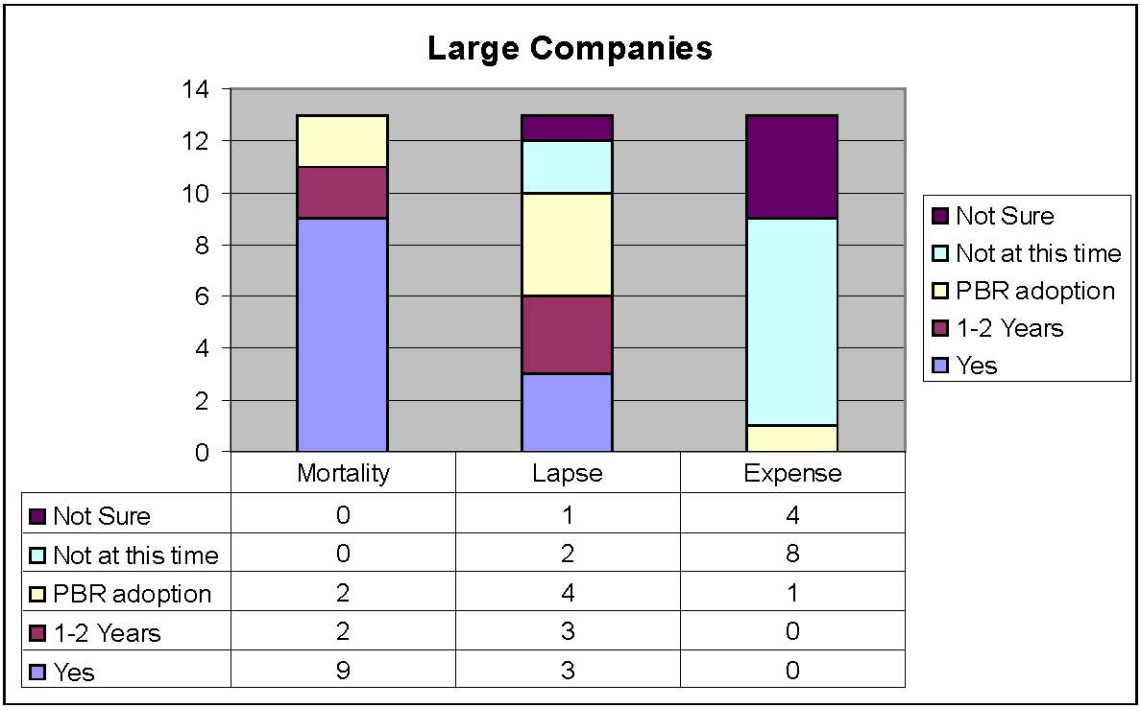


	Mortality	Lapse	Expense
Not Sure	0	1	1
Not at this time	1	1	2
PBR adoption	2	2	1
1-2 Years	0	0	0
Yes	1	0	0

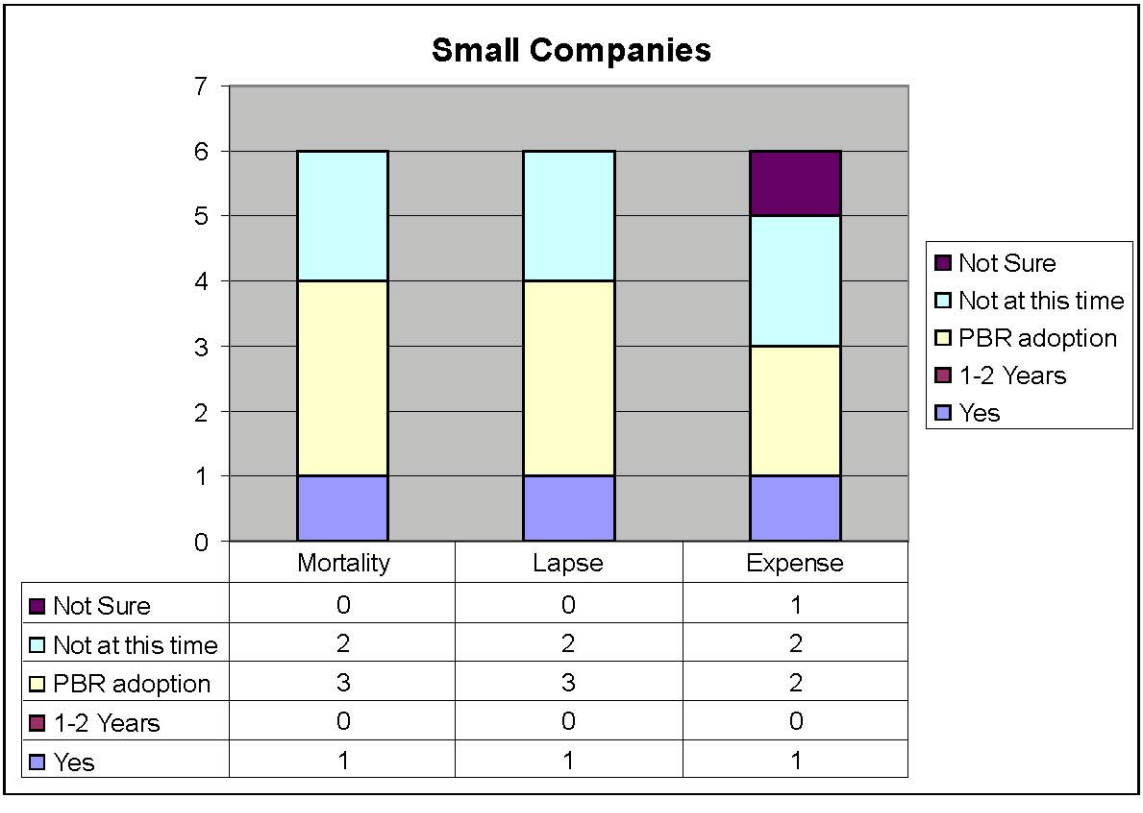
Appendix A: Analysis of Survey Responses

Question 2: Does your company have a standard for applying Credibility Theory?

FINANCIAL SIZE



	Mortality	Lapse	Expense
Not Sure	0	1	4
Not at this time	0	2	8
PBR adoption	2	4	1
1-2 Years	2	3	0
Yes	9	3	0

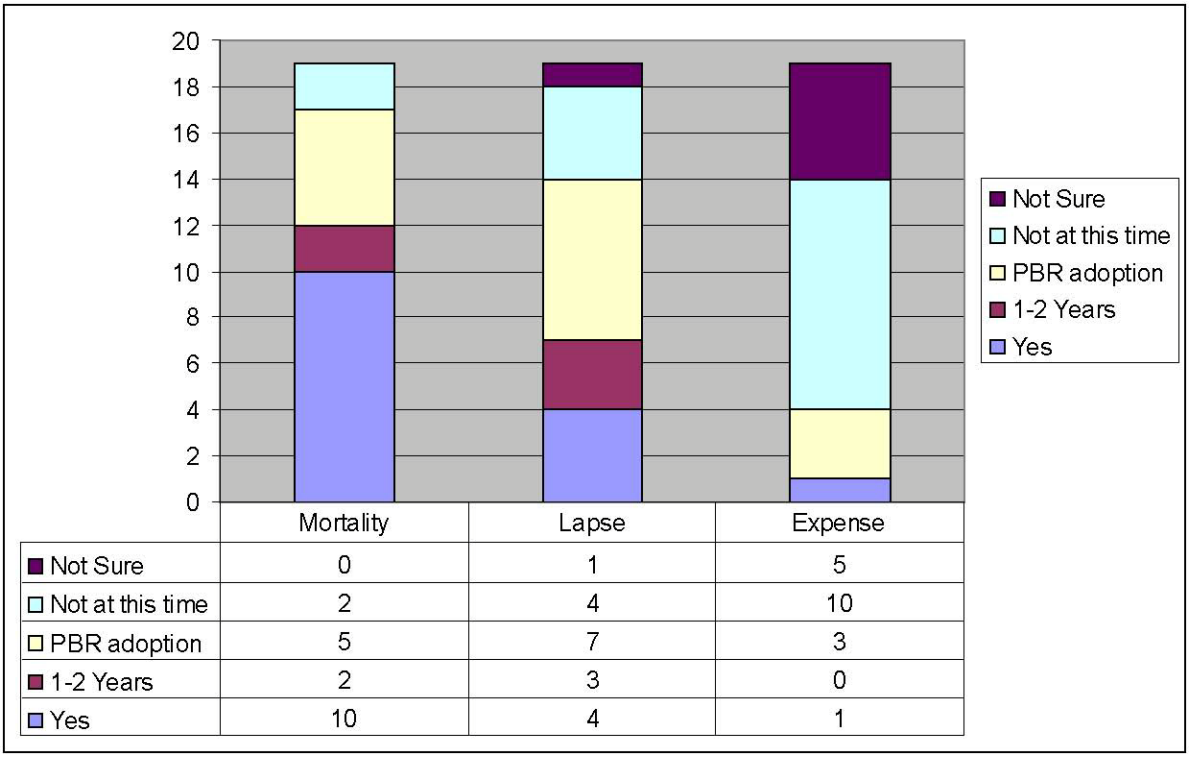


	Mortality	Lapse	Expense
Not Sure	0	0	1
Not at this time	2	2	2
PBR adoption	3	3	2
1-2 Years	0	0	0
Yes	1	1	1

Appendix A: Analysis of Survey Responses

Question 2: Does your company have a standard for applying Credibility Theory?

THE WHOLE



	Mortality	Lapse	Expense
Not Sure	0	1	5
Not at this time	2	4	10
PBR adoption	5	7	3
1-2 Years	2	3	0
Yes	10	4	1

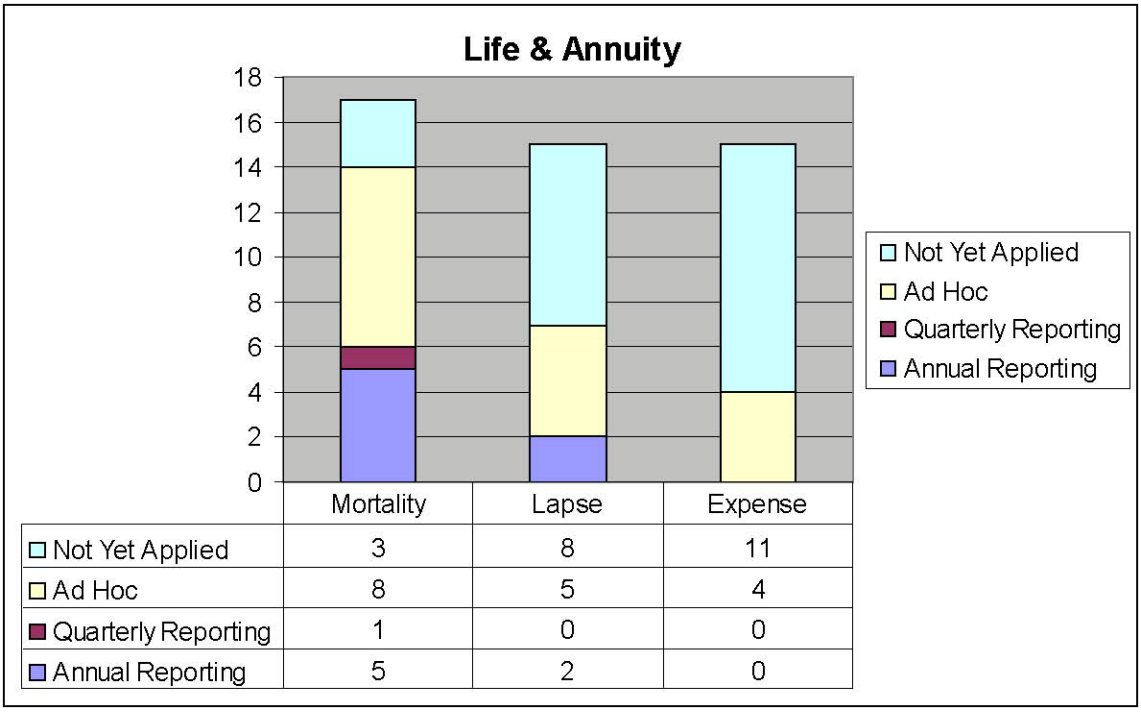
Question 3: Comments pertaining to Question 2

- "I wish there were more easily available guidance on how to apply credibility to real life insurance situations."
- "Expense factors are 100% based on actual company experience."
- "Currently use mortality ratio confidence intervals as part of credibility analysis for evaluating experience."

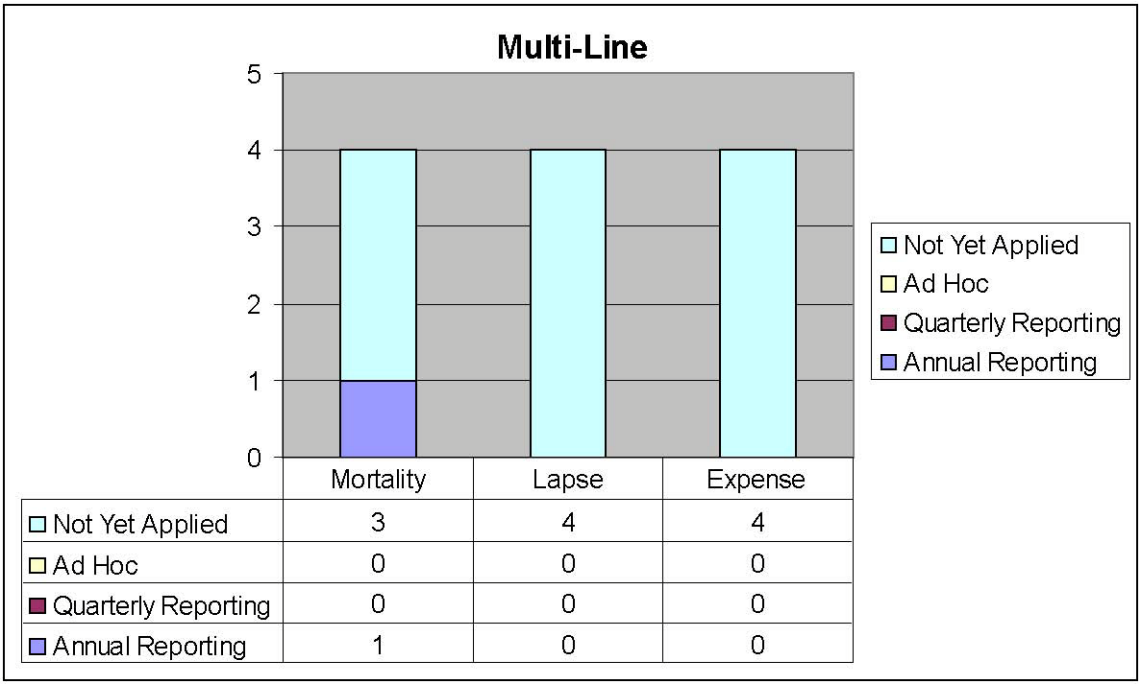
Appendix A: Analysis of Survey Responses

Question 4: When do you or your company apply Credibility Theory?

LIFE & ANNUITY VS. MULTI-LINE



	Mortality	Lapse	Expense
Not Yet Applied	3	8	11
Ad Hoc	8	5	4
Quarterly Reporting	1	0	0
Annual Reporting	5	2	0

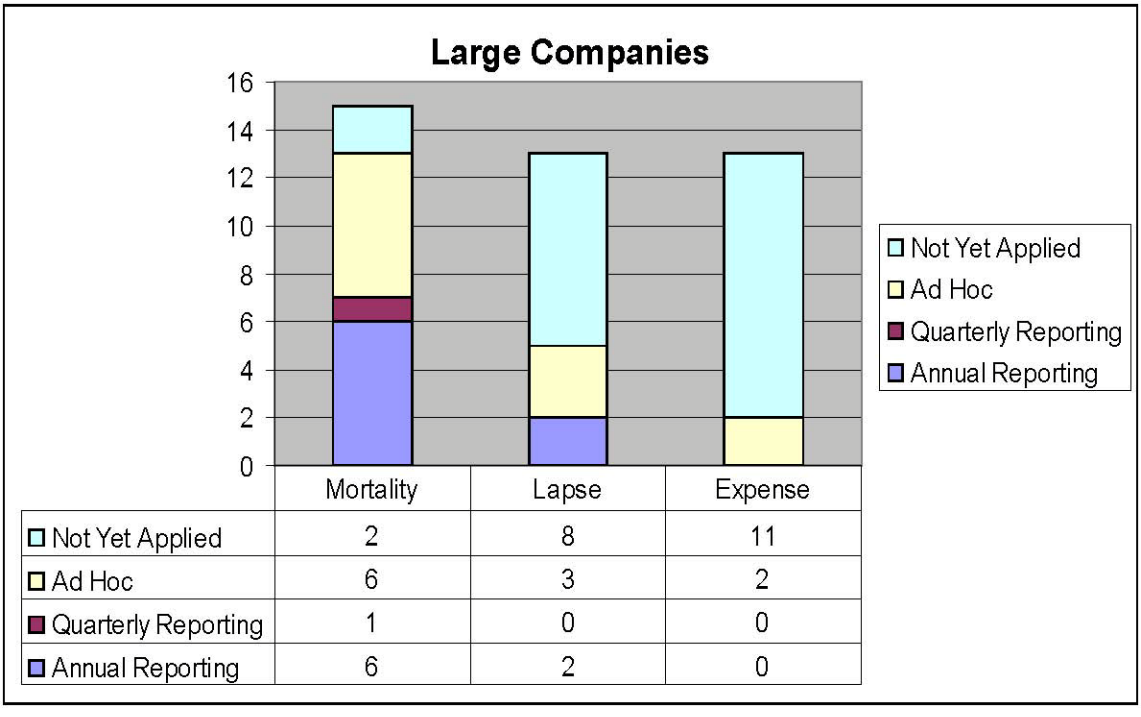


	Mortality	Lapse	Expense
Not Yet Applied	3	4	4
Ad Hoc	0	0	0
Quarterly Reporting	0	0	0
Annual Reporting	1	0	0

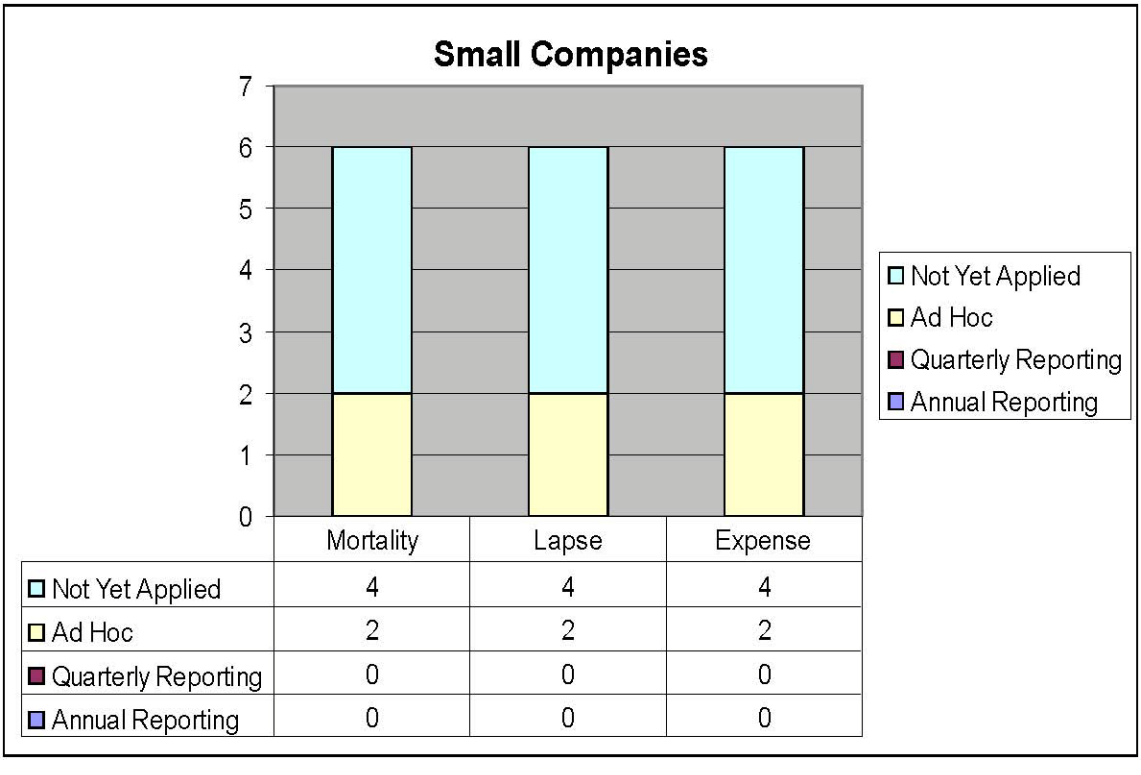
Appendix A: Analysis of Survey Responses

Question 4: When do you or your company apply Credibility Theory?

FINANCIAL SIZE



	Mortality	Lapse	Expense
Not Yet Applied	2	8	11
Ad Hoc	6	3	2
Quarterly Reporting	1	0	0
Annual Reporting	6	2	0

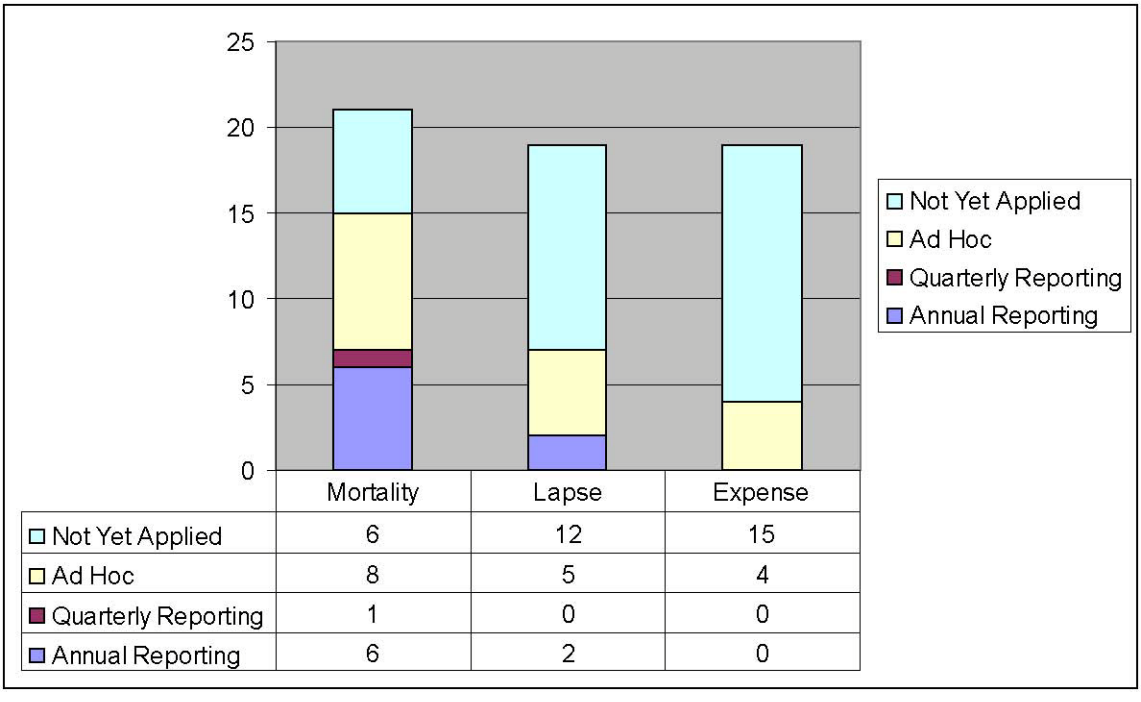


	Mortality	Lapse	Expense
Not Yet Applied	4	4	4
Ad Hoc	2	2	2
Quarterly Reporting	0	0	0
Annual Reporting	0	0	0

Appendix A: Analysis of Survey Responses

Question 4: When do you or your company apply Credibility Theory?

THE WHOLE



	Mortality	Lapse	Expense
Not Yet Applied	6	12	15
Ad Hoc	8	5	4
Quarterly Reporting	1	0	0
Annual Reporting	6	2	0

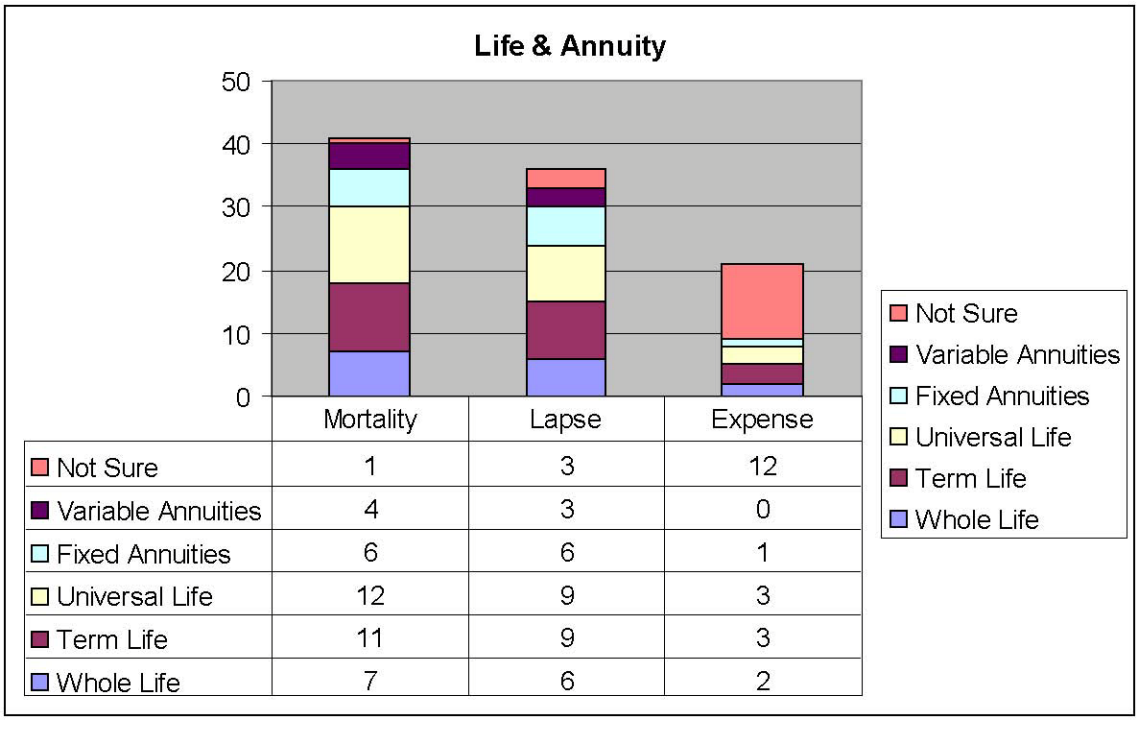
Question 5: Comments pertaining to Question 4

- "We use credibility in our annual unlocking process for DAC and related balances. These assumptions are then used for other purposes."
- "Annual pricing studies"
- "It is used in setting mortality assumptions used in pricing individual life products."
- "When setting new assumptions for pricing or updating projection assumptions."
- "Routine management reports analyzing emerging mortality experience."

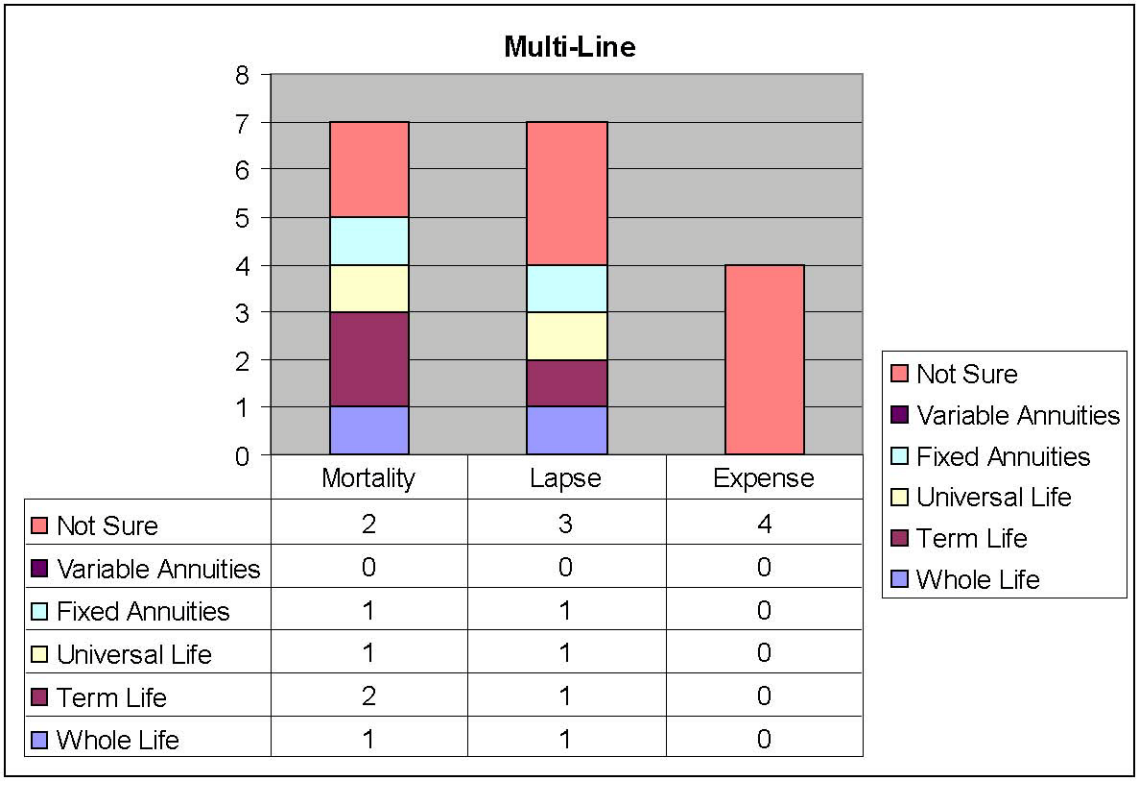
Appendix A: Analysis of Survey Responses

Question 6: To which products might you be applying Credibility Theory?

LIFE & ANNUITY VS. MULTI-LINE



	Mortality	Lapse	Expense
Not Sure	1	3	12
Variable Annuities	4	3	0
Fixed Annuities	6	6	1
Universal Life	12	9	3
Term Life	11	9	3
Whole Life	7	6	2

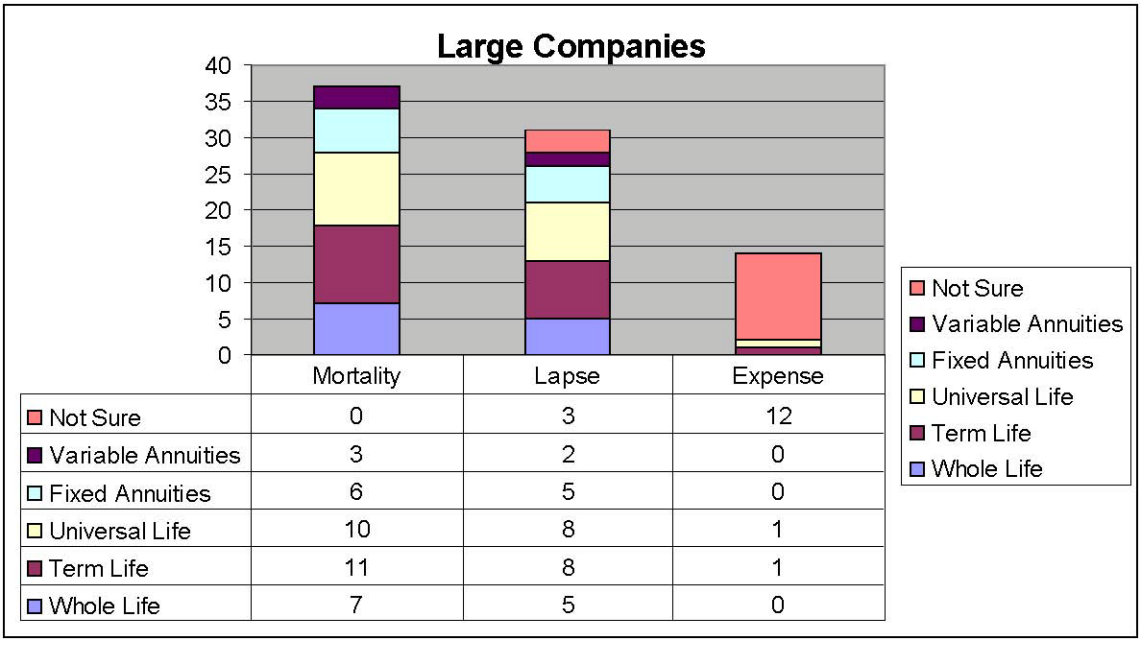


	Mortality	Lapse	Expense
Not Sure	2	3	4
Variable Annuities	0	0	0
Fixed Annuities	1	1	0
Universal Life	1	1	0
Term Life	2	1	0
Whole Life	1	1	0

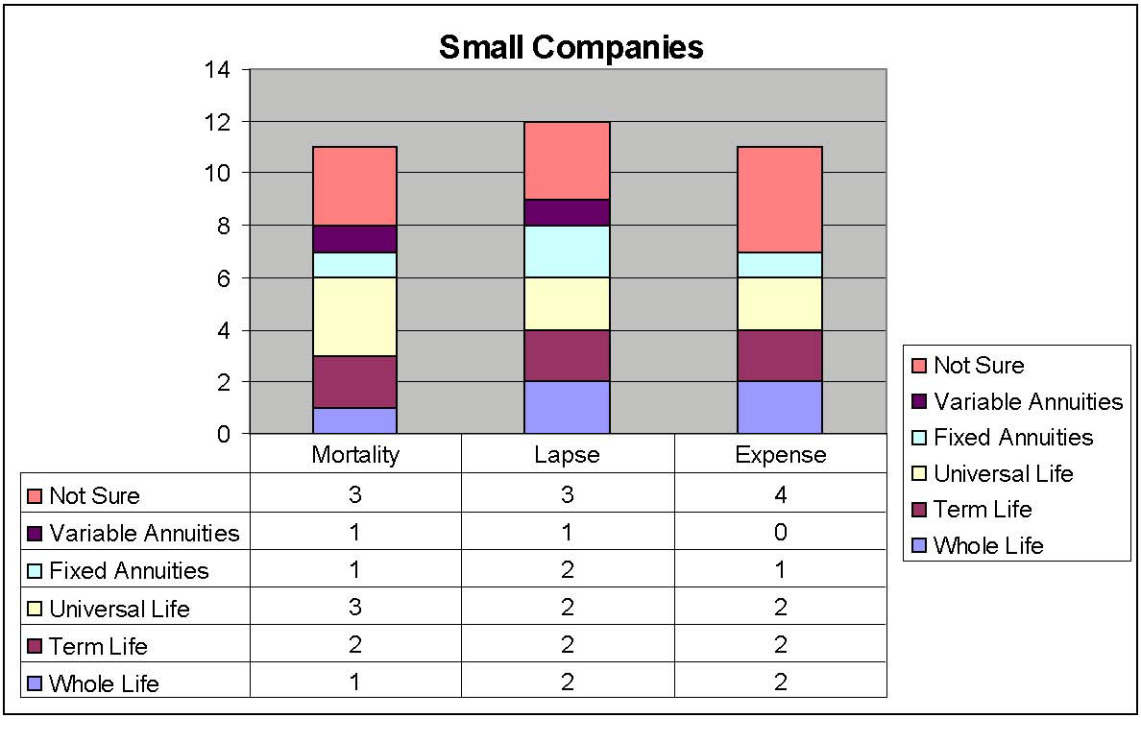
Appendix A: Analysis of Survey Responses

Question 6: To which products might you be applying Credibility Theory?

FINANCIAL SIZE



	Mortality	Lapse	Expense
Not Sure	0	3	12
Variable Annuities	3	2	0
Fixed Annuities	6	5	0
Universal Life	10	8	1
Term Life	11	8	1
Whole Life	7	5	0

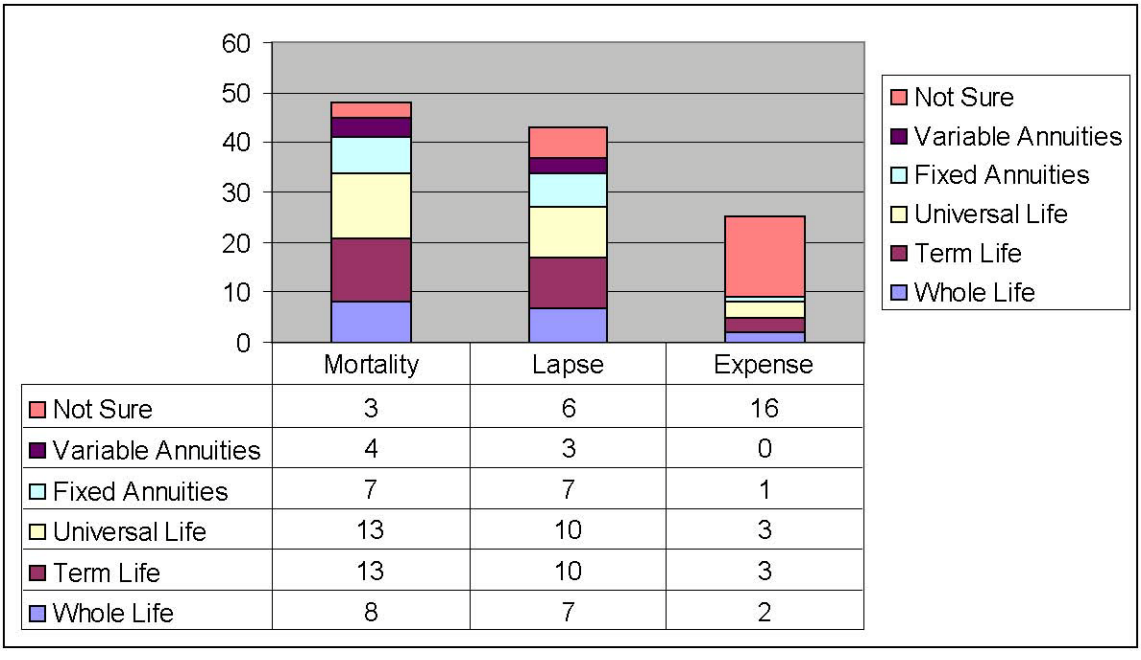


	Mortality	Lapse	Expense
Not Sure	3	3	4
Variable Annuities	1	1	0
Fixed Annuities	1	2	1
Universal Life	3	2	2
Term Life	2	2	2
Whole Life	1	2	2

Appendix A: Analysis of Survey Responses

Question 6: To which products might you be applying Credibility Theory?

THE WHOLE



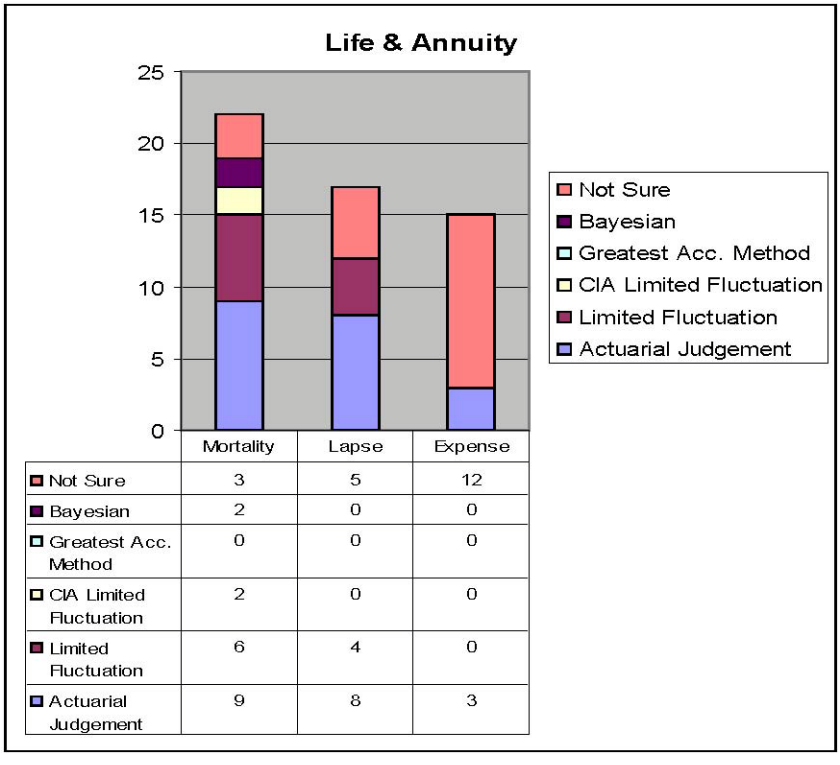
	Mortality	Lapse	Expense
Not Sure	3	6	16
Variable Annuities	4	3	0
Fixed Annuities	7	7	1
Universal Life	13	10	3
Term Life	13	10	3
Whole Life	8	7	2

**No comments pertaining to this question.

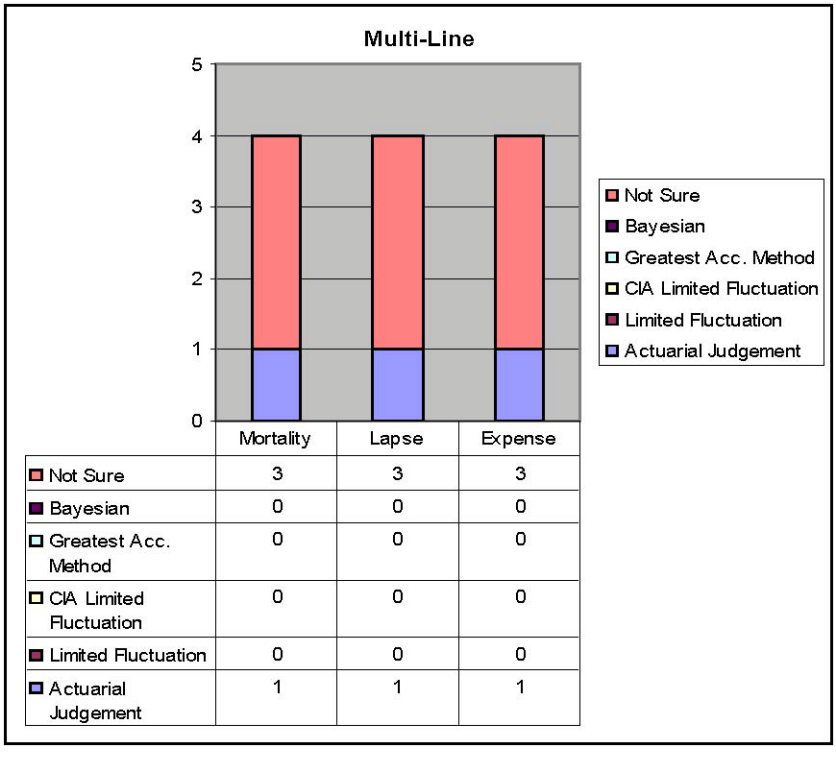
Appendix A: Analysis of Survey Responses

Question 7: Which Credibility approaches are used (or are planned)?

LIFE & ANNUITY VS. MULTI-LINE



	Mortality	Lapse	Expense
Not Sure	3	5	12
Bayesian	2	0	0
Greatest Acc. Method	0	0	0
CIA Limited Fluctuation	2	0	0
Limited Fluctuation	6	4	0
Actuarial Judgement	9	8	3

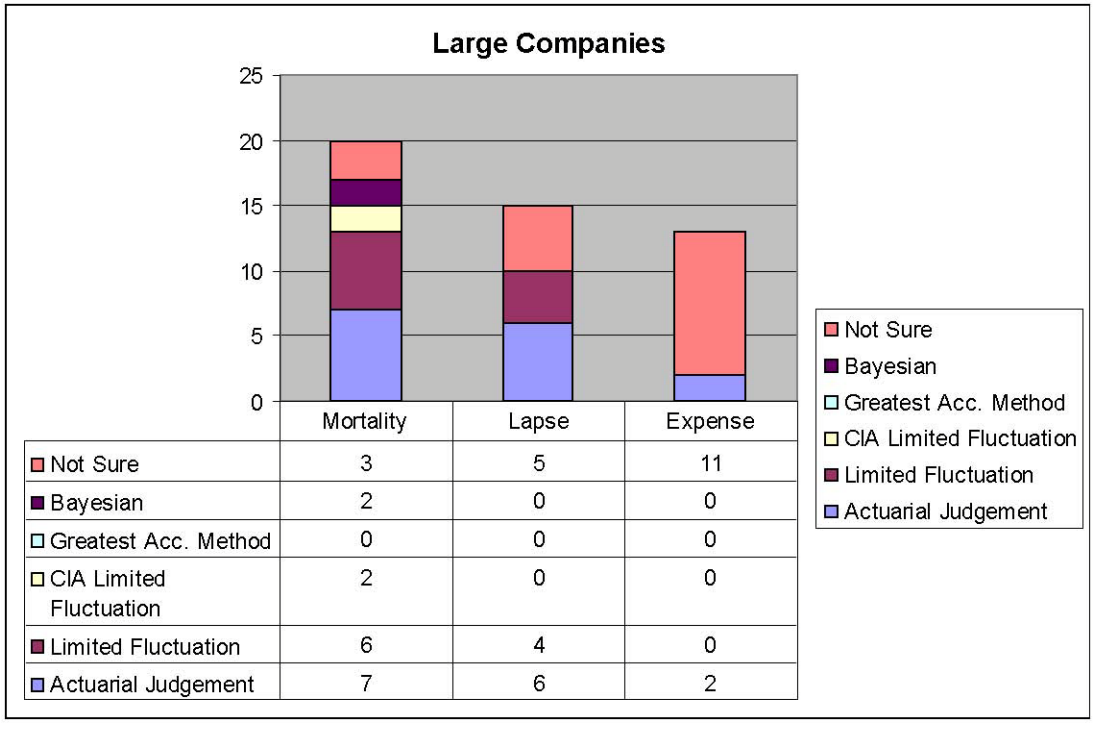


	Mortality	Lapse	Expense
Not Sure	3	3	3
Bayesian	0	0	0
Greatest Acc. Method	0	0	0
CIA Limited Fluctuation	0	0	0
Limited Fluctuation	0	0	0
Actuarial Judgement	1	1	1

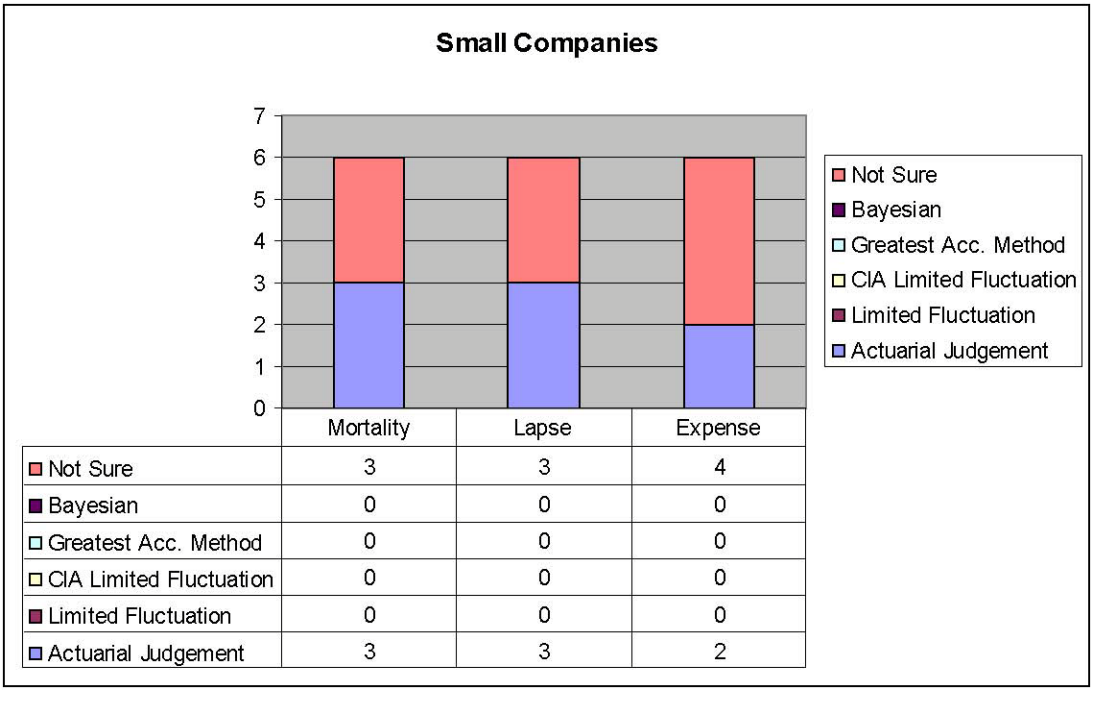
Appendix A: Analysis of Survey Responses

Question 7: Which Credibility approaches are used (or planned)?

FINANCIAL SIZE



	Mortality	Lapse	Expense
Not Sure	3	5	11
Bayesian	2	0	0
Greatest Acc. Method	0	0	0
CIA Limited Fluctuation	2	0	0
Limited Fluctuation	6	4	0
Actuarial Judgement	7	6	2

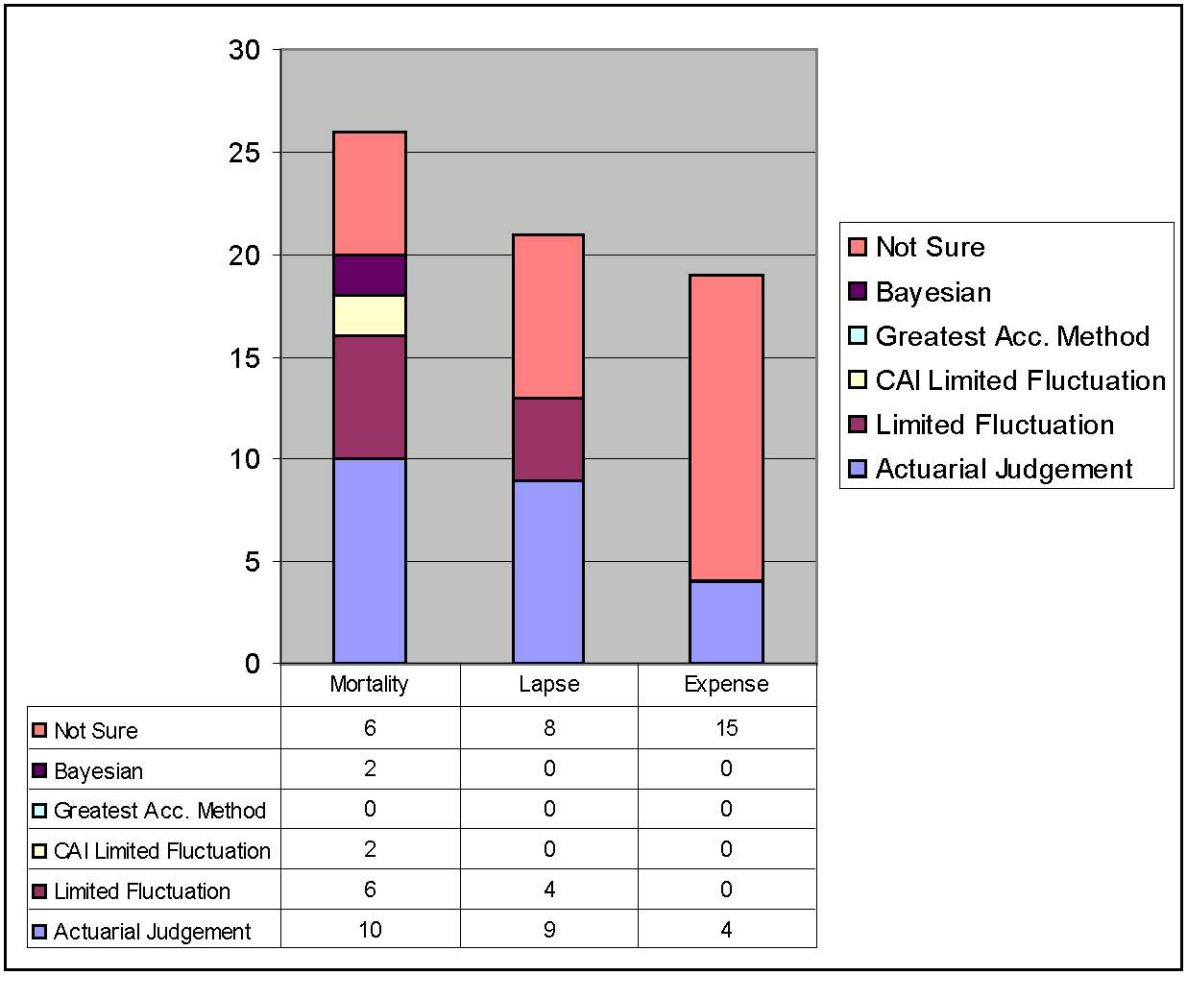


	Mortality	Lapse	Expense
Not Sure	3	3	4
Bayesian	0	0	0
Greatest Acc. Method	0	0	0
CIA Limited Fluctuation	0	0	0
Limited Fluctuation	0	0	0
Actuarial Judgement	3	3	2

Appendix A: Analysis of Survey Responses

Question 7: Which Credibility approaches are used (or planned)?

THE WHOLE



	Mortality	Lapse	Expense
Not Sure	6	8	15
Bayesian	2	0	0
Greatest Acc. Method	0	0	0
CAI Limited Fluctuation	2	0	0
Limited Fluctuation	6	4	0
Actuarial Judgement	10	9	4

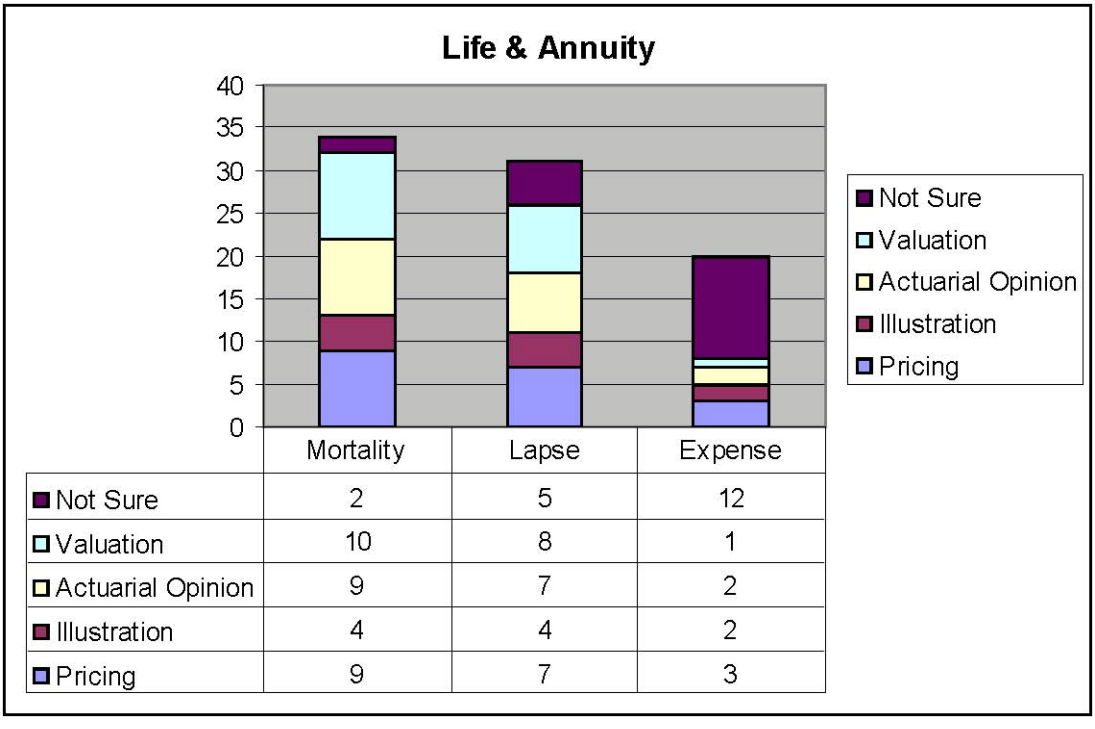
Question 8: Comments pertaining to Question 7

- "We don't use credibility for expenses."
- "We use an approach based on confidence intervals. Don't know if that is one of those listed above or not."

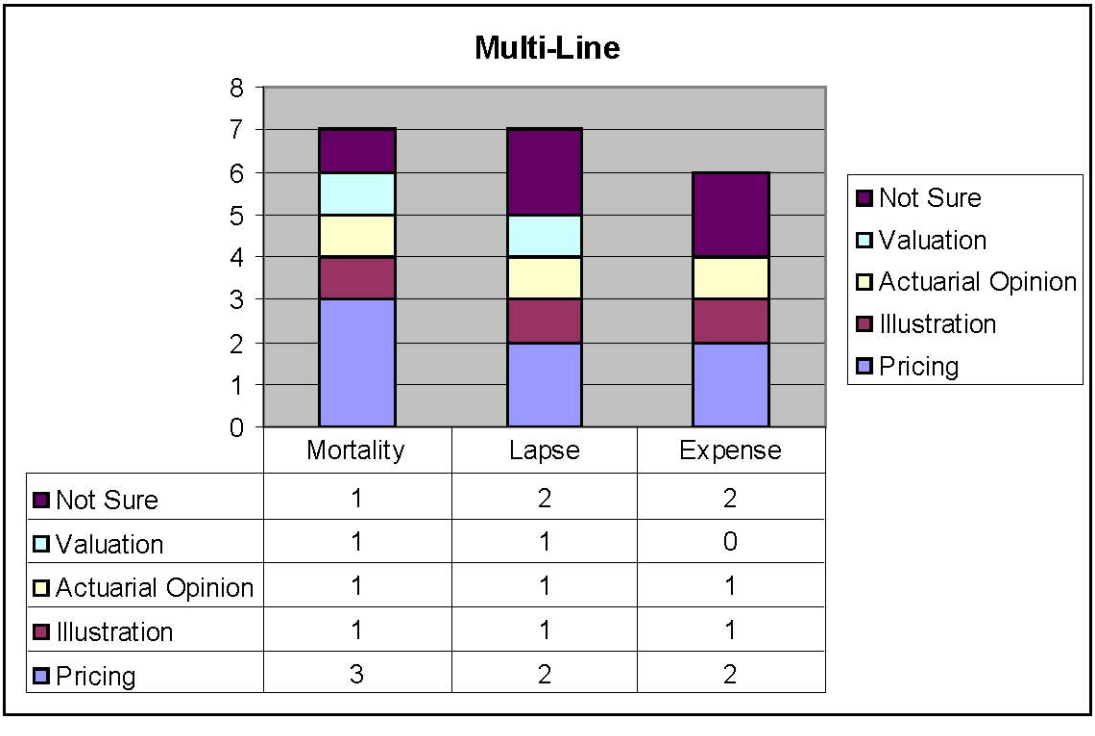
Appendix A: Analysis of Survey Responses

Question 9: Which functional areas use the results of credibility analysis?

LIFE & ANNUITY VS. MULTI-LINE



	Mortality	Lapse	Expense
Not Sure	2	5	12
Valuation	10	8	1
Actuarial Opinion	9	7	2
Illustration	4	4	2
Pricing	9	7	3

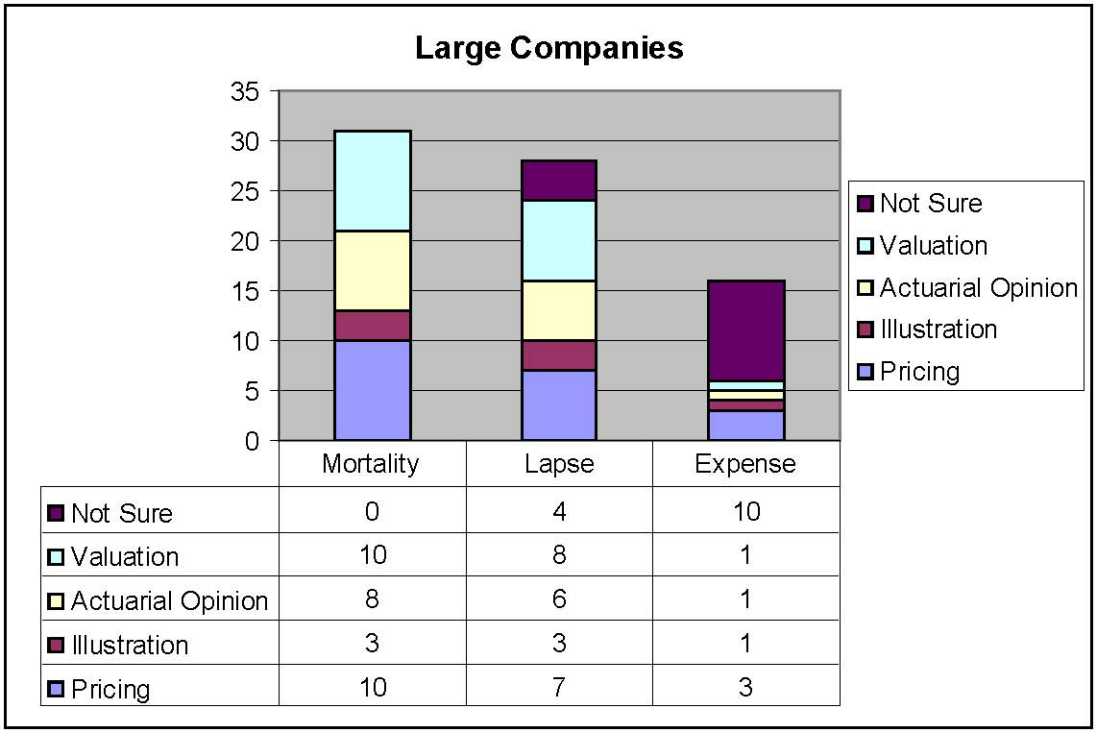


	Mortality	Lapse	Expense
Not Sure	1	2	2
Valuation	1	1	0
Actuarial Opinion	1	1	1
Illustration	1	1	1
Pricing	3	2	2

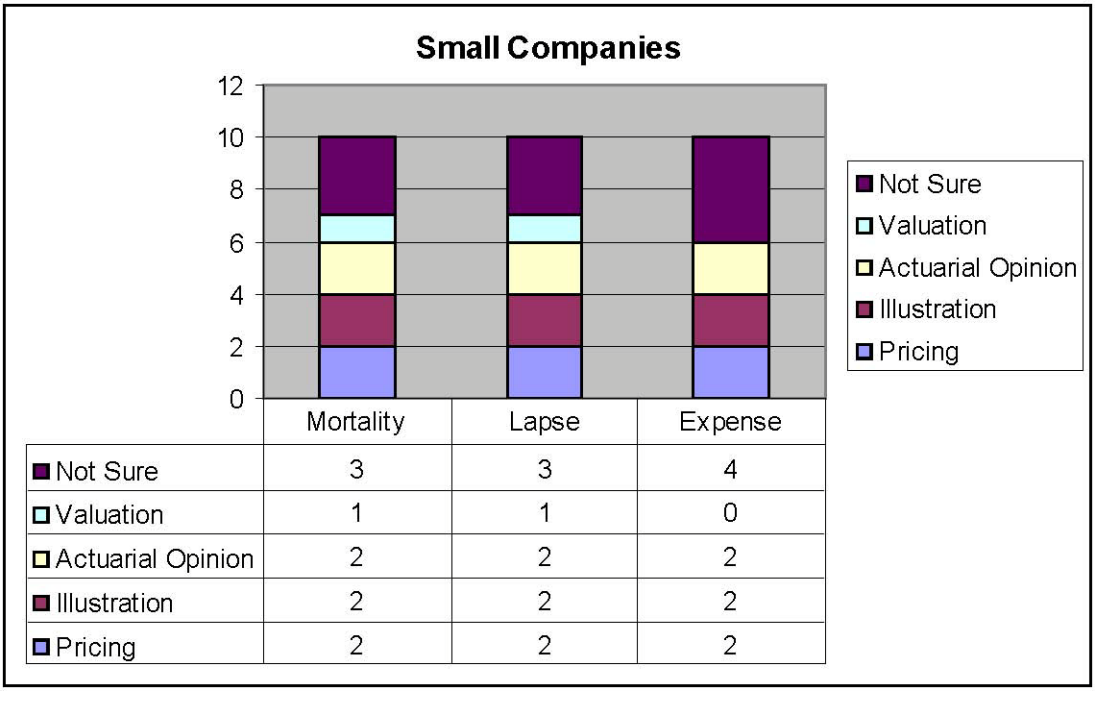
Appendix A: Analysis of Survey Responses

Question 9: Which functional areas use the results of credibility analysis?

FINANCIAL SIZE



	Mortality	Lapse	Expense
Not Sure	0	4	10
Valuation	10	8	1
Actuarial Opinion	8	6	1
Illustration	3	3	1
Pricing	10	7	3

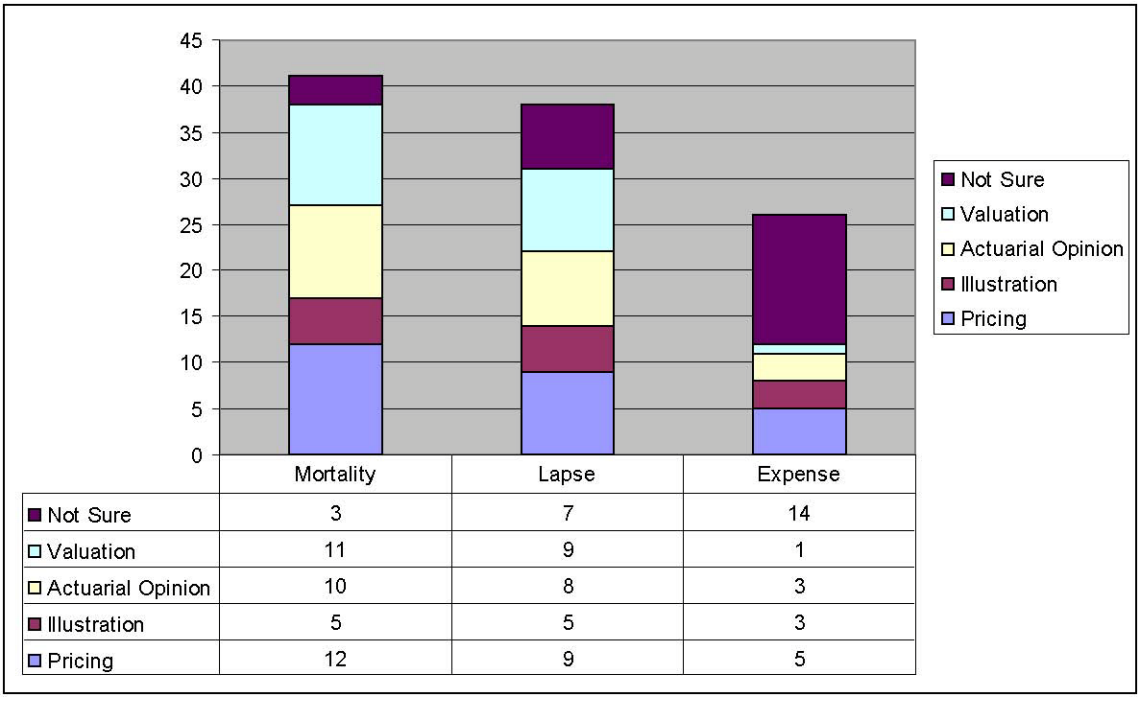


	Mortality	Lapse	Expense
Not Sure	3	3	4
Valuation	1	1	0
Actuarial Opinion	2	2	2
Illustration	2	2	2
Pricing	2	2	2

Appendix A: Analysis of Survey Responses

Question 9: Which functional areas use the results of credibility analysis?

THE WHOLE



	Mortality	Lapse	Expense
Not Sure	3	7	14
Valuation	11	9	1
Actuarial Opinion	10	8	3
Illustration	5	5	3
Pricing	12	9	5

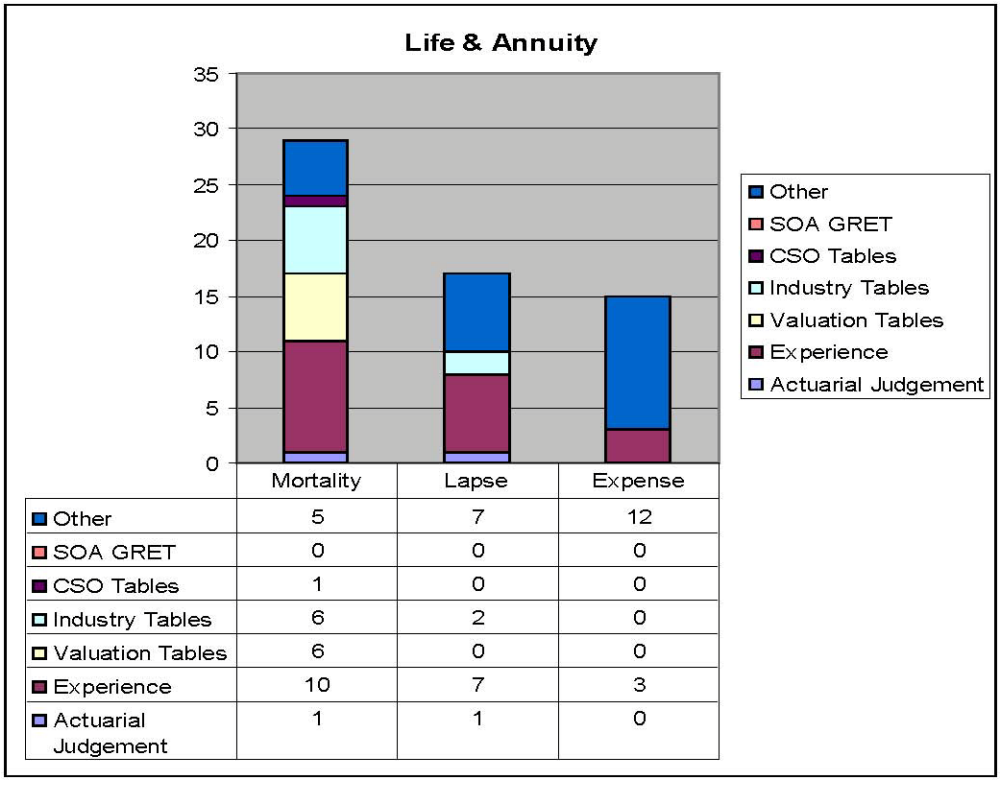
Question 10: Comments pertaining to Question 9

- "We don't use credibility for expenses."

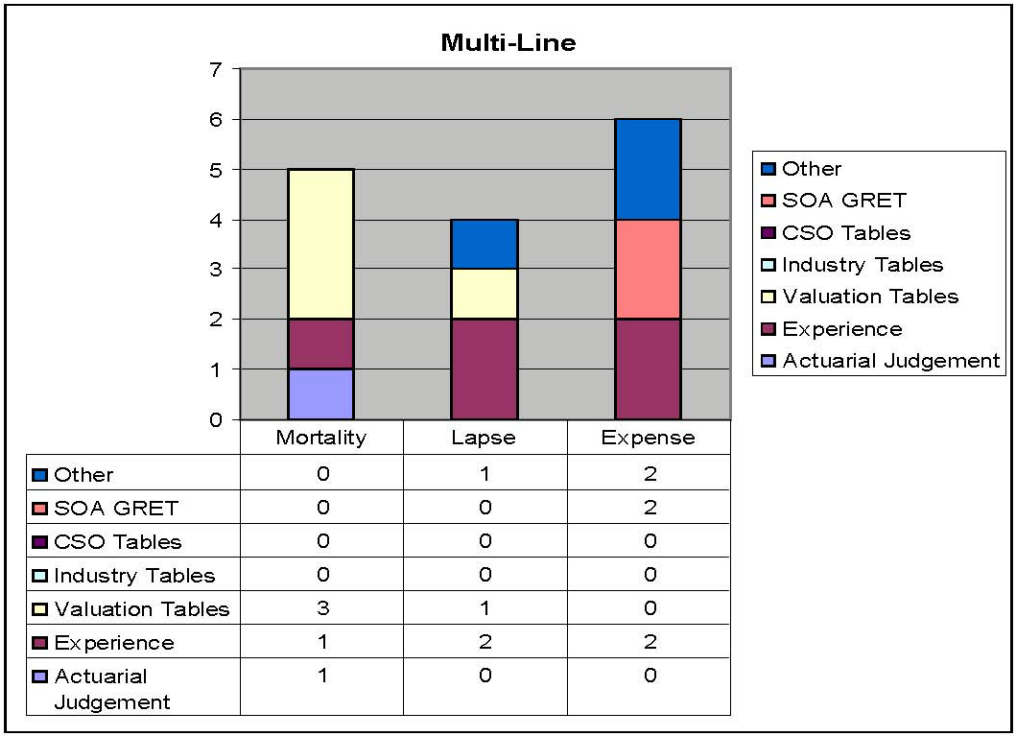
Appendix A: Analysis of Survey Responses

Question 11: Which functional areas use the results of credibility analysis?

LIFE & ANNUITY VS. MULTI-LINE



	Mortality	Lapse	Expense
Other	5	7	12
SOA GRET	0	0	0
CSO Tables	1	0	0
Industry Tables	6	2	0
Valuation Tables	6	0	0
Experience	10	7	3
Actuarial Judgement	1	1	0

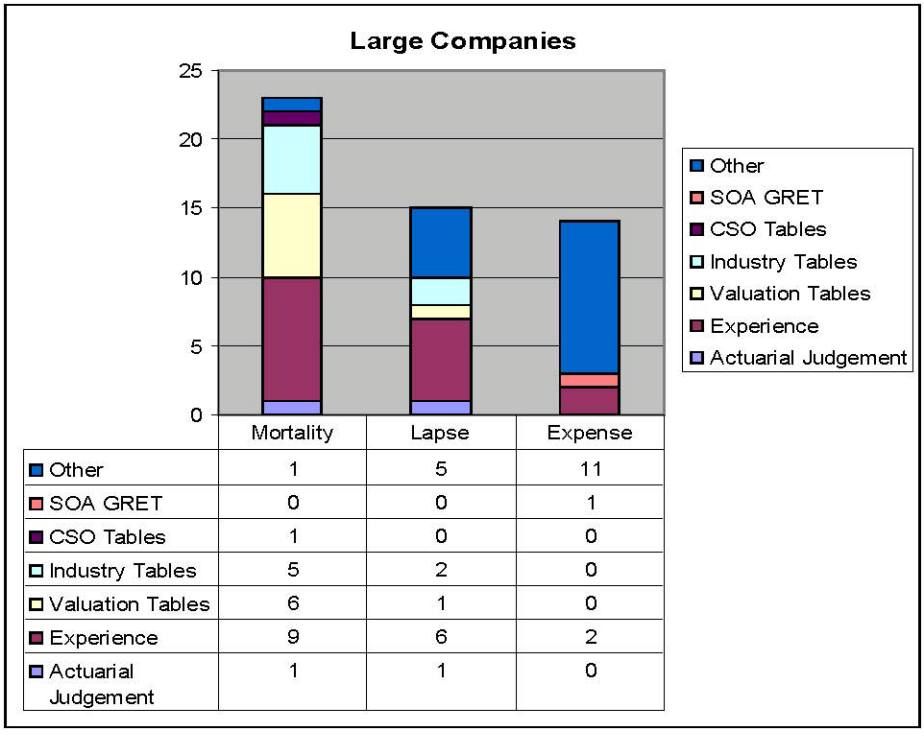


	Mortality	Lapse	Expense
Other	0	1	2
SOA GRET	0	0	2
CSO Tables	0	0	0
Industry Tables	0	0	0
Valuation Tables	3	1	0
Experience	1	2	2
Actuarial Judgement	1	0	0

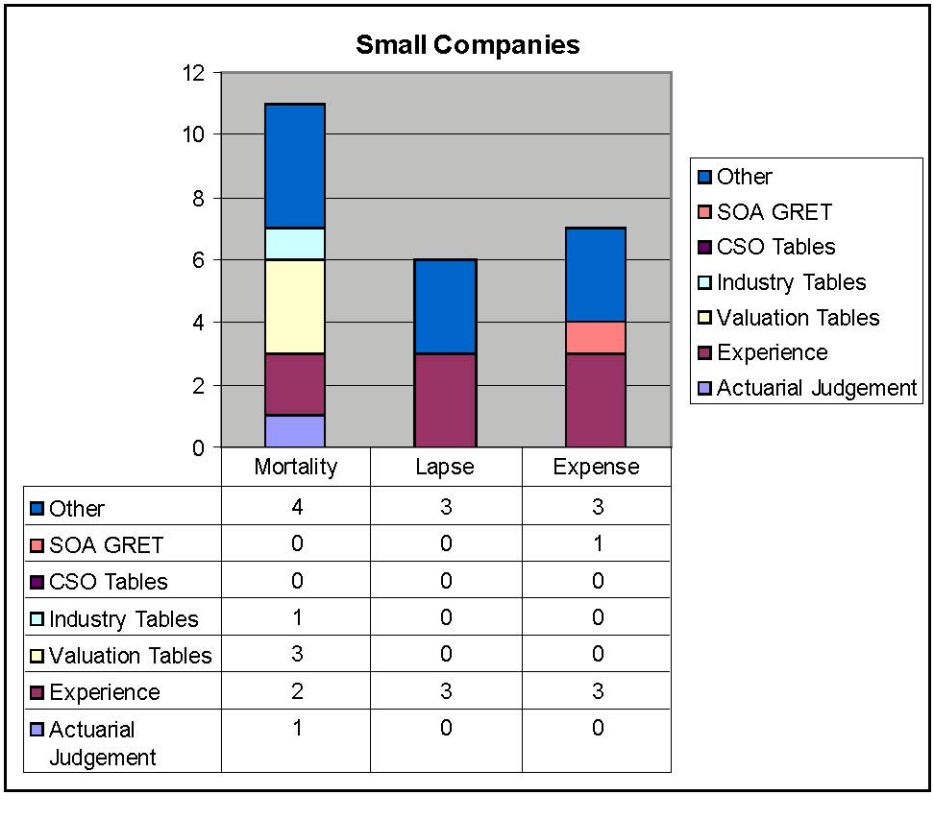
Appendix A: Analysis of Survey Responses

Question 11: Which functional areas use the results of credibility analysis?

FINANCIAL SIZE



	Mortality	Lapse	Expense
Other	1	5	11
SOA GRET	0	0	1
CSO Tables	1	0	0
Industry Tables	5	2	0
Valuation Tables	6	1	0
Experience	9	6	2
Actuarial Judgement	1	1	0

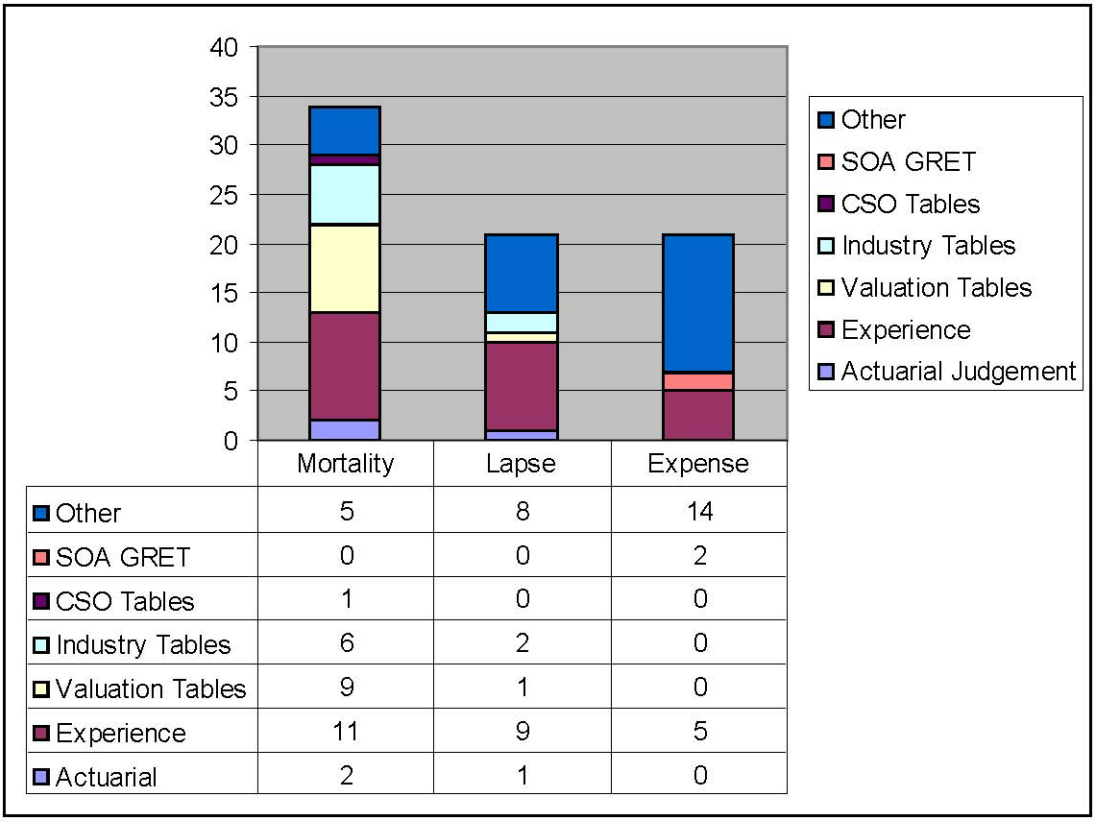


	Mortality	Lapse	Expense
Other	4	3	3
SOA GRET	0	0	1
CSO Tables	0	0	0
Industry Tables	1	0	0
Valuation Tables	3	0	0
Experience	2	3	3
Actuarial Judgement	1	0	0

Appendix A: Analysis of Survey Responses

Question 11: Which functional areas use the results of credibility analysis?

THE WHOLE



	Mortality	Lapse	Expense
Other	5	8	14
SOA GRET	0	0	2
CSO Tables	1	0	0
Industry Tables	6	2	0
Valuation Tables	9	1	0
Experience	11	9	5
Actuarial	2	1	0

Question 12: Comments pertaining to Question 11

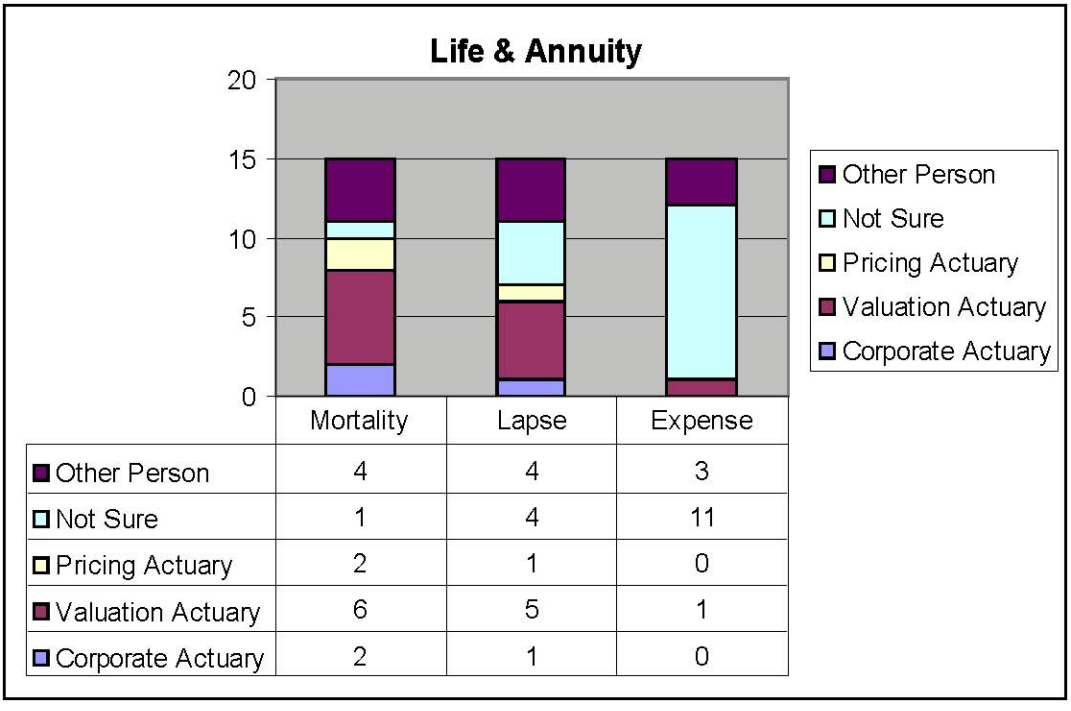
• "Typically, we blend actual experience on a block with the original mortality assumptions - where original mortality may have been based on 75-80 or vbt or on industry experience provided by reinsurers. For lapse, we again simply update the prior assumptions (usually based on a combination of actuarial judgment and experience) based on actual experience."

• "Not currently using credibility approach."

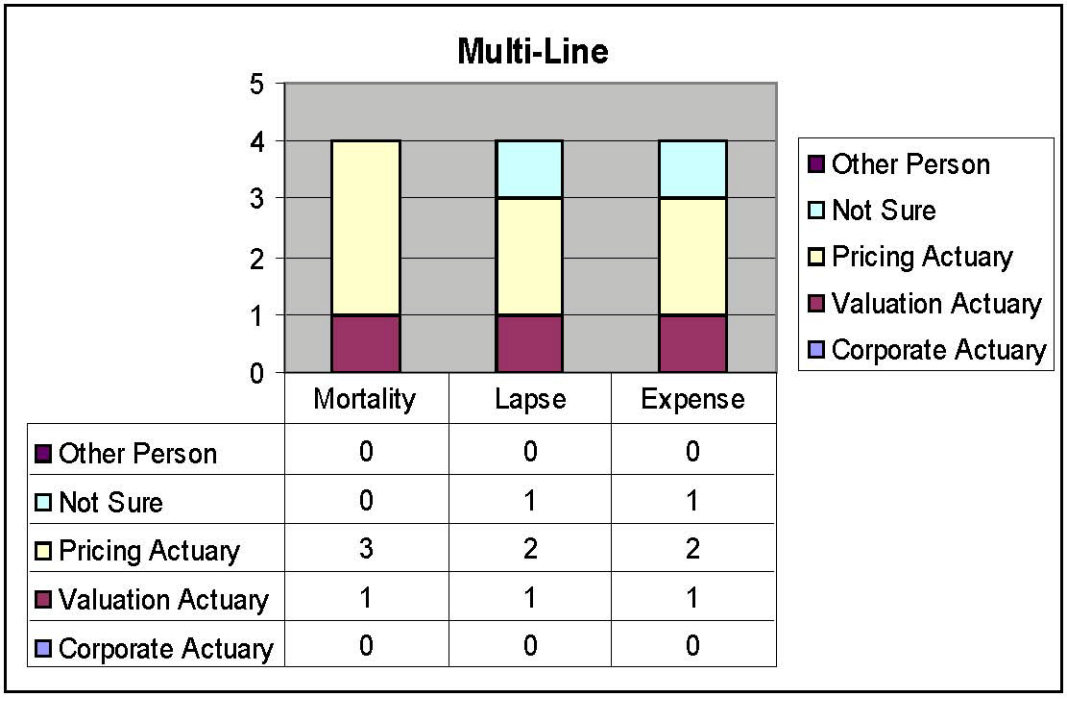
Appendix A: Analysis of Survey Responses

Question 13: Who determines the methods of the credibility analysis?

LIFE & ANNUITY VS. MULTI-LINE



	Mortality	Lapse	Expense
Other Person	4	4	3
Not Sure	1	4	11
Pricing Actuary	2	1	0
Valuation Actuary	6	5	1
Corporate Actuary	2	1	0

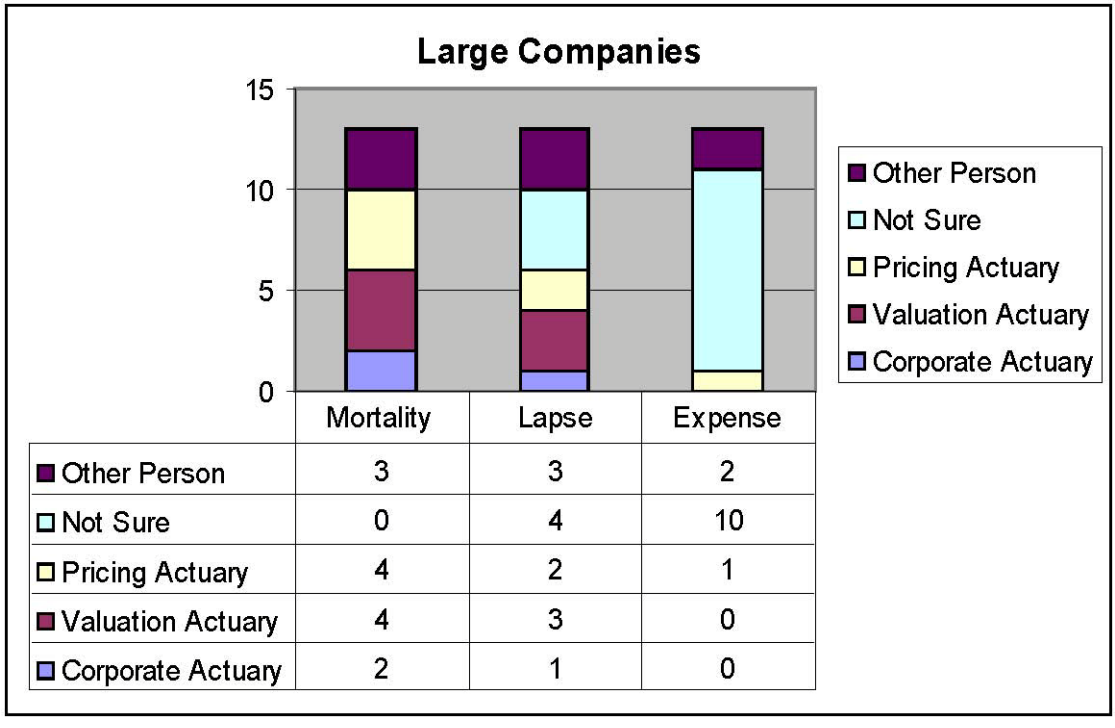


	Mortality	Lapse	Expense
Other Person	0	0	0
Not Sure	0	1	1
Pricing Actuary	3	2	2
Valuation Actuary	1	1	1
Corporate Actuary	0	0	0

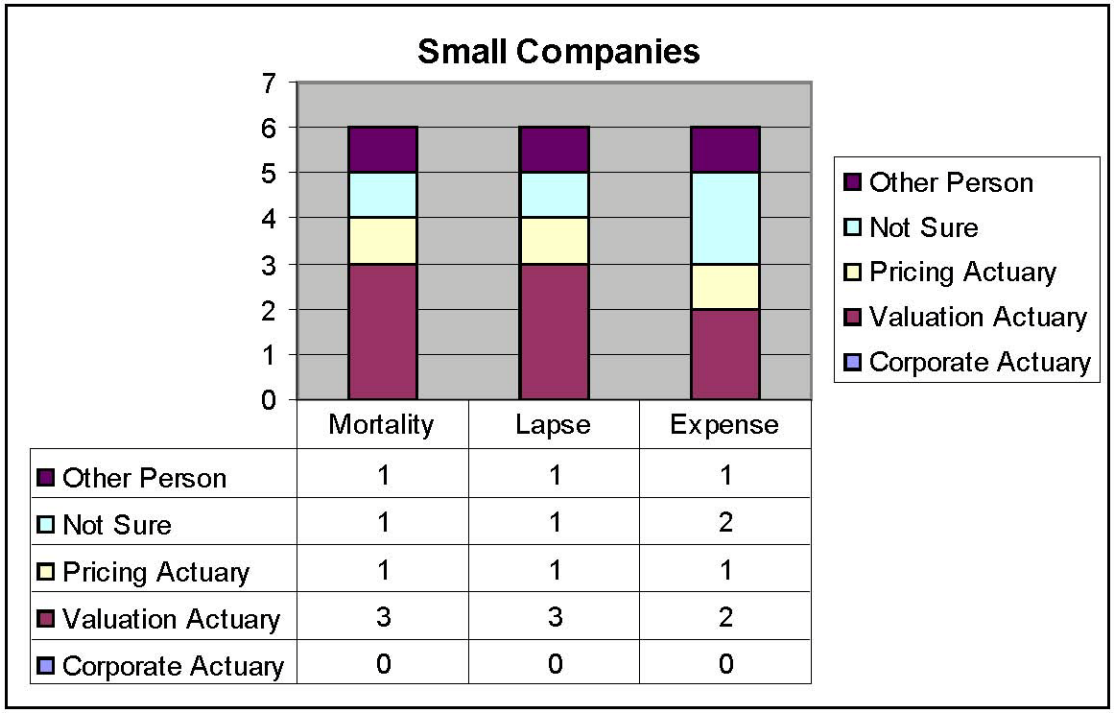
Appendix A: Analysis of Survey Responses

Question 13: Who determines the methods of the credibility analysis?

FINANCIAL SIZE



	Mortality	Lapse	Expense
Other Person	3	3	2
Not Sure	0	4	10
Pricing Actuary	4	2	1
Valuation Actuary	4	3	0
Corporate Actuary	2	1	0

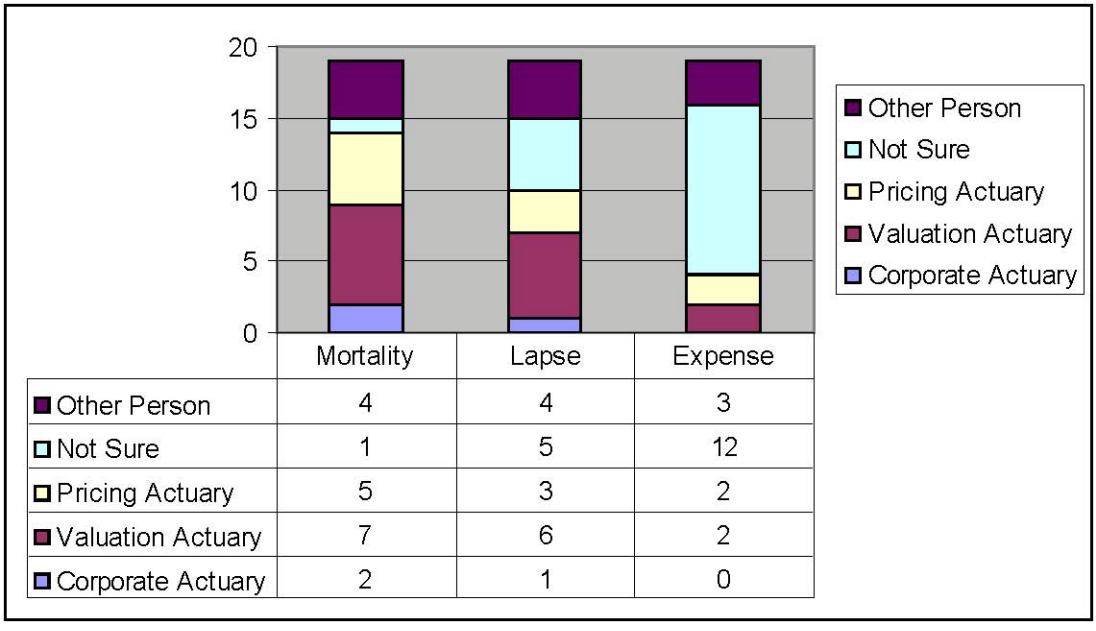


	Mortality	Lapse	Expense
Other Person	1	1	1
Not Sure	1	1	2
Pricing Actuary	1	1	1
Valuation Actuary	3	3	2
Corporate Actuary	0	0	0

Appendix A: Analysis of Survey Responses

Question 13: Who determines the methods of the credibility analysis?

THE WHOLE



	Mortality	Lapse	Expense
Other Person	4	4	3
Not Sure	1	5	12
Pricing Actuary	5	3	2
Valuation Actuary	7	6	2
Corporate Actuary	2	1	0

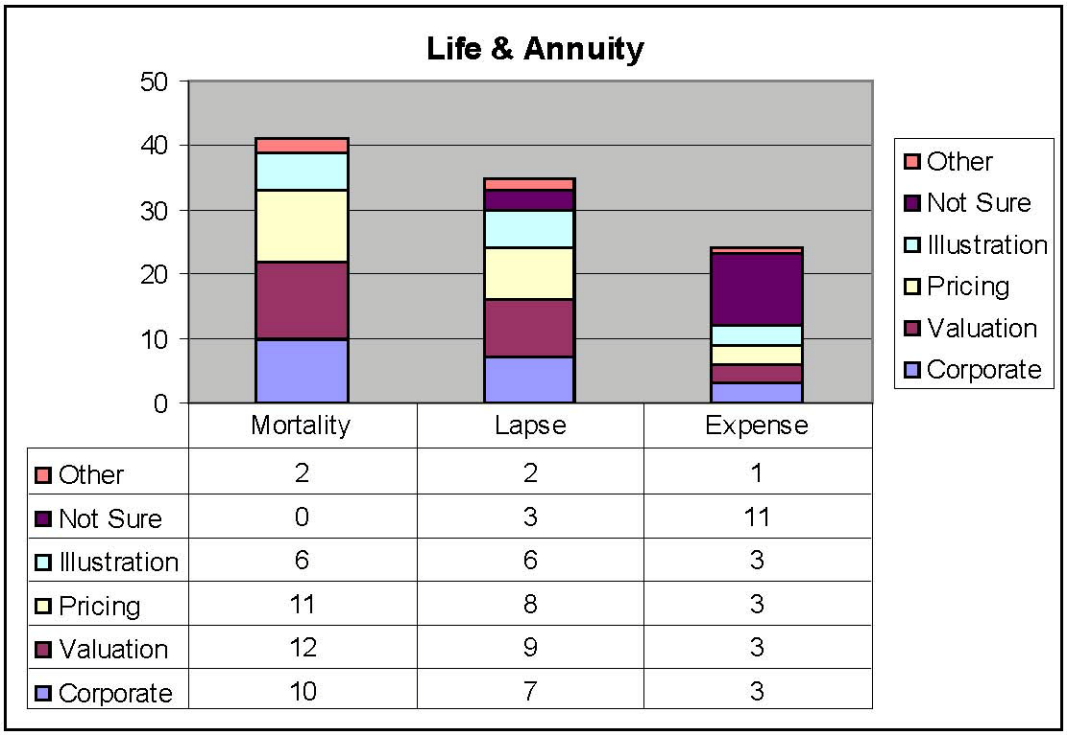
Question 14: Comments pertaining to Question 13

- "The experience actuary makes recommendations to a team that includes corporate, valuation and pricing actuaries."
- "Chief Actuary."
- "Multiple (Pricing, Corporate, Valuation Actuaries) as needed."
- "Methods are approved by the Assumptions Committee. The Corporate Actuary, Valuation Actuary and Pricing Actuary are all members of the committee."
- "Expense Actuary"

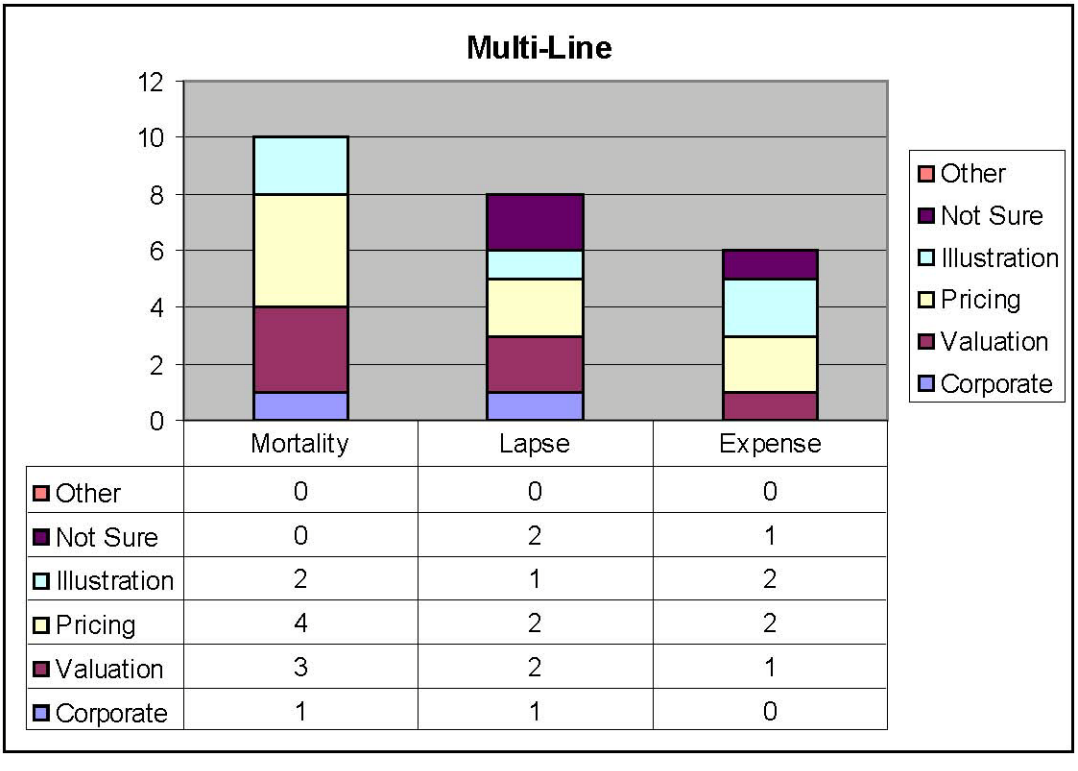
Appendix A: Analysis of Survey Responses

Question 15: Who uses the results of the credibility analysis?

LIFE & ANNUITY VS. MULTI-LINE



	Mortality	Lapse	Expense
Other	2	2	1
Not Sure	0	3	11
Illustration	6	6	3
Pricing	11	8	3
Valuation	12	9	3
Corporate	10	7	3

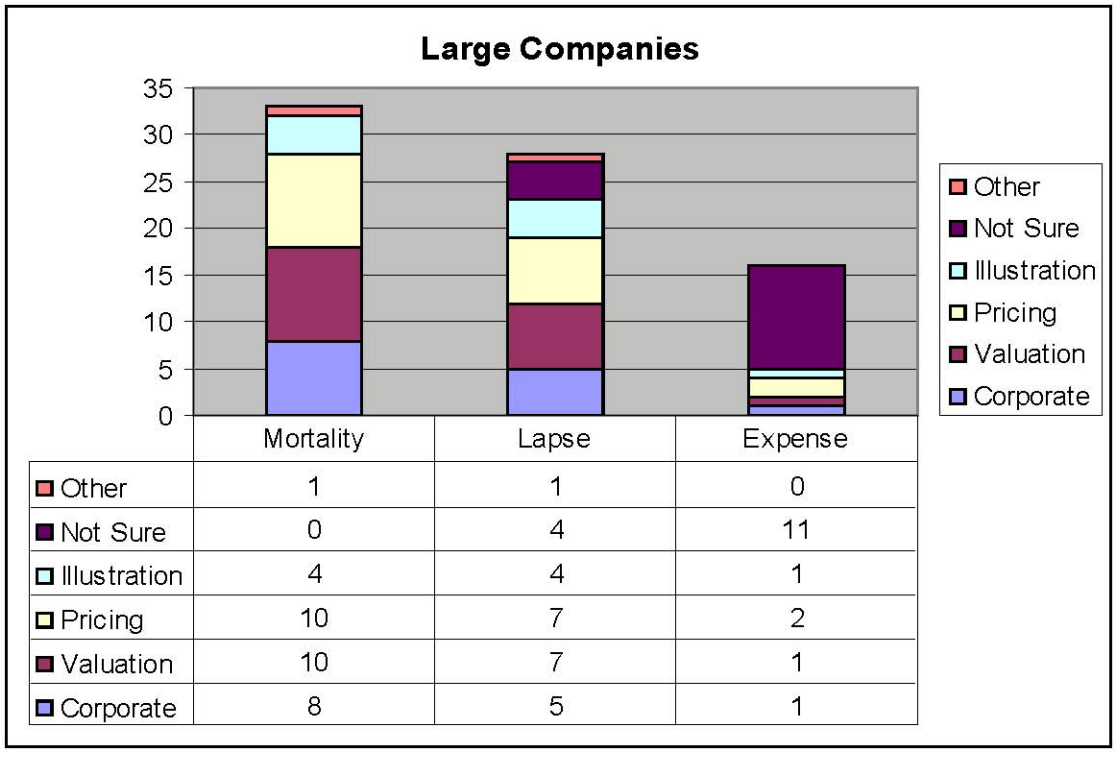


	Mortality	Lapse	Expense
Other	0	0	0
Not Sure	0	2	1
Illustration	2	1	2
Pricing	4	2	2
Valuation	3	2	1
Corporate	1	1	0

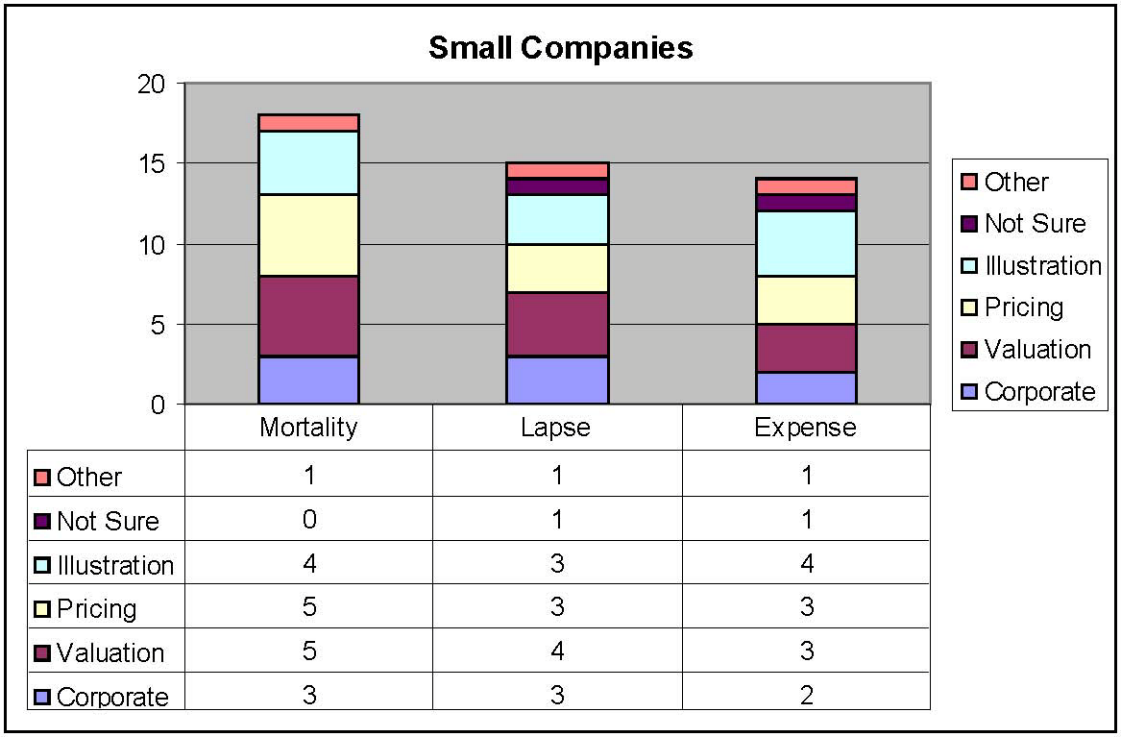
Appendix A: Analysis of Survey Responses

Question 15: Who uses the results of the credibility analysis?

FINANCIAL SIZE



	Mortality	Lapse	Expense
Other	1	1	0
Not Sure	0	4	11
Illustration	4	4	1
Pricing	10	7	2
Valuation	10	7	1
Corporate	8	5	1

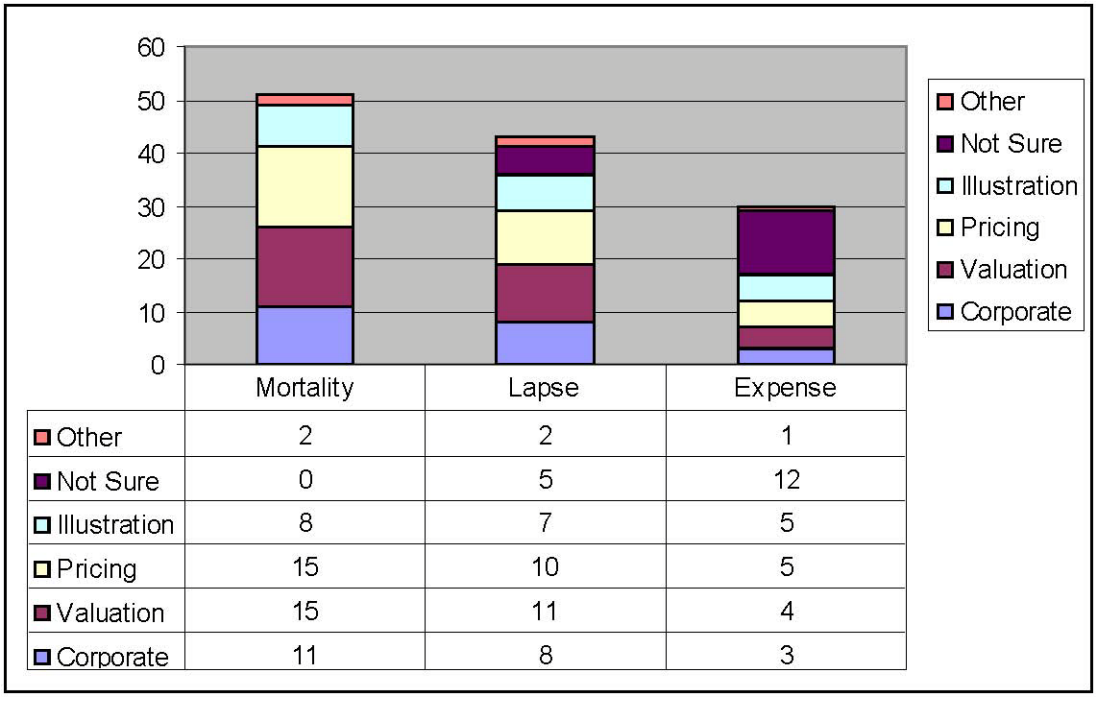


	Mortality	Lapse	Expense
Other	1	1	1
Not Sure	0	1	1
Illustration	4	3	4
Pricing	5	3	3
Valuation	5	4	3
Corporate	3	3	2

Appendix A: Analysis of Survey Responses

Question 15: Who uses the results of the credibility analysis?

THE WHOLE



	Mortality	Lapse	Expense
Other	2	2	1
Not Sure	0	5	12
Illustration	8	7	5
Pricing	15	10	5
Valuation	15	11	4
Corporate	11	8	3

Question 16: Comments pertaining to Question 15

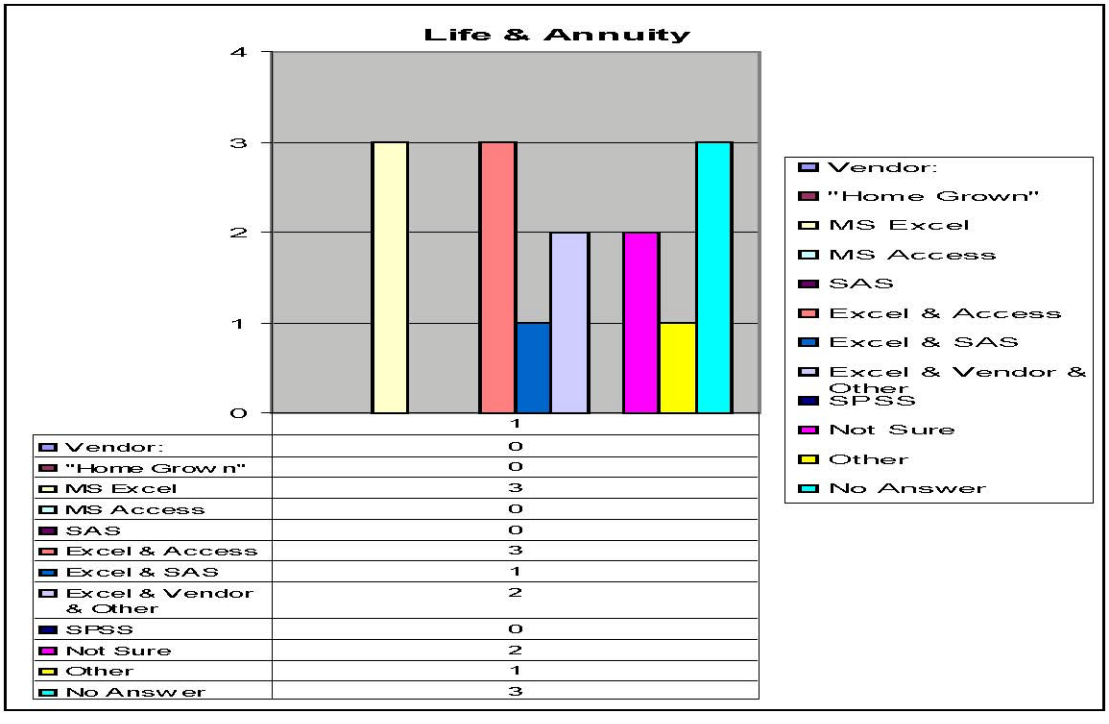
• "The experience actuary uses the results to create proposals for the team that includes corporate, valuation, and pricing actuaries. No assumptions go into production in any area without being vetted by that team."

• "Small company without separate areas everyone uses same analysis."

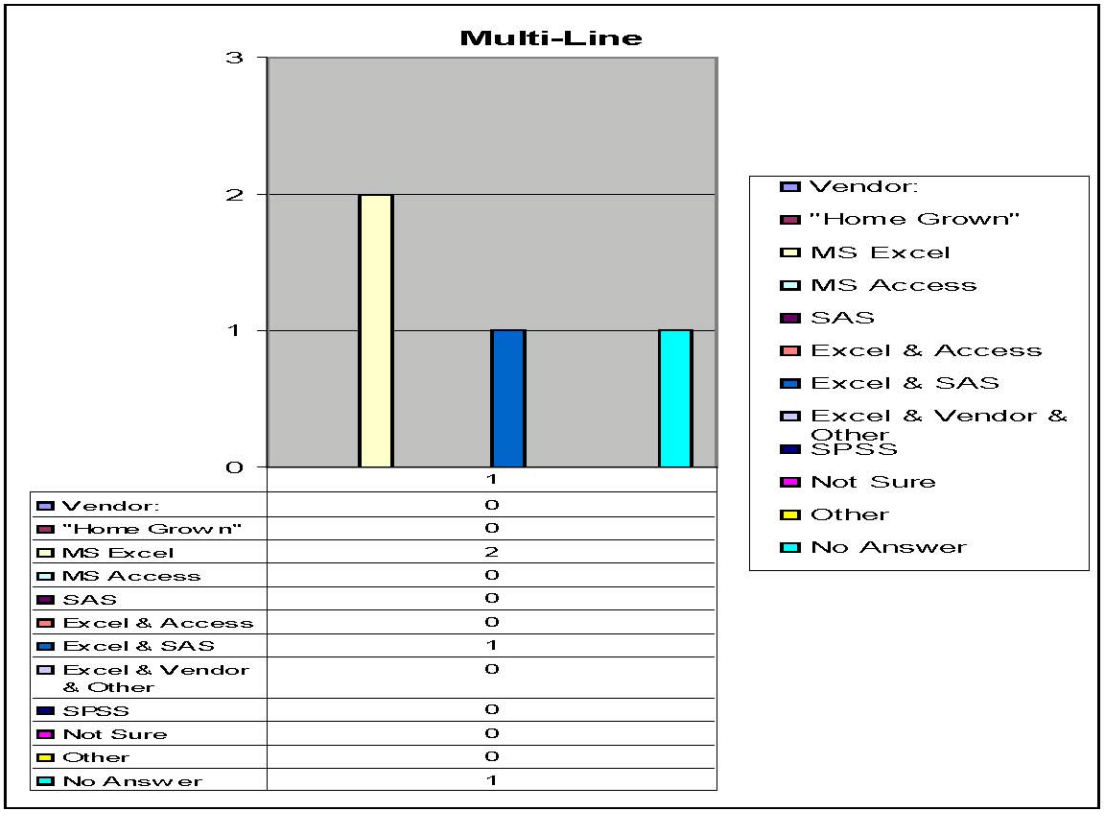
Appendix A: Analysis of Survey Responses

Question 18: What software does your company use in credibility analysis?

LIFE & ANNUITY VS. MULTI-LINE



	1
Vendor:	0
"Home Grown"	0
MS Excel	3
MS Access	0
SAS	0
Excel & Access	3
Excel & SAS	1
Excel & Vendor & Other	2
SPSS	0
Not Sure	2
Other	1
No Answer	3

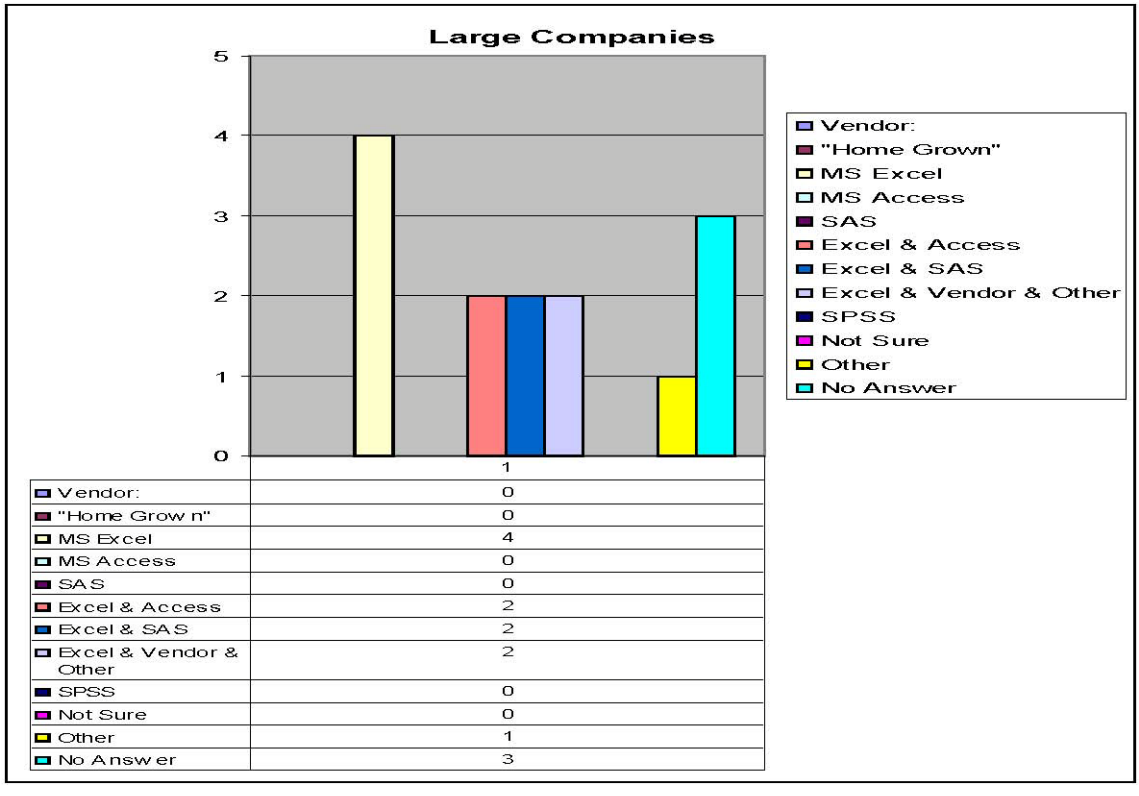


	1
Vendor:	0
"Home Grown"	0
MS Excel	2
MS Access	0
SAS	0
Excel & Access	0
Excel & SAS	1
Excel & Vendor & Other	0
SPSS	0
Not Sure	0
Other	0
No Answer	1

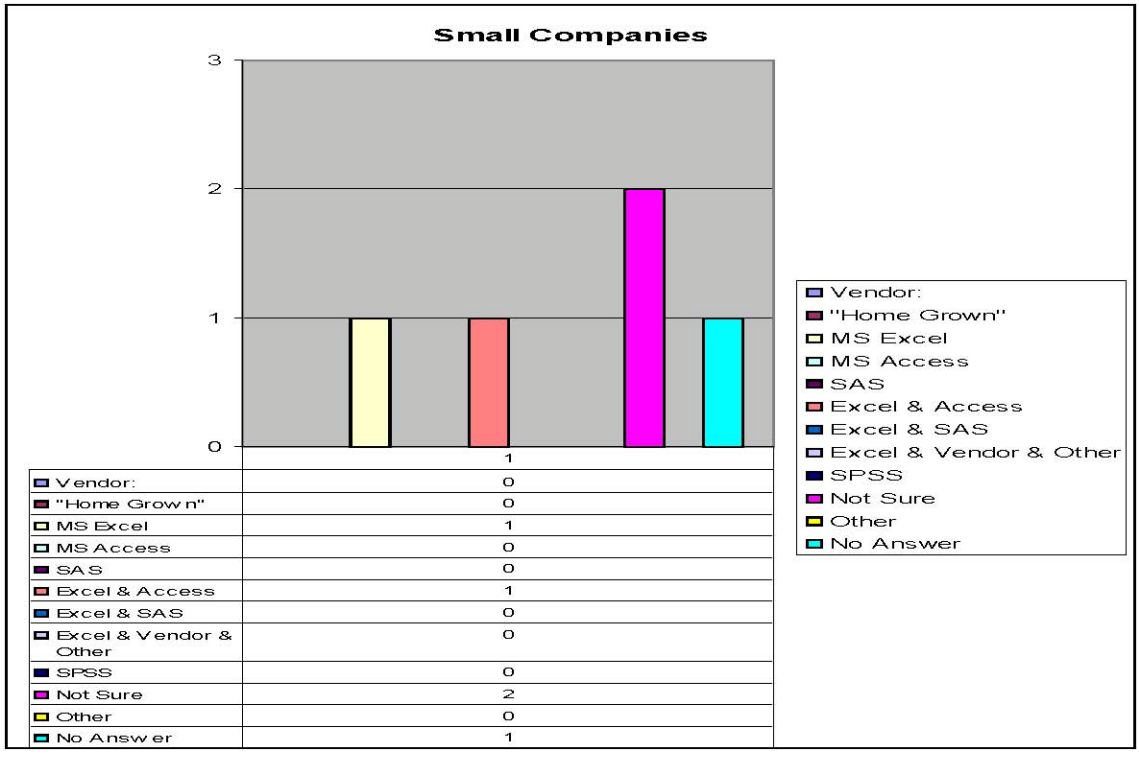
Appendix A: Analysis of Survey Responses

Question 18: What software does your company use in credibility analysis?

FINANCIAL SIZE



	1
Vendor:	0
"Home Grown"	0
MS Excel	4
MS Access	0
SAS	0
Excel & Access	2
Excel & SAS	2
Excel & Vendor & Other	2
SPSS	0
Not Sure	0
Other	1
No Answer	3

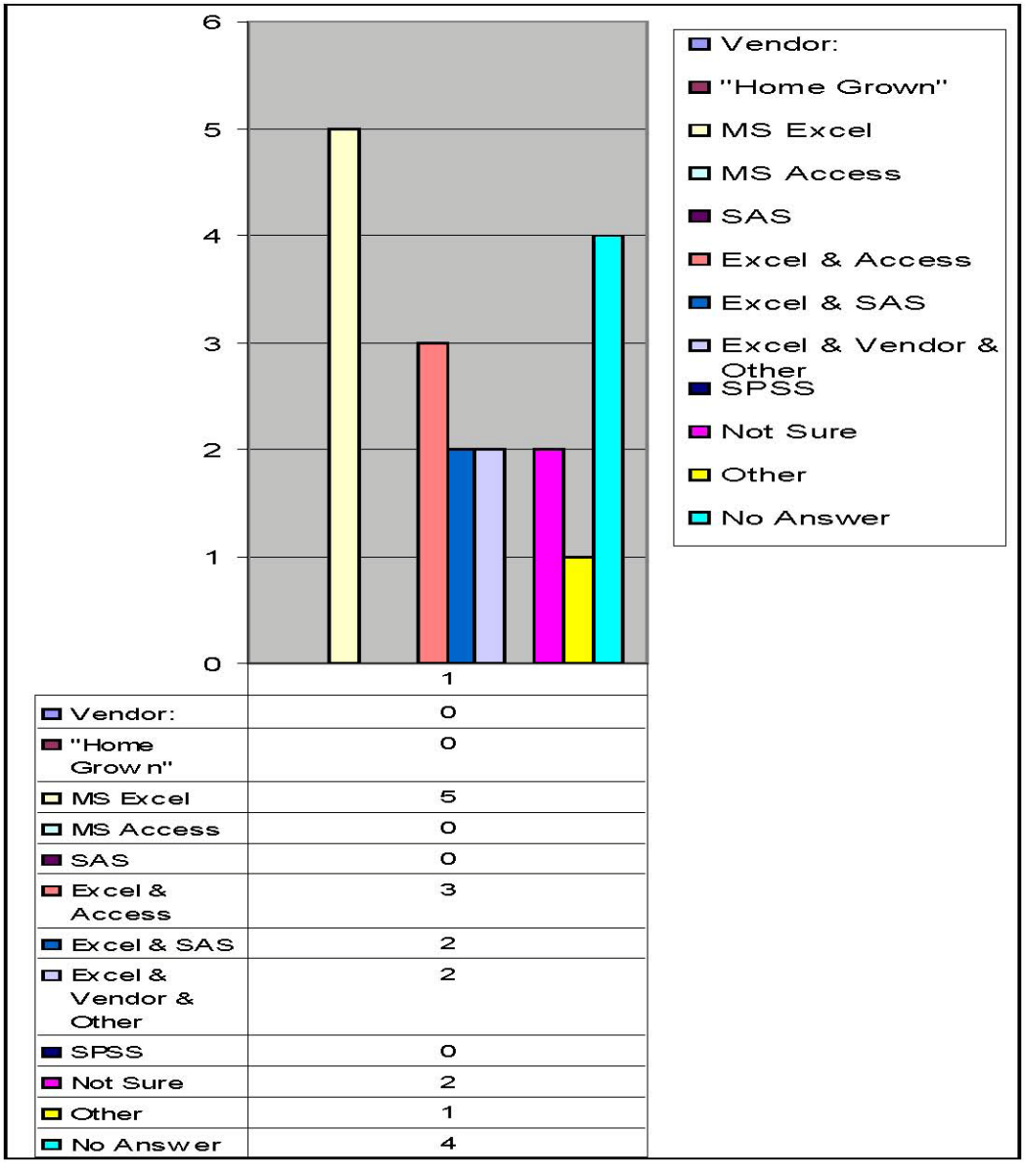


	1
Vendor:	0
"Home Grown"	0
MS Excel	1
MS Access	0
SAS	0
Excel & Access	1
Excel & SAS	0
Excel & Vendor & Other	0
SPSS	0
Not Sure	2
Other	0
No Answer	1

Appendix A: Analysis of Survey Responses

Question 18: What software does your company use in credibility analysis?

THE WHOLE



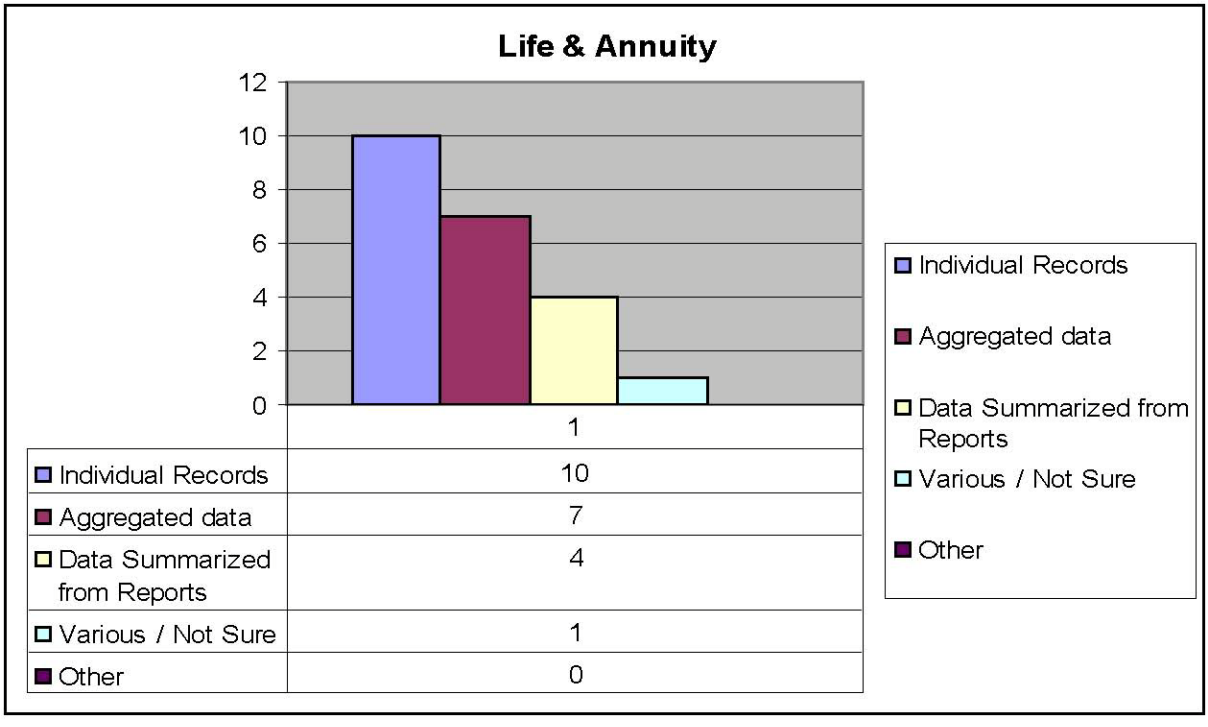
	1
Vendor:	0
"Home Grown"	0
MS Excel	5
MS Access	0
SAS	0
Excel & Access	3
Excel & SAS	2
Excel & Vendor & Other	2
SPSS	0
Not Sure	2
Other	1
No Answer	4

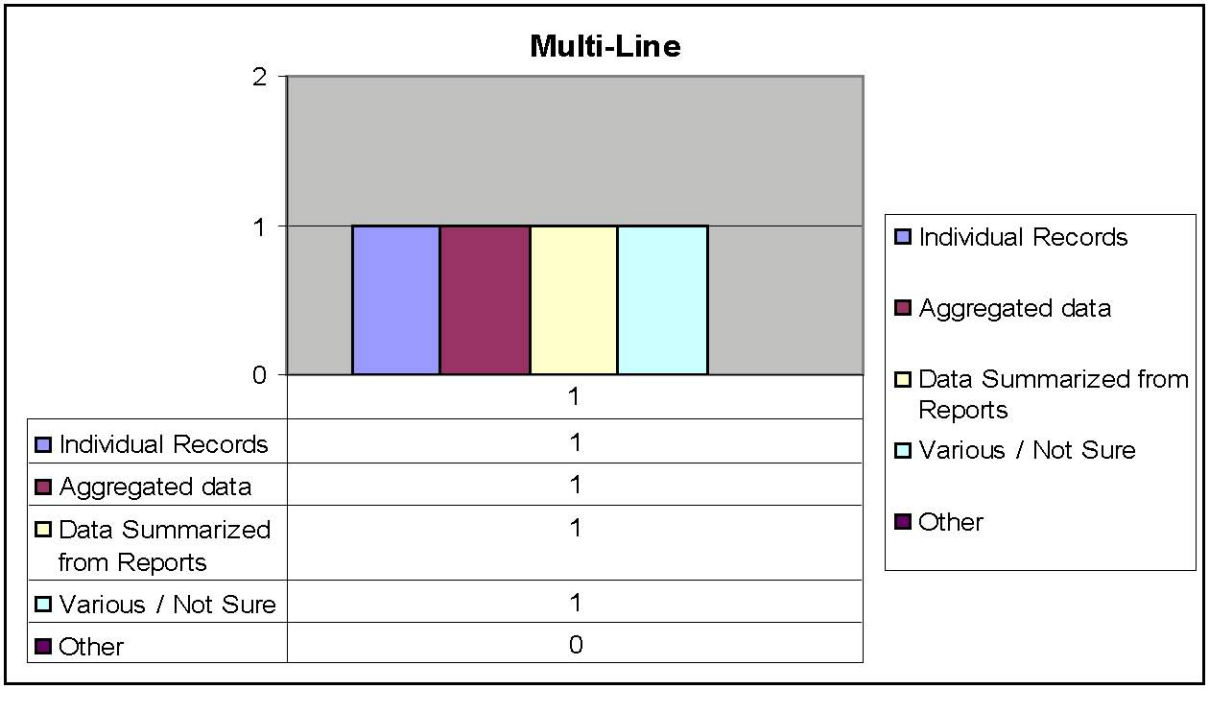
***No Comments Associated with this Question

Appendix A: Analysis of Survey Responses

Question 19: What software does your company use in credibility analysis?

LIFE & ANNUITY VS. MULTI-LINE

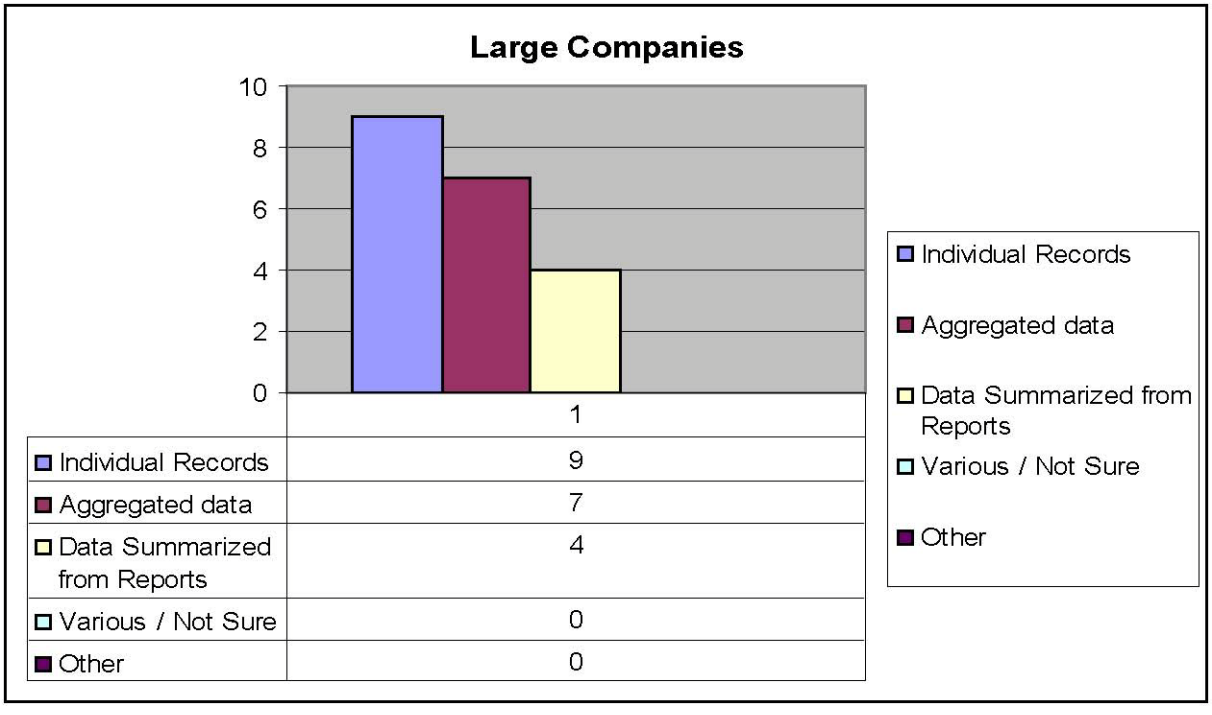


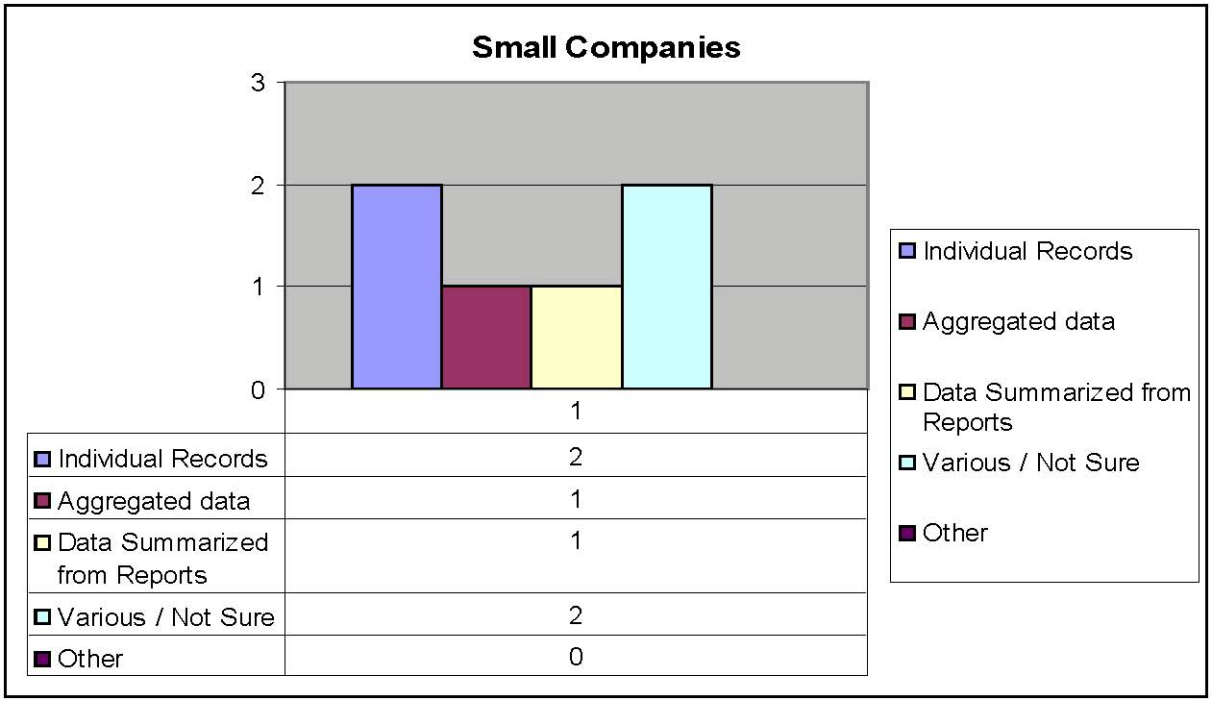


Appendix A: Analysis of Survey Responses

Question 19: What software does your company use in credibility analysis?

FINANCIAL SIZE

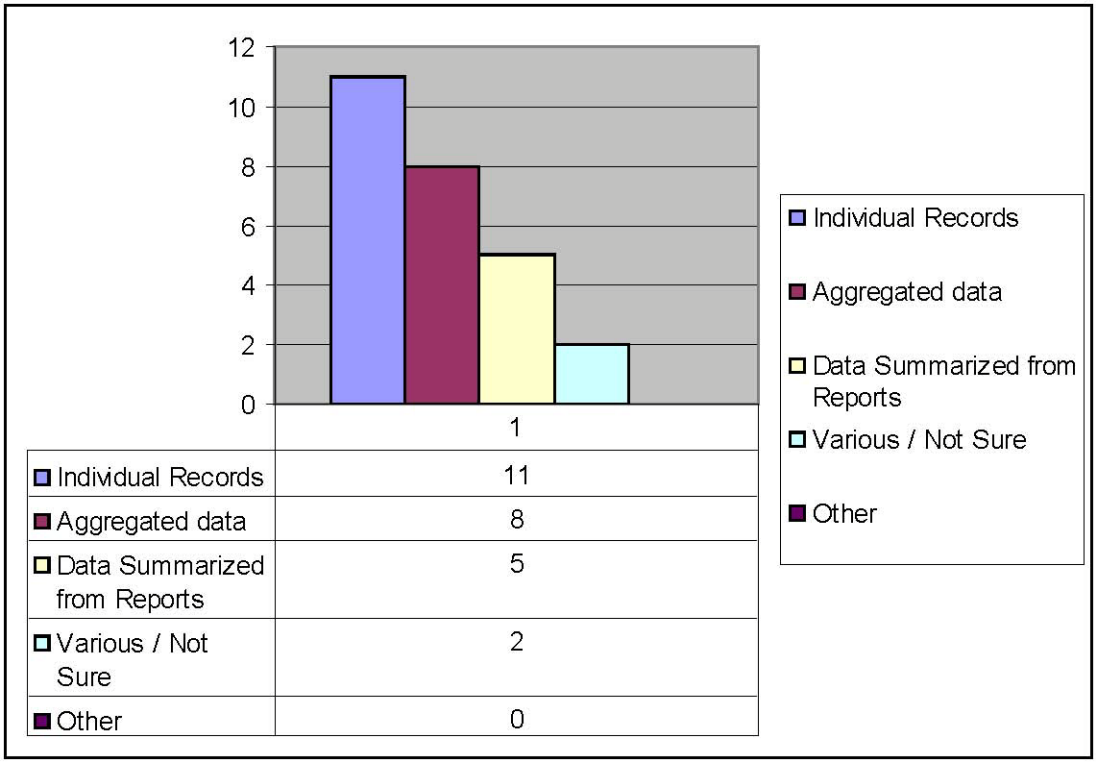




Appendix A: Analysis of Survey Responses

Question 19: What software does your company use in credibility analysis?

THE WHOLE



	1
Individual Records	11
Aggregated data	8
Data Summarized from Reports	5
Various / Not Sure	2
Other	0

***No Comments pertaining to this Question

APPENDIX B: Companies invited to participate in 2009 Credibility Theory Survey

The following US life insurance companies were invited to participate in the survey of current practices and use of Credibility Theory by way of an email sent to the Chief Actuary on record at the Society of Actuaries. An initial email was sent March 3, 2009, with reminder emails sent to the same distribution (excluding those whose companies had already submitted a completed survey) on March 12 and March 19. Participation was closed on March 29, 2009.

Companies denoted in italics submitted responses that were used in survey tabulations.

COMPANIES

AEGON Direct Marketing Services Inc
AEGON USA Inc
AFLAC
AGL Life Assurance Co
AIG American General
AIG SunAmerica Life Assurance Company
AIG VALIC
Alfa Insurance Co
Allianz Life Insurance Co of NA
Allstate Life Insurance Co
Allstate Life Insurance Co of NY
Amalgamated Life Insurance Co
American Equity Investment Life Ins Co
American Express
American Family Life Insurance Co
American Fidelity Group
American Life Insurance Company
American National Insurance Company
American Republic Insurance Co
Americo Life Inc
Amerigroup
Ameriprise Financial Inc
Ameritas Holding Company
AmerUs Group
Amica Mutual Insurance Co
Assurant Employee Benefits
Aviva USA
AXA Equitable Life Insurance Co
AXA Financial MONY
Baltimore Life
Bankers Life and Casualty Company
Banner Life Insurance Co
BCS Insurance Group
Beneficial Life Insurance Company
Berkshire Life Insurance Company
Best Life & Health Insurance Co
Boston Mutual Life Insurance Co
Central States Indemnity Co
CIGNA
Citi Assurance Services
Citizens Inc
CNA Insurance Companies
Colonial Supplemental Insurance
Columbian Mutual Life Insurance Co
Combined Ins Co of America
Commerce Insurance Company
Commonwealth Annuity & Life Insurance Co
Conseco Insurance Companies
Continental American Insurance
Country Insurance & Financial Services
CUNA Mutual Insurance Society
Deseret Mutual
EMC National Life Company
EPIC Life Insurance Co
Equitable Life
EquiTrust Life Insurance Company
Erie Insurance
Family Heritage Life Insurance Co
Farm Bureau Life Insurance Company
Farm Bureau Life of Michigan
Farm Family Life Insurance Co.
Farmers New World Life Insurance Co
FCCI Insurance Group
Federal Life Insurance Co (Mutual)
Federated Mutual Insurance Co
Fidelity Investments Life
First Investors Life Insurance Company
Foresters

APPENDIX B (continued)

Forethought Financial Services Inc
Fort Dearborn Life Insurance Company
Genworth Financial
Gerber Life Insurance Co
Government Personnel Mutual Life
Grange Insurance Group
Great American Life Insurance Company
Great-West Life & Annuity Insurance Co
Guarantee Trust Life Insurance
Guardian Life Insurance Co.
Hartford Life Insurance Company
Health Care Svc Corp Mut Legal Reserve
Homesteaders Life Company
Horace Mann Insurance Companies
HSBC Insurance North America
IAC Group
Illinois Mutual Life Ins Co
Indiana Farm Bureau Insurance
ING Life & Annuity
ING US Financial Services
Jackson National Life Insurance Co
James River Group
John Hancock Life Insurance Company
Legion Insurance Co
Liberty Bankers Life Insurance Co
Liberty Life Insurance Co
Liberty Mutual Group
Liberty National Life Insurance Co
Life Insurance Company of the Southwest
LifeCare Assurance Co
Life of the South
Lincoln Financial Group
Lincoln National Life Insurance
Londen Insurance Group
Madison National Life Insurance Co
Manulife Financial
Massachusetts Mutual Life Insurance Co
Merit Life Insurance Co
Merrill Lynch Life Insurance Company
Metropolitan Life Insurance Co
Midland National Life Insurance Company
Modern Woodmen of America
Monarch Life Insurance Co
Monumental Life Insurance Company
Mutual of America Life Insurance Company
Mutual of Omaha Insurance Co
NACOLAH
National Benefit Life Ins Co
National Guardian Life Ins Co
National Life Insurance Co
National Planning Holdings
National States Ins Co
National Teachers Association
National Western Life Insurance Company
Nationwide Financial
New England Financial
New Era Life Ins Co
New York Life Insurance Co
North American Company for L & H Ins
Northwestern Mutual
Ohio National Financial Services
Old Mutual Financial Network
Old Republic General Services Inc
OneAmerica Financial Partners
Oregon Mutual Insurance Co
Oxford Life Ins Co
Pacific Guardian Life Insurance
Pacific Life Insurance Company
Pan-American Life Insurance Co
Penn Mutual Life Insurance Co
Penn National Ins
Phoenix Life Insurance Company
Presidential Life Ins Co
Pretime Financial
Primerica Life Insurance Co
Principal Financial Group
Protective Life Corp.
Prudential Insurance Company
ReliaStar Life Insurance Company
Reserve National Ins Co
RGA Reinsurance Company
Royal Neighbors of America
Sagicor Life Insurance Co
Savings Bank Life Insurance Co of MA
SBLI USA Mutual Life Ins Co Inc

APPENDIX B (continued)

Scottish Re (US) Inc
Securian Financial Group
Security Benefit Life Insurance Company
Security Mutual Life Ins Co of NY
Security National Life Insurance Co
Sentry Insurance Group
Shenandoah Life Ins Co
Southern Farm Bureau Life Insurance
State Farm Life Insurance Co
Sun Life Assurance Company of CA (US)
Sun Life Financial
Sun Life of Canada
Symetra Life Insurance Co
Teachers Protective Mutual Life Insurance Co
Texas Life Ins Co
Thrivent Financial for Lutherans
TIAA-CREF
Torchmark Corp
Transamerica Life Insurance Company
Travelers Life & Annuity
Union Central Life Insurance Co
United American Ins Co
United Concordia Cos
United Farm Family Life Ins Co
United Heritage
United Investors Life Insurance Co
Unity Mutual Life Ins Co
Universal American Corp
UnumProvident Corporation
USAA Life
VantisLife Ins Co
West Coast Life Ins Co
Western & Southern Financial Group
Western Reserve Life Assurance Co of OH
Western United Life Assurance Co
White Mountains Re America
XL Insurance America Inc

2009 Credibility Theory Survey

Page 1 - Survey Introduction

MIB Solutions, on behalf of the Society of Actuaries' Credibility Theory Practices Project Oversight Group, requests your responses to a brief, confidential survey regarding the use and practice of Credibility Theory at your company individual life and annuities. Actuaries from over 180 U.S. Life Insurance Companies are being asked to participate; aggregate results will provide an understanding of the degree to which Credibility Theory is practiced across the industry.

All survey information will remain confidential to the SOA and will only be used for the purposes of this study. No company's proprietary methods or practices will be disclosed, except as non-identifiable information within the context of the analysis of survey responses.

The survey covers the use of Credibility Theory for the following product types (individual lines only):

Whole Life
Term Life
Universal & Variable Universal Life
Fixed & Variable Annuities

If you or your company does handle the any of these lines of business, please let us know so we can remove your name from our mailing list.

In this survey the application of credibility theory refers to the methods and practices employed in determining the level of reliance that can be placed on company experience and/or the approaches used in blending company data with industry or other data. An in-depth description of Credibility Theory can be found at http://www.actuary.org/pdf/practnotes/life_credibility08.pdf .

The Survey is brief and should take no more than 10 minutes to complete. In consideration of your time, all respondents will receive a complementary MIB Solutions / SOA USB drive and an early look at aggregate results when they become available.

If you have any questions, please contact the survey coordinator, Jan Palmbach, Senior Data Coordinator at jpalmbach@mib.com, or call 781-751-6128.

If others in your company are better suited to answer this survey, please feel free to forward the originating email to the appropriate person.

1. **Does your company either use or plan to use Credibility Theory on your Mortality, Lapse or Expense data for individual life or annuities?**

   ◯ Yes

   ◯ No

Please note that a response of "No" to Question 1 will direct you to the final page of the survey.

Page 2 - General Information

The following clarifying questions should be responded to only with regard to the practice of credibility theory in the individual life or annuity business. These questions survey how credibility theory is or could be applied.

If you have any questions, please contact the survey coordinator Jan Palmbach, Senior Data Coordinator, at jpalmbach@mib.com, or call 781-751-6128 [direct].

Check all answers that apply. Please note that all questions with '*' require at least one answer per row.

2. **Does your company have a standard of applying credibility theory to the evaluation of its experience or operating results?**

	Yes	No, but plan to do so in 1-2 years	No, but plan to do so as PBR is adopted	No, not at this time	Not sure
Mortality	○	○	○	○	○
Lapse	○	○	○	○	○
Expense	○	○	○	○	○

3. **Additional comments on applying a standard to credibility theory.**

4. **When do you or your company apply credibility theory to the evaluation of experience or operating results? (Check all that apply).**

	During the annual reporting cycle	During quarterly financial reporting	Ad hoc	Credibility theory not currently applied
Mortality	☐	☐	☐	☐
Lapse	☐	☐	☐	☐
Expense	☐	☐	☐	☐

5. **If your company applies credibility theory at other times, please specify or comment.**

6. **To which products might you be applying Credibility Theory? (Check all that apply and only check where Credibility Theory is applied.)**

	Whole Life	Term Life	Universal Life	Fixed Annuities	Variable Annuities	Not sure
Mortality	☐	☐	☐	☐	☐	☐

Lapse	☐	☐	☐	☐	☐	☐
Expense	☐	☐	☐	☐	☐	☐

Page 3 - Clarifying Questions

The following clarifying questions should be responded to only with regard to the practice of credibility theory in the individual life business. These questions survey how credibility theory is or could be applied.

If you have any questions, please contact the survey coordinator Jan Palmbach, Senior Data Coordinator, at jpalmbach@mib.com, or call 781-751-6128 [direct].

Check all answers that apply. Please note that all questions marked with an (*) require at least one answer.

7. **Which credibility approaches are used (or are planned)? (Check all that apply).**
(In this survey we will consider CIA Limited Fluctuation Method to be with Normalization.)

	Actuarial Judgment	Limited Fluctuation Method	CAI Limited Fluctuation Method	Greatest Accuracy Method/ Buhlmann	Bayesian Approach	Not sure
Mortality	☐	☐	☐	☐	☐	☐
Lapse	☐	☐	☐	☐	☐	☐
Expense	☐	☐	☐	☐	☐	☐

8. **Additional comments on credibility theory approaches.**

9. **Which functional areas use the results of credibility analysis? (Check all that apply).**

	Pricing	Illustration	Actuarial Opinion Memoran-dum Report	Valuation	Not sure
Mortality	☐	☐	☐	☐	☐
Lapse	☐	☐	☐	☐	☐
Expense	☐	☐	☐	☐	☐

10. **Please specify other functional areas using the results of credibility analysis.**

11. **What standard table does your company use to blend your experience with the credibility approach? (Check all that apply.)**

Actuarial Judgement	Comany experi-ence	Valuation Basic Tables	Industry Tables	CSO Tables	SOA GRET	Other/ Not sure

Mortality	☐	☐	☐	☐	☐	☐	☐
Lapse	☐	☐	☐	☐	☐	☐	☐
Expense	☐	☐	☐	☐	☐	☐	☐

12. **Please specify other tables or bases that your company may use in the credibility approach.**

13. **Who determines the methods of the credibility analysis?**

	Corporate Actuary	Valuation Actuary	Pricing Actuary	Not sure	Other person
Mortality	○	○	○	○	○
Lapse	○	○	○	○	○
Expense	○	○	○	○	○

14. **If another person determines the methods, please specify**

15. **Who uses the results of the credibility analysis? (Check all that apply).**

	Corporate Actuary	Valuation Actuary	Pricing Actuary	Illustration Actuary	Not sure	Other person
Mortality	☐	☐	☐	☐	☐	☐
Lapse	☐	☐	☐	☐	☐	☐
Expense	☐	☐	☐	☐	☐	☐

16. **Please specify if 'Other person' was selected.**

17. **Please describe your role in applying credibility theory to the evaluation of your company's experience or operating results. (Limit: 1000 characters)**

Page 4 - Technical Questions

The following questions pertain to technical aspects of the use of credibility theory in the life insurance business. These questions survey the technology behind the use of creditability theory.

If you have any questions, please contact the survey coordinator Jan Palmbach, Senior Data Coordinator, at jpalmbach@mib.com, or call 781-751-6128 [direct].

Check all answers that apply. Please note that all questions marked with an (*) require at least one answer.

18. **What software does your company use in credibility analysis? (Check all that apply).**

- [ ] Vendor software (please include vendor name below)
- [ ] "Home Grown" software application
- [ ] MS Excel
- [ ] MS Access
- [ ] SAS
- [ ] SPSS
- [ ] Not sure
- [ ] Other, please specify (e.g., APL, etc.)

19. **What data is used in the credibility approach? (Check all that apply).**

- [ ] Individual/ seriatim policy records
- [ ] Aggregated data
- [ ] Data summarized from reports
- [ ] Various/Not sure
- [ ] Other, please specify

Page 5 - THANK YOU FOR PARTICIPATING

20. **Please include your contact information to enable us contact you if we need clarification of some your answers. In addition we would like to send you an acknowledgement of your participation, and aggregate results when they become available.**

Name	
Title	
Company	
Work Phone	
Email Address	

If you have any questions, please contact Jan Palmbach, MIB Solutions' Survey Coordinator at jpalmbach@mib.com or call 781-751-6128 [direct].

Annotated Credibility Bibliography

Papers of historical interest (listed chronologically)

Mowbray, A. H. (1914), "How Extensive a Payroll Exposure Is Necessary to Give a Dependable Pure Premium?" *Proceedings of the Casualty Actuarial Society*, **I**, 24-30. [Introduced the concept of limited fluctuation credibility]

Whitney, A.W. (1918), "The Theory of Experience Rating," *Proceedings of the Casualty Actuarial Society*, **IV**, 274-292. [First paper to discuss what is now called greatest accuracy credibility]

Bailey, A. (1950), "Credibility Procedures," *Proceedings of the Casualty Actuarial Society*, **XXXVII**, 7-23, 94-115. [A summary of the start of the art at that time, but the mathematics is ad-hoc]

Longley-Cook, L. (1962), "An Introduction to Credibility Theory," *Proceeding of the Casualty Actuarial Society*, **XLIX**, 194-221. [This was on the CAS exam syllabus for many years]

Bühlmann, H. (1967), "Experience Rating and Credibility," *ASTIN Bulletin*, **4**, 199-207. [Put greatest accuracy on a sound mathematical footing, considered to be the most important paper on the subject]

Bühlmann, H. and Straub, E. (1970), "Glaubwürdigkeit für Schadensätze (credibility for loss ratios)," *Mitteilungen der Vereinigung Schweizerischer Versicherungs-Mathematiker*, **70**, 111-133. [Provided formulas for the estimation of the parameters in the greatest accuracy model, is the basis for the development of the formulas in this paper]

Kahn, P., ed. (1975), *Credibility: Theory and Applications*, New York: Academic Press [A compilation of papers presented at the 1974 Actuarial Research Conference, several extend credibility to more complex settings].

Sundt, B. (1986), Special issue on credibility theory, *Insurance: Abstracts and Reviews*, **2**. [Contains an extensive bibliography as of that date]

Klugman, S. (1987), "Credibility for Classification Ratemaking Via the Hierarchical Linear Model," *Proceedings of the Casualty Actuarial Society*, **LXXIV**, 272-321. [An application of the full Bayesian method (as compared to the empirical Bayesian method promoted in this paper]

Current textbooks on credibility (presented alphabetically by author), those marked (*) are on the syllabus for Society of Actuaries Exam C

Bühlmann, H. and Gisler, A. (2005), *A Course in Credibility Theory and its Applications*, The Netherlands: Springer

Goovaerts, M. J. and Hoogstad, W. J. (1987), *Credibility Theory, Surveys of Actuarial Studies No. 4*, Rotterdam: Nationale-Nederlanden.

(*)Herzog, T. (1999), *Introduction to Credibility Theory*, 3rd ed., Winsted, CT: ACTEX.

(*)Klugman, S., Panjer, H. and Willmot, G. (2008), *Loss Models, 3rd ed*, New York: Wiley.

(*)Mahler, H. and Dean, C. (2001), "Credibility," in *Foundations of Casualty Actuarial Science, 4th ed*, Arlington, VA: Casualty Actuarial Society. [For Exam C this material is supplemented by a student note "Topics in Credibility, available at www.soa.org/files/pdf/c-24-05.pdf]

Waters, H. R. (1993), *Credibility Theory*, Edinburgh: Department of Actuarial Mathematics & Statistics, Heriot-Watt University. [Used by the Faculty and Institute of