# Estimation of Employee Stock Option Exercise Rates and Firm Cost*

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**Abstract**

This paper is the first to perform a comprehensive estimation of employee stock option exercise behavior and option cost to firms. We develop a GMM-based methodology, robust to heteroskedasticity and correlation across exercises, for estimating the rate of voluntary option exercise as a function of the stock price path and of various firm and option holder characteristics. We use it to estimate an exercise function for a sample of 1.3 million employee-option grants to 530,266 employees at 103 publicly-traded firms between 1981–2009. We use the estimated exercise functions in a simulation based valuation model to analyze the effect of different firm and option holder characteristics on option value, and show that the true value of these options can differ substantially from values calculated using the usual modified Black-Scholes approximation.

JEL classification: G14.

With the explosive growth of employee stock options in corporate compensation, investors, auditors, and regulators have become increasingly concerned about the cost of these options to shareholders. Regulation requiring firms to recognize option cost has intensified the demand for suitable valuation methods. The difficulty is that these are long-lived American options, so their value depends crucially on how employees exercise them. Yet, because employees face hedging constraints, standard option theory does not directly apply. For example, evidence indicates that employees systematically exercise options on non-dividend paying stocks well before expiration (see, for example, Huddart and Lang (1996), Bettis, Bizjak, and Lemmon (2005)), which substantially reduces their value.

Pricing by no arbitrage is still possible as long as the exercise decision generates an option payoff that is subject only to hedgeable risks, such as stock price risk, and diversifiable risks, such as uncertainties that are idiosyncratic across employees. The option valuation problem then reduces to accurately characterizing the option payoffs, that is, the exercise policies of executives. Until recently, however, full-blown estimation has not been possible because of insufficient data and inadequate methodology. Detailed employee option grant, exercise, and cancellation data are proprietary and very difficult to obtain for a large number of firms. In addition, traditional hazard rate models are not suitable for describing voluntary option exercises, where partial and repeated exercise of options from a given grant is the norm.

This paper is the first to perform a complete empirical estimation of employee stock option exercise behavior and option cost to firms.

Reliable estimation of any option exercise model requires a large sample that includes a wide variety of stock price paths. We estimate our model using a comprehensive sample of option exercise grant and exercise data for 1.3 million option grants to 530,266 employees at 103 publicly-traded firms between 1981–2009. The proprietary data were provided by corporate participants in a sponsored research project that was funded by the Society of Actuaries. The methodology presented in this paper is the first step in developing an actuarial science for valuing compensatory stock options, similar to that for pension liabilities.

In our estimation we find that the rate of voluntary option exercise is positively related to the level of the stock price, the imminence of a dividend, and negatively related to the stock return correlation with the S&P 500 Composite Index, consistent with the theory of optimal employee option exercise in the presence of a hedging asset. We also find the exercise rates are negatively related to stock return volatility and option time to expiration, consistent with standard option theory. In addition, the exercise rate is higher when the stock price is in the 90th percentile of its distribution over the past year or in the two weeks after a vesting date. In addition, holding all else equal, men are more likely to exercise their options than women, and exercise rates are decreasing with employee age.

The estimated exercise function, together with a model for involuntary terminations, can be combined with Monte Carlo simulation to calculate the value of these options to shareholders, taking the employees' exercise and termination behavior into account. This approach is similar to the prepayment modeling and valuation methods developed for mortgage-backed securities (see, for example, Schwartz and Torous (1989)). We show that option cost increases with volatility, decreases with the dividend rate, increases with correlation, decreases with the employment termination rate, and is nonmonotonic with respect to the length of the option vesting period.

We compare the prices based on our estimation with the modified Black-Scholes (MBS) method suggested as an approximate valuation technique by the Financial Accounting Standards Board (FASB) and show how the approximation error varies with firm and option characteristics. We find that the MBS approximation can exhibit significant pricing errors, which are even greater for underwater options than at-the-money options.

# 1 Previous Literature

The principles of employee option stock valuation and the need to study exercise behavior are well-understood in the literature. One approach that has been taken is to model the exercise decision theoretically. The employee presumably chooses an option exercise policy as part of a greater utility maximization problem that includes other decisions, such as portfolio, consumption, and effort choice, and this typically leads to early exercise for the purpose of diversification. Papers that develop utility-maximizing models and then calculate the implied cost of options to shareholders include Huddart (1994), Detemple and Sundaresan (1999), Ingersoll (2006), Leung and Sircar (2009), and Carpenter, Stanton, and Wallace (2009).

Combining theory and data, papers such as Carpenter (1998) and Bettis et al. (2005) calibrate utility-maximizing models to mean exercise times and stock prices in the data, and then infer option value. However, these papers provide no formal estimation and the approach relies on the validity of the utility-maximizing models used. Huddart and Lang (1996) and Heath, Huddart, and Lang (1999) provide more flexible empirical descriptions of option exercise patterns, but do not go as far as option valuation. Two recent approaches, Armstrong, Jagolinzer, and Larcker (2006) and Klein and Maug (2009) estimate exercise behavior using a hazard model, but this specification is inappropriate for option valuation because employees exercise random fractions of outstanding option grants.

A number of analytic methods for approximating executive stock option value have also been proposed in the literature. The FASB currently permits using the Black-Scholes formula with the expiration date replaced by the option's expected life. Jennergren and Näslund

(1993), Carr and Linetsky (2000)), and Cvitanić, Wiener, and Zapatero (2004) derive analytic formulas for option value assuming exogenously specified exercise boundaries and stopping rates. Hull and White (2004) propose a model in which exercise occurs when the stock price reaches an exogenously specified multiple of the stock price and forfeiture occurs at an exogenous rate. However, until the accuracy of these methods can be determined, the usefulness of these approximations cannot be assessed.

# 2 Modeling Exercise Behavior

## 2.1 Hazard Rates

At first sight, it seems natural to use hazard rates to model the exercise of employee stock options, since they have often been used in the finance literature to model apparently similar events, such as mortgage prepayment (see Schwartz and Torous (1989)) and corporate bond default (see, for example, Duffie and Singleton (1999)).[1] However, whereas it makes sense to think of the prepayment of one mortgage as independent of the prepayment of another, conditional on the level of interest rates, ESOs are typically exercised in blocks. As a result, the exercise of one option in a given grant held by an individual is extremely highly correlated with the exercise of another option in the same grant held by the same individual. It is also quite highly correlated with the exercise of options in other grants held by the same individual. This high degree of correlation between options makes it difficult to use standard econometric techniques, which assume independence between events, to estimate hazard rates at the individual option level.[2]

One attempt to solve this problem was suggested by Armstrong et al. (2006). Instead of using a hazard rate to model the exercise of individual options, they use a hazard rate to model the exercise behavior of an entire grant of options held by an individual. Aggregating in this way gets around the problem of correlation between individual option exercises, but it introduces a new problem. Whereas a hazard rate describes an event with two states – either something has happened, or it has not – the proportion of an option grant that is exercised in a given period is essentially a continuous variable, which can take on any value between zero and one. Armstrong et al. (2006) work with the dummy variable $\text{Exercise}_{i,k,t}$, which

[1] A hazard rate is defined as the likelihood (per period) of an option's being exercised in the next instant, conditional on not having being exercised previously. For good introductions to hazard rate analysis, see Cox (1972) or Kalbfleisch and Prentice (1980).

[2] This issue also arises in modeling corporate bond default. One popular solution, when the number of firms involved is small, has been to use "copula functions", which explicitly model this correlation [See, for example, Li (2001)]. However, in our case the number of options (and hence the number of correlation coefficients) is too high to be feasible.

indicates whether or not employee $i$ exercises at least 25% of the vested and unexercised options in grant $k$ on day $t$ (and at least 10% of all options from the grant). This addresses some of the correlation issues described above, but introduces new problems of its own. First, unlike, say, death from a disease, this variable can equal one more than once, so standard hazard rate estimation techniques may not immediately apply. Second, important information is lost in this aggregation process. For example, consider two option holders who have the same likelihood of exercising on any given date, however, option holder 1 always exercises 25% of the remaining grant whenever he exercises, whereas option holder 2 always exercises 100% of his remaining options. The conditional probability of a given option's being exercised at any instant is four times as high for options held by option holder 2 versus option 1, so their options will have very different values, yet the Exercise variable modeled by Armstrong et al. (2006) would behave exactly the same way for the two option holders. Their valuation methodology assumes 100% of a given vesting tranche is exercised at the hazard rate estimated for exercises in excess of 25%, an inconsistency which would appear to overstate the rate of early exercise and understate option value. Klein and Maug (2009) use a similar approach, counting exercises as events if the fraction exercised out of a given vesting tranche exceeds a pre-specified threshold, and thus fail to model the distribution of the fraction exercised. Moreover, they do not appear to account for the correlation between exercises of different vesting tranches from the same grant.

## 2.2 Modeling Fractional Exercise

A solution to all of the problems above is to abandon the hazard rate approach altogether and instead to model the *fraction* of each grant exercised each period. Heath et al. (1999) follow this approach, regressing the fraction of each grant exercised against various explanatory variables. However, their regression approach has some problems. In particular, it may generate expected exercise fractions that are negative or greater than one, both of which cause problems for valuation.[3] One possible solution is to transform the proportion exercised, such as by using a logistic transformation,

$$\log\left(\frac{y}{1-y}\right),$$

which can take on any value between $-\infty$ and $+\infty$, and use this on the right hand side of the regression. Unfortunately, by Jensen's inequality, the expected proportion exercising is not just the inverse transformation of the expected transformed proportion. More important,

[3] Attempting to remedy this, for example by truncating the variables, will lead to biases.

this approach cannot handle the numerous dates on which no options are exercised at all. Heath et al. (1999) also aggregate across individuals, thus discarding potentially important information about the differences in exercise behavior across individuals.

Like Heath et al. (1999), we also model the fraction of each grant exercised by each holder each period, but we do so in a manner that generates consistent estimates of expected exercise rates that are guaranteed to be between zero and one, while explicitly handling the correlation between option exercises within and between different grants held by the same individual. Our approach, based on the fractional logistic approach of Papke and Wooldridge (1996), also allows for arbitrary heteroskedasticity in the exercise rates.

Let $y_{ijt}$ be the fraction exercised at time $t$ of grant $j$ held by individual $i$, and write

$$y_{ijt} = G(X_{ijt}\beta) + u_{ijt}, \tag{1}$$

where $X_t$ is some set of covariates in $I_t$, the information set at date $t$, where $G$, the expected fraction exercised at date $t$, is a function satisfying $0 < G(z) < 1$, and where

$$\begin{aligned} E(u_{ijt} \mid I_t) &= 0, \\ E(u_{ijt}\, u_{i'j't'}) &= 0 \quad \text{if } i \neq i' \text{ or } t \neq t'. \end{aligned}$$

From now on, we shall use the logistic function,

$$G(X_{ijt}\beta) = \frac{\exp(X_{ijt}\beta)}{1 + \exp(X_{ijt}\beta)},$$

which takes on only values between zero and one. Note that, while we are assuming the residuals $\epsilon_{ijt}$ are uncorrelated between individuals and across time periods, we are allowing for $\epsilon_{ijt}$ to be arbitrarily correlated between different grants held by the same individual at a given point in time, and we are not making any further assumptions about the exact distribution of $\epsilon_{ijt}$, or even about its variance. In particular, unlike assuming a beta distribution for $y_{ijt}$ (see Mullahy (1990) or Ferrari and Cribari-Neto (2004)), we are allowing a strictly positive probability that $y_{ijt}$ takes on the extreme values zero or one.

As in Papke and Wooldridge (1996), we estimate the parameter vector $\beta$ using quasi-maximum likelihood (see Gouriéroux, Monfort, and Trognon (1984)) with the Bernoulli log-likelihood function,

$$l_{ijt}(\beta) = y_{ijt} \log\left[G(X_{ijt}\beta)\right] + (1 - y_{ijt}) \log\left[1 - G(X_{ijt}\beta)\right]. \tag{2}$$

Estimation involves solving

$$\max_{\beta} \sum_{i,j,t} l_{ijt}(\beta).$$

The $K$ first order conditions, corresponding to the $K$ elements of $\beta$, are given by

$$\begin{aligned}
\sum_{i,j,t} \frac{dl_{ijt}(\beta)}{d\beta} &= \sum_{i,j,t} X_{ijt} \left[ G'(X_{ijt}\beta) \left( \frac{y_{ijt}}{G(X_{ijt}\beta)} - \frac{1 - y_{ijt}}{1 - G(X_{ijt}\beta)} \right) \right] \\
&= \sum_{i,j,t} X_{ijt} \left( y_{ijt} - G(X_{ijt}\beta) \right) \qquad (3) \\
&= 0.
\end{aligned}$$

Equation (1) implies (using iterated expectations) that the population expectation of these first order conditions is zero, hence this QML estimator, $\widehat{\beta}$, is a (consistent) GMM estimator of $\beta$, with no assumptions other than Equation (1). Following the notation in Papke and Wooldridge (1996), define the residual

$$\widehat{u}_{ijt} \equiv y_{ijt} - G(X_{ijt}\widehat{\beta}),$$

and define

$$\widehat{g}_{ijt} \equiv G'(X_{ijt}\widehat{\beta}) \; .$$

To allow for heteroskedasticity and for correlation between option grants held by a given individual, write

$$\text{var}(u) = \Omega = \begin{pmatrix} \Sigma_1 & & \dots & & 0 \\ & \ddots & & & \\ \vdots & & \Sigma_i & & \vdots \\ & & & \ddots & \\ 0 & & \dots & & \Sigma_I \end{pmatrix},$$

where each $\Sigma$ block corresponds to all of the option grants held by a given individual on a particular date. Then the asymptotic covariance matrix of $\widehat{\beta}$ takes the "sandwich" form (see Gouriéroux et al. (1984)),

$$\text{var}\left(\widehat{\beta}\right) = \widehat{\mathbf{A}}^{-1}\widehat{\mathbf{B}}\widehat{\mathbf{A}}^{-1},$$

where

$$\begin{aligned}\widehat{\mathbf{A}} &= \sum_{i,j,t} \frac{\partial^2 l_{ijt}(\widehat{\beta})}{\partial\beta\partial\beta'} \\ &= \sum_{i,j,t} \widehat{g}_{ijt} X_{ijt} X'_{ijt},\end{aligned} \tag{4}$$

and

$$\widehat{\mathbf{B}} = \mathbf{X}'\widehat{\Omega}\mathbf{X},$$

where $\mathbf{X}$ is a matrix containing all of the stacked $X_{ijt}$ values, and $\widehat{\Omega}$ is a consistent estimator of $\Omega$ given by

$$\widehat{\Omega} = \begin{pmatrix} \widehat{\Sigma}_1 & & \dots & & 0 \\ & \ddots & & & \\ \vdots & & \widehat{\Sigma}_i & & \vdots \\ & & & \ddots & \\ 0 & & \dots & & \widehat{\Sigma}_I \end{pmatrix}$$

where

$$\widehat{\Sigma}_i = \widehat{u}_i \widehat{u}'_i.$$

This covariance matrix is robust both to arbitrary heteroskedasticity and to arbitrary correlation between the residuals in a given block.[4]

# 3 Data

As discussed above, our estimation strategy is carried out using a proprietary data set comprising complete histories of employee stock option grants, vesting structures, and option exercise and cancellation events for all employees who received options at 103 publicly traded corporations between 1981 and 2009.[5] As shown in Table 1, there is considerable heterogeneity in the sample of firms in terms of their industry type, reported at one-digit Standard Industrial Classification (SIC) codes, firm size, as measured by market cap and numbers of employees, revenue growth over the period, and stock return volatility. Sample firm dividend rates are low and many firms pay no dividend, so early exercise is driven by

[4]For further discussion of calculating standard errors in the presence of clustering, see Rogers (1993), Baum, Schaffer, and Stillman (2003), Wooldridge (2003) and Petersen (2008).

[5]The data were obtained as part of a research grant written by the authors and funded by the Society of Actuaries. In addition, we thank Terrence Adamson at AON Consulting who also provided data for this study.

other factors. Sample firm's return correlation with S&P Composite Index average from 25% to 37% across sectors, suggesting employees have some scope for hedging their option compensation by reducing their market exposure in their outside portfolios.

## 3.1 Proprietary option data

Our unit of analysis is an employee-grant-day. For each option grant we merge the appropriate path of daily split-adjusted stock prices and dividends, starting at the initial grant date, to the path of outstanding option vesting and exercise events for all grants and employees. These daily paths are constructed using detailed information on the contractual option vesting structure, the exercise events, and the cancellation events recorded for each grant. We track the employee-grant-days and a series of time-varying covariates until the options in the grant are fully exercised, the options are cancelled, or we reach the end of the sample period of December 31, 2009.

Table 2 summarizes the size and structure of the sample of option data by industry and in aggregate. In total there are 22,694,875 option exercises across 1,314,724 grants to 530,266 employees. On average, there are 2.5 grants per employee, but there is considerable variability across firms and employees, with some employees receiving dozens of option grants. For the firms for which we have the employee ranking of the employee, the largest grant recipients are typically the CEO or senior managers.

Table 3 summarizes the size, vesting structure, and maturity of option grants in the sample. The average grant has $49,984 worth of underlying shares at the grant date, but this varies widely across industry, with the greatest mean and variance of grant size in the finance industry. The combined effects of the potentially large number of grants per employee and the size of these grants implies that individual employees may hold large inventories of options with different strikes, expiration dates, and vesting structures. This feature of the data introduces significant correlation across the exercise decisions of individual employees. There is likely to be high correlation in the exercise decisions across grants that are held by the same individual. A particular strength of our fractional logistic estimator is that it does not require assumptions of independence across exercise events. We also pool by employee and correct our standard errors to account for our pooled structure.

Vesting structures also vary widely, both across and within firms in our sample, and can be complex. The average grant has 4.13 vesting dates, but some have as many as 60 vesting dates. An example of a vesting structure that would lead to a large maximum would be a grant with a 25% vest at the end of the first year and then 2.08% monthly vests over the next 36 months. The minima are generated by "cliff vests" where all the options in a given grant vest on the same day. Another feature of the grants that exhibits important heterogeneity across firms is the percentage of options that vest on the first vesting dates, with industry means ranging from 33% in SIC 3, which includes technology firms, to 92% in SIC 4, transportation, communications, and utilities.

The only homogeneous contractual feature of employee stock option grants across firms is the maturity in months from the issuance date to the date of expiry. The term of executive stock options is quite uniformly ten years although there are some twenty-year and one-year maturity options granted on the part of some firms. At the employee-level, the employees in our sample are in some cases managing as many as ten different contractual option vesting structures in their inventory of options.

Table 4 summarizes exercise patterns in the sample. Options are exercised very early. At the time of exercise, the average option has 5.6 years remaining to expiration and has only been in the money and vested for 292 days. These patterns are consistent with those documented by Huddart and Lang (1996). On average, the option is 439% in the money at the time of exercise, and more than half the time, the stock price is near its annual high. At the time of exercise, an average of 85% of vested options are exercised. This sample includes grants to all firm employees, many of which are very small. Among larger grants, fractional exercise is much more pervasive, and very small fractions are exercised in some cases, which motivates the development of our fractional logistic estimation strategy.

In summary, there are three features of the stock option exercise patterns observed in our sample. First, many employees hold more than one option grant and make exercise decisions over more than one vested option at any given time. For this reason, estimation strategies must account for the correlated decision structure of employee option exercise. Second, both the contractual vesting structure and the exogenous price paths appear to have strong effects on option exercise patterns, thus careful controls for both of these feature on a daily basis must be included in a successful estimation strategy. Finally, many option positions are exercised fractionally, that is the proportion of the outstanding options that are exercised at exercise events can be substantially less than one. For this reason, a successful econometric methodology must account for path dependent fractional exercise behavior or risk introducing significant misspecification bias and inaccurate forecasts of exercise timing.

## 3.2 The covariates

Employees may voluntarily choose to exercise options or they may be forced to do so because of impending employment termination or option expiration. To estimate the model of voluntary exercise, we begin with the sample of employee-grant-days on which the option is in the money and vested and then eliminate those days that are within six months of the grant expiration date or six months of a cancellation of any option by that employee, because most cancellations are associated with employment termination. The remaining employee-grant-days are treated as days on which the employee has a choice about whether and how many options to exercise. To explain the fraction of options exercised by a given employee from a given grant on a given day, as specified by Equation (1), we choose as covariates variables drawn from optimal option exercise theory, such as Carpenter et al. (2009), as well as behavioral variables identified in empirical studies such as Heath et al. (1999).

Since employee stock options are non-transferable, the optimal exercise policy for these options can look quite different from that for standard American call options, as theoretical models of the optimal employee exercise policy have shown. In particular, the need for diversification can lead an employee to exercise much earlier than standard theory would predict. In virtually every model of optimal exercise, however, the degree to which the option is in the money is an important determinant of the exercise decision, though the nature of the relationship can vary. In both standard option theory, and in many models of employee option exercise, the option holders exercises once the stock price rises above a critical boundary. Intuition also suggests that in practice, exercise become more attractive as the option gets deeper in the money and more of its total value shifts to its exercise value. The variable *Price-to-strike ratio*, the employee-grant-day ratio of the split-adjusted price of the stock to the split-adjusted option strike price captures the degree to which the option is in the money.

Carpenter et al. (2009) prove very generally that the dividend effect for employee option exercise is qualitatively the same for employee option exercise decisions as for standard, transferable options. That is, a higher dividend makes early exercise more attractive, all else equal. The variable *Dividend in next two weeks* is the product of an indicator that a dividend will be paid within the next 14 calendar days and the ratio of the dividend payment to the current stock price.

The theoretical effect of higher stock return volatility on the exercise decision is more complicated for employee options than the simple negative effect from standard theory. Employee risk aversion and the convexity of the option payoff have offsetting effects on employees' attitudes toward volatility, and the net effect is an open empirical question. The variable *Volatility* is the daily volatility estimated from the stock return over the 66 trading days prior to the given employee-grant-day.

Unlike in standard theory, the degree to which the employee can hedge the option position in an outside portfolio is an important theoretical determinant of the exercise decision, and Carpenter et al. (2009) and others have shown that the higher the correlation between the stock return and the return on a tradeable asset, the lower the propensity to exercise early. The variable *Correlation* is the correlation between the stock return and the return on the S&P 500 Composite Index estimated from daily returns over the three months prior to the given employee-grant-day.

The theoretical effect of more time to expiration on the exercise decision can also be more complicated for employee options than the simple negative effect from standard theory, and is thus also an open empirical question. The variable *Time to expiration* is the number of calendar days from the given employee-grant-day to the expiration date of the grant.

Recent empirical studies of employee stock option exercise report links between behavioral indicators, or "rules of thumb", that employees appear to rely upon in making their option exercise decisions. Armstrong et al. (2006) find a statistically significant association between the timing of vesting events and option exercise. They argue that recent exercise events both mechanically affect an employees' ability to exercise their options and may also serve as a periodic reminder to employees to evaluate the value of their option positions. Heath et al. (1999) and Armstrong et al. (2006) also find a statistically significant positive association between option exercise and the occurrence of the current stock price exceeding the 90th percentile of the past year's price distribution. They argue that this association is driven by cognitive benchmarks that employees use in their decision rules. Given the importance of these variables in prior studies, we also include them as controls in all of our specifications. To capture the vesting structure of the grant, the variable *Vesting event in past two weeks* indicates whether the given employee-grant-day is within 2 weeks since a vesting date for that grant. Our cognitive benchmark proxy is the variable *Price $\geq$ 90th percentile of prior year distribution*, which indicates whether the current stock price is greater than or equal to 90th percentile of the stock price distribution over the prior year of trading.

A prior empirical literature has found evidence that older individuals are more risk averse in financial decision making than younger individuals and that females appear to be more risk averse than males in their financial decisions (See Bajtelsmit and Bernasek (2001); Bellante and Green (2004); and Armstrong et al. (2006)). Carpenter et al. (2009) prove that less risk averse employees are likely to exercise later and consequently the cost of their options is greater. For 62 firms in our sample, we have information on the age and gender of the employee. Table 5 shows that the average employee is 42-years old and 56% of employees are male.

Employee wealth and undiversifiable portfolio risk can also have a theoretically important effect on the exercise decision. For five firms in the sample, we have information about employee salary and rank, which may correlate with these variables. Table 5 shows that the mean salary in this subsample is $298,124 and 1% of employees are top executives.

In summary, the covariates used in the fractional logistic specification include the salient state variables related to stock price paths, volatility, and market risk that have been the focus of the recent theoretical literature on employee stock option valuation and cost. In addition, we proxy for factors such as risk aversion and possible cognitive benchmarks using the covariates gender, age, salary, and employment status. We use this rich set of covariates to explore a set of theoretically motivated null hypotheses that have appeared in the recent literature. Our predictions are: 1) the deeper in the money, the more likely the option is to be exercised; 2) higher dividend rates should make option exercise more likely; 3) higher volatility is an empirical question, since theoretically it could lead to either earlier or later exercise in a utility maximizing framework; 2) more risk aversion should make early option exercise more likely; and 4) higher correlation with the market makes earlier exercise less likely. We report the results of these tests in the next section of the paper.

# 4 Estimation Results

We estimate four alternative specifications of Equation (1), using either the full sample of voluntary exercises from all 103 firms or the subsample of 62 firms for which demographic data are available, and with or without industry one-digit SIC industry fixed effects. Table 6 reports coefficients and standard errors in parentheses below the coefficient estimates. The estimator clusters at the level of the individual employee.

As Table 6 shows, the results are similar across the alternative specifications, and consistent with predictions of optimal exercise theory. The rate of voluntary option exercise is strongly positively related to the level of the stock price as expected. Exercise rates are also significantly higher when a dividend payment is imminent. Also in line with the theory of employee option exercise, exercise rates are consistently lower when the stock return is more highly correlated with S&P Composite Index, so that a greater fraction of the stock risk can be hedged with reductions in exposure to the market portfolio.

As discussed above, Carpenter et al. (2009) show that stock return volatility and time to expiration do not have clearly signed theoretical effects on an employee's optimal exercise policy, so the empirical effects are an open question. The results reported in Table 6 indicate that increased levels of stock return volatility and more time to expiration are associated with smaller fractions of options exercised, which is consistent with the theory for ordinary American options.

Table 6 also shows that employees exercise a significantly larger fraction of outstanding options when the stock price is greater than or equal to the $90^{th}$ percentile its distribution over the prior calendar year and in the two weeks following a vesting date of the grant in question. As discussed previously, these results are consistent with the earlier empirical studies of Heath et al. (1999) and Armstrong et al. (2006), who argue that employees may tie their exercise decisions to cognitive benchmarks as a means of reducing monitoring costs.

Based on the subsample of firms for which information on age and gender are available, the results in Table 6 indicate that male employees have a greater propensity to exercise their options than female employees, and older employees are less likely to exercise options early. These results appear to be inconsistent with the notion that women and older people are more risk averse.

# 5 Option Cost to the Firm

For an individual option, the exercise function describes the expected proportion of each outstanding option grant to be voluntarily exercised at a given time and state, conditional on having survived to that point. If the event that the option is actually exercised is sufficiently independent across option holders with identical exercise functions, conditional on the given time and state, then in a large enough pool of such option holders, the fraction of options exercised voluntarily will exactly equal the exercise function. Similarly, the termination rate describes the fraction of options stopped through termination in a diversified pool. We assume that such diversification is possible, or, more generally, that the conditional variance in the number of options actually exercised around the expected value is not a priced risk in the market, so that option valuation proceeds as if perfect diversification were possible.

Given the estimated voluntary exercise rate per period, $G(X\beta)$, and termination rate $\lambda$, the value of the option is given by its expected risk-neutral discounted payoff,

$$\begin{aligned} O_t \quad = \quad & \mathrm{E}_t^*\{\int_{t\vee t_v}^{T} e^{-r(\tau-t)}\,(S_\tau-K)^+\,(G_\tau+\lambda)e^{-\int_t^\tau (G_s+\lambda)\,ds}\,d\tau \\ & +e^{-r(T-t)}e^{-\int_t^T (G_s+\lambda)\,ds}(S_T-K)^+\}\ , \end{aligned} \tag{5}$$

where $t_v$ is the vesting date. To understand the intuition for this expression, note that $G+\lambda$ measures the expected fraction of a grant exercising or canceling, measured as a fraction of the options still unexercised one period earlier. To calculate the expected fraction of *today's* options that exercise or cancel at date $t$, we therefore need to multiply by the proportion of the grant outstanding today that has not exercised prior to $t$, given by

$$e^{-\int_t^\tau \hat{G}_s+\lambda\,ds}\,d\tau.$$

We estimate this value with Monte Carlo simulation, using antithetic variates and importance sampling to increase precision. Tables 7-8 report option values, labeled ESO Value, for a variety of parameterizations. The tables assuming the option holder voluntarily exercises according to the estimated exercise rate function in Table 6, and in addition terminates employment at a constant rate. For the base case, we set the employee age and termination rate equal to their sample average values. We use SIC 3 as the base case industry and set the firm volatility to 50%, dividend rate to zero, which is representative of a technology firm. We set the firm beta to one, the vesting period to two years, and the option expiration date to ten years.

For comparison, the column labeled Modified Black-Scholes (MBS) gives option value approximated as the probability of vesting times the Black-Scholes value adjusted for dividends, with contractual expiration date replaced by the option's expected term, conditional on vesting. While new methodologies are developing, the FASB accepts this approximation for accounting valuation, and it is used by the vast majority of firms. Like ESO Value, we compute the option's expected life using Monte Carlo simulation assuming the option holder follows the estimated exercise rate function and terminates employment at a constant rate. This expectation is with respect to the true probability measure, so it depends on the true expected return on the stock. We assume the mean stock return is determined by the CAPM, with a 6% excess expected return on the market.

In theory, the MBS approximation can either understate or overstate the true option value, depending on the exercise policy. To understand why, consider two special cases, and for simplicity assume immediate vesting. First, if the option holder follows the value-maximizing exercise policy in the presence of dividends, as in standard theory, then the true option value will be greater than the Black-Scholes value to any deterministic expiration date, so it will exceed the MBS approximation. Alternatively, suppose the option is stopped, either through exercise or cancellation, at a purely exogenous rate, independent of the stock price, without regard to whether it is in or out of the money. Then the true option value is the average Black-Scholes value over possible stopping dates, while the MBS approximation is Black-Scholes value to the average stopping date, so since the Black-Scholes value tends to be concave in the option expiration date, the MBS approximation will overstate the true value, by Jensen's inequality. The exercise policies followed in practice contain elements of both of these examples, and the MBS approximation can either overstate or understate the true ESO cost.

The left side of Tables 7 and 8 focuses on the case of at-the-money options at their grant date. The right side considers so-called "underwater" options, two years after grant, vested, but 40% out of the money. After the steep decline in the stock market during the financial crisis, most firms found that the options they granted to employees before the crash were deeply out of the money during 2008 and 2009. Many firms offered their employees equal-present-value exchanges of at-the-money options for the old out-of-the-money options, perhaps in an effort to restore performance incentives. The last column of Tables 7-8 shows the exchange ratio. In general, the MBS value overstates option value for the parameterizations considered here. The overstatement is even greater for the underwater options than for the at-the-money options.

Table 7 shows the effects of changing firm stock return characteristics. The first panel presents volatility effects. As Table 6 shows, increasing volatility reduces the estimated exercise rate, and this also increases the option's expected life. Both the true ESO value and its MBS value increase with volatility. True ESO value increases more slowly with volatility than the MBS approximation does, so the overstatement increases significantly with volatility. This is because the higher volatility, the more quickly Black-Scholes option value increases to its maximum as a function of time to expiration, so the Jensen's inequality between Black-Scholes to average life and average Black-Scholes becomes more pronounced.

The second panel shows the effect of increasing the dividend rate. This increases the estimated exercise rate, conditional on the option being vested and in the money, but uniformly reduces future possible stock prices, and ESO value declines with the the dividend rate. The option's expected term, conditional on vesting, actually increases slightly, because more stock price paths stay out of the money longer, but the MBS option value declines even faster than the true option value, so the error declines in algebraic value. This may be because the value-maximizing policy calls for some early exercise prior to a dividend payment, and the estimated empirical exercise policy comes closer to that than the deterministic-time exercise policy implicit in the MBS approximation.

The third panel of Table 7 shows the effect of increasing stock return correlation with the market. This reduces the estimated exercise rate, as shown in Table 6, which increases option value. It also increases the option's average life, which increase the MBS value. The approximation error remains relatively constant.

Table 8 considers certain employee, contract, and market effects. The first panel shows how option value varies with the termination rate. A higher termination rate increases the chance of pre-vesting forfeiture, the chance of post-vesting cancellation, and the rate of suboptimal early exercise, so it reduces option value. It also reduces option life, so the MBS approximation also declines.

The second panel of Table 8 illustrates the effect of increasing the vesting period. A longer vesting period increases the risk of pre-vesting forfeiture, which reduces option value. Conditional on vesting, the option stopping time has less room to vary, so the difference between the option value and the MBS approximation shrinks.

The third panel of Table 8 shows how the MBS approximation varies with the stock return premium. This has no effect ESO value, given the estimated exercise function. However, it reduces the true expected option life, as it increases the rate at which the option gets in the money, so it reduces the MBS value and approximation error.

# 6 Conclusions

This paper is the first to perform a complete empirical estimation of employee stock option exercise behavior and option cost to firms. We develop a methodology for estimating option exercise and cancellation rates as a function of the stock price path, time to expiration, and firm and option holder characteristics. Our estimation is based on a fractional logistic approach, and accounts for correlation between exercises by the same executive. Valuation proceeds by using the estimated exercise rate function to describe the option's expected payoff along each stock price path, and then computing the present value of the payoff. The estimation of empirical exercise rates also allows us to test the predictions of theoretical models of option exercise behavior.

We apply our estimation technique to the largest dataset yet analyzed in the literature, consisting of a comprehensive sample of option exercise grant and exercise data for all employees at 103 publicly traded firms from 1981 to 2009. Our results indicate that using standard pricing approximations, such as the adjusted Black-Scholes method suggested by FASB, can lead to significant errors. The proprietary data used in this study were provided by corporate participants in a sponsored research project that was funded by the Society of Actuaries, who hope that the results of our study will eventually be used as the standard set of exercise assumptions to be used in calculating ESO values on firms' income statements.

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Table 1: Summary Statistics for Sample Firms

This table provides financial performance summary statistics for the 103 firms in our sample, grouped by one-digit SIC code. Start Sample is the earliest option grant date in our sample and End Sample is the last day that we track all grants and exercise events in the sample for all of the firms. Employees is the sample period mean of the annual number of employees for the firms as reported in Compustat as *"Employees: Thousands"* (DATA29). Market Cap is the sample period mean of *"Common Shares Outstanding: Millions"* in Compustat (DATA25) multiplied by *"Price minus Fiscal Year Close"* in Compustat (DATA199). Market-to-Book is the sample period mean of the the Market Cap scaled by common equity as reported in Compustat (DATA60). Revenue is the sample period mean of *"Sales/Turnover"* in Compustat (DATA12). Revenue Growth is the sample period mean of ($Revenue_t/Revenue_{t-1} - 1$). Net Income is the sample period mean of *"Net Income"* in Compustat (DATA172). Volatility is the average stock return volatility over the sample period. Dividend rate is the average dividend rate over the sample period. Correlation is the average correlation between the firm return and return on the CRSP S&P Composite Index over the sample period.

| | Construction & Manufacturing | | Transportation, Communications & Utilities | Retail | Finance, Insurance, & Real Estate | Services | |
|---|---|---|---|---|---|---|---|
| On Digit SIC | 1 & 2 | 3 | 4 | 5 | 6 | 7 | 8 & 9 |
| Number of Firms | 19 | 29 | 3 | 14 | 14 | 17 | 7 |
| Start Sample | 3/31/1982 | 12/31/1981 | 12/31/1992 | 6/28/1985 | 3/31/1982 | 3/31/1986 | 9/30/1991 |
| End Sample | 12/31/2007 | 12/31/2009 | 11/1/2007 | 6/11/2008 | 12/27/2007 | 12/31/2009 | 11/1/2007 |
| Employees (000's) | 8.43 | 5.94 | 19.15 | 24.31 | 11.85 | 8.83 | 3.57 |
| Market Cap ($ Millions) | 2,309.49 | 1,748.69 | 9,578.27 | 2,675.09 | 5,314.62 | 1,859.17 | 646.31 |
| Market-to-Book | 5.06 | 11.66 | 2.52 | 4.16 | 2.37 | 5.73 | 7.30 |
| Revenue ($ Millions) | 1,965.02 | 866.05 | 3,446.75 | 4,962.56 | 3,569.51 | 1,029.78 | 355.50 |
| Net Income ($ Millions) | 101.71 | 33.73 | 250.82 | 101.63 | 316.19 | 25.19 | 22.23 |
| Volatility | 0.540 | 0.556 | 0.317 | 0.460 | 0.286 | 0.603 | 0.587 |
| Dividend Rate | 0.007 | 0.003 | 0.022 | 0.004 | 0.009 | 0.037 | 0.065 |
| Correlation | 0.310 | 0.346 | 0.372 | 0.358 | 0.354 | 0.334 | 0.253 |

Table 2: Summary Statistics for Proprietary Option Data: Sample Size and Structure

| | Construction & Manufacturing | | Transportation, Communications & Utilities | Retail | Finance, Insurance, & Real Estate | Services | | All |
|---|---|---|---|---|---|---|---|---|
| SIC (one digit) | 1 & 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
| Panel A. Aggregate Number of Employees, Grants, and Exercise Events | | | | | | | | |
| Number of Employees | 184,235 | 121,832 | 11,874 | 36,648 | 115,152 | 53,995 | 6,530 | 530,266 |
| Number of Grants | 367,121 | 353,826 | 18,123 | 127,090 | 274,937 | 156,531 | 15,096 | 1,312,724 |
| Number of Exercises | 6,262,230 | 6,461,873 | 227,587 | 2,189,262 | 4,880,661 | 2,415,307 | 257,955 | 22,694,875 |
| Panel B. Number of Grants per Employee | | | | | | | | |
| Mean | 2.00 | 2.91 | 1.58 | 3.46 | 2.40 | 2.80 | 2.31 | 2.47 |
| Median | 2.00 | 1.00 | 1.00 | 2.00 | 1.00 | 2.00 | 1.00 | 2.00 |
| Standard Deviation | 1.40 | 3.38 | 1.44 | 3.26 | 2.77 | 2.89 | 2.22 | 2.62 |
| Minimum | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Maximum | 53.00 | 35.00 | 16.00 | 27.00 | 74.00 | 47.00 | 26.00 | 74.00 |

Table 3: Summary Statistics for Proprietary Option Data: Grant Size and Contractual Features

| | Construction & Manufacturing | | Transportation, Communications & Utilities | Retail | Finance, Insurance, & Real Estate | Services | | All |
|---|---|---|---|---|---|---|---|---|
| SIC (one digit) | 1 & 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
| Panel A. Dollar Value of Underlying Shares (per Grant) | | | | | | | | |
| Mean | 25,005.49 | 46,940.67 | 19,411.61 | 53,095.84 | 105,886.94 | 60,905.62 | 41,322.36 | 49,984.32 |
| Median | 5,700.00 | 12,825.00 | 3,211.95 | 11,562.53 | 25,362.69 | 26,125.00 | 10,976.00 | 10,187.50 |
| Standard Deviation | 190,762.67 | 282,423.83 | 178,102.47 | 298,297.88 | 712,098.88 | 467,002.78 | 201,517.12 | 384,240.47 |
| Minimum | 2.49 | 3.16 | 7.32 | 14.81 | 9.89 | 3.60 | 1.02 | 1.02 |
| Maximum | 52,500,000.00 | 45,238,135.50 | 29,600,592.00 | 58,600,000.00 | 154,440,000.00 | 179,600,000.00 | 14,587,065.12 | 179,600,000.00 |
| Panel B. Maximum Number of Vesting Periods per Grant | | | | | | | | |
| Mean | 2.67 | 7.32 | 1.31 | 3.30 | 2.91 | 4.87 | 3.26 | 4.13 |
| Median | 1.00 | 4.00 | 1.00 | 3.00 | 3.00 | 4.00 | 3.00 | 3.00 |
| Standard Deviation | 7.36 | 9.46 | 0.86 | 1.34 | 1.13 | 5.88 | 1.25 | 6.75 |
| Minimum | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Maximum | 60.00 | 49.00 | 7.00 | 60.00 | 8.00 | 49.00 | 5.00 | 60.00 |
| Panel C. Percent of Options that Vest on the First Vesting Date (per Grant) | | | | | | | | |
| Mean | 91.02 | 33.63 | 92.41 | 38.73 | 48.11 | 37.06 | 35.87 | 56.88 |
| Median | 100.00 | 25.00 | 100.00 | 33.33 | 33.33 | 25.00 | 33.33 | 33.33 |
| Standard Deviation | 25.01 | 26.77 | 20.63 | 26.77 | 29.14 | 28.46 | 14.37 | 36.66 |
| Minimum | 1.67 | 2.08 | 1.15 | 1.67 | 12.50 | 2.08 | 20.00 | 1.15 |
| Maximum | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| Panel D. Number of Months from Grant to Full Vesting (per Grant) | | | | | | | | |
| Mean | 19.48 | 45.08 | 11.05 | 40.99 | 33.21 | 41.26 | 39.17 | 32.44 |
| Median | 12.00 | 48.00 | 0.00 | 36.00 | 36.00 | 48.00 | 36.00 | 36.00 |
| Standard Deviation | 12.62 | 19.61 | 17.94 | 14.91 | 17.79 | 18.49 | 15.01 | 21.10 |
| Minimum | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Maximum | 85.00 | 84.00 | 85.00 | 84.00 | 85.00 | 72.00 | 60.00 | 85.00 |
| Panel E. Number of Months from Grant to Expiration (per Grant) | | | | | | | | |
| Mean | 103.33 | 78.95 | 93.47 | 122.43 | 120.17 | 120.00 | 120.00 | 102.25 |
| Median | 120.00 | 120.00 | 120.00 | 120.00 | 120.00 | 120.00 | 120.00 | 120.00 |
| Standard Deviation | 31.56 | 53.14 | 39.89 | 11.81 | 0.38 | 0.00 | 0.00 | 38.47 |
| Minimum | 1.00 | 1.00 | 3.00 | 120.00 | 120.00 | 120.00 | 120.00 | 1.00 |
| Maximum | 240.00 | 120.00 | 240.00 | 180.00 | 121.00 | 120.00 | 120.00 | 240.00 |

Table 4: Summary Statistics for Proprietary Option Data: Exercise Patterns

| | Construction & Manufacturing | | Transportation, Communications & Utilities | Retail | Finance, Insurance, & Real Estate | Services | | All |
|---|---|---|---|---|---|---|---|---|
| SIC (one digit) | 1 & 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
| Panel A. Number of Days Remaining to Expiration on Exercise Date (per Exercise Event) | | | | | | | | |
| Mean | 1,572.58 | 2,107.55 | 2,447.68 | 2,216.45 | 2,290.60 | 2,593.91 | 2,496.51 | 2,054.82 |
| Median | 1,675.00 | 2,439.00 | 2,530.00 | 2,357.00 | 2,397.00 | 2,739.00 | 2,736.50 | 2,360.00 |
| Standard Deviation | 1,203.41 | 1,049.64 | 612.45 | 875.40 | 736.23 | 661.37 | 699.13 | 1,041.61 |
| Minimum | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Maximum | 3,643.00 | 3,649.00 | 3,478.00 | 5,292.00 | 3,650.00 | 3,627.00 | 3,525.00 | 5,292.00 |
| Panel B. Number of Days the Option was In-the-Money from Vesting Date to Exercise Date (per Exercise Event) | | | | | | | | |
| Mean | 450.66 | 247.85 | 1,105.94 | 263.24 | 329.82 | 237.24 | 237.30 | 292.37 |
| Median | 72.00 | 67.00 | 223.00 | 96.00 | 149.00 | 31.00 | 128.00 | 86.00 |
| Standard Deviation | 763.04 | 409.32 | 1,372.86 | 431.58 | 474.37 | 440.52 | 323.88 | 494.97 |
| Minimum | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Maximum | 3,665.00 | 2,929.00 | 3,990.00 | 3,527.00 | 3,217.00 | 3,646.00 | 2,563.00 | 3,990.00 |
| Panel C. Ratio of Stock Price to Strike Price at Exercise Date (per Exercise Event) | | | | | | | | |
| Mean | 3.65 | 3.50 | 1.92 | 4.45 | 5.76 | 5.45 | 6.67 | 4.39 |
| Median | 3.12 | 2.21 | 1.81 | 2.05 | 2.61 | 2.98 | 2.17 | 2.86 |
| Standard Deviation | 2.05 | 5.87 | 0.67 | 5.16 | 7.50 | 9.29 | 14.67 | 6.12 |
| Minimum | 1.01 | 1.02 | 1.01 | 1.03 | 1.01 | 1.02 | 1.01 | 1.02 |
| Maximum | 101.92 | 197.97 | 9.44 | 95.65 | 147.58 | 192.27 | 215.26 | 215.26 |
| Panel D. Whether Stock Price $\geq 90^{th}$ Percentile of Prior Year Distribution (per Exercise Event) | | | | | | | | |
| Mean | 0.49 | 0.61 | 0.49 | 0.56 | 0.57 | 0.56 | 0.51 | 0.55 |
| Median | 0.00 | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Standard Deviation | 0.50 | 0.49 | 0.50 | 0.50 | 0.49 | 0.50 | 0.50 | 0.50 |
| Minimum | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Maximum | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Panel E. Fraction of Grant's Vested Options that are Exercised (per Exercise Event) | | | | | | | | |
| Mean | 0.96 | 0.85 | 0.94 | 0.74 | 0.81 | 0.71 | 0.94 | 0.85 |
| Median | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Standard Deviation | 0.15 | 0.28 | 0.19 | 0.34 | 0.29 | 0.37 | 0.17 | 0.28 |
| Minimum | 0.001 | 0.001 | 0.012 | 0.001 | 0.001 | 0.001 | 0.003 | 0.001 |
| Maximum | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

Table 5: Employee Characteristics

This table presents summary statistics for the demographic information that is reported by a subset of the firms in the sample. We summarize the information by employees over the sample period.

| | Number of Employees | Mean | Median | Standard Deviation | Minimum | Maximum |
|---|---|---|---|---|---|---|
| Age | 248225 | 41.94 | 41.50 | 10.18 | 20.00 | 85.00 |
| Male | 241076 | 0.56 | 1.00 | 0.50 | 0.00 | 1.00 |
| Executive | 329058 | 0.01 | 0.00 | 0.11 | 0.00 | 1.00 |
| Salary | 52743 | 298,124 | 39,144 | 3,631,444 | 10,001 | 368,352,000 |

Table 6: Estimation Results

This table presents the results for alternative specifications of the fractional logistic estimator. Specifications 1 and 2 are estimated on the full sample. In specification 1, we exclude the sex, age, and industry fixed effects. Specification 2 includes industry (one-digit SIC code) fixed effects and again excludes the sex and age covariates. Specifications 3 and 4 are estimated with a smaller subsample of firms that reported information on sex and age. Specification 3 includes sex and age and excludes industry fixed effects. Specification 4 includes sex, age and industry fixed effects. The standard errors are reported in parentheses below the coefficient estimates. The estimator clusters at the level of the individual employee.

| | Alternative Specifications | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| Covariates | | | | |
| Constant | -5.5973 | -4.6373 | -5.4517 | -5.8975 |
| | (0.0084) | (0.0094) | (0.0186) | (0.0253) |
| Price-to-strike ratio | 0.0054 | 0.0064 | 0.0152 | 0.0158 |
| | (0.0001) | (0.0001) | (0.0001) | (0.0001) |
| Dividend in next two weeks | 2.2369 | 3.9773 | 14.7871 | 18.4637 |
| | (0.0554) | (0.0661) | (1.2119) | (1.1888) |
| Correlation | -0.8754 | -0.5849 | -0.6590 | -0.5975 |
| | (0.0096) | (0.0101) | (0.0137) | (0.0145) |
| Volatility | -13.5554 | -18.7566 | -13.1809 | -18.0737 |
| | (0.1829) | (0.2073) | (0.2933) | (0.3445) |
| Time to expiration | -0.0004 | -0.0003 | -0.0004 | -0.0004 |
| | (0.000002) | (0.000002) | (0.000003) | (0.000004) |
| Vesting event in past two weeks | 2.8675 | 2.7265 | 3.0740 | 3.0441 |
| | (0.0052) | (0.0054) | (0.0064) | (0.0067) |
| Price $\geq 90^{th}$ percentile of prior year distribution | 0.3940 | 0.3993 | 0.4286 | 0.4322 |
| | (0.0041) | (0.0041) | (0.0054) | (0.0054) |
| Age | | | -0.0081 | -0.0066 |
| | | | (0.0002) | (0.0002) |
| Male | | | 0.2122 | 0.1831 |
| | | | (0.0056) | (0.0058) |
| Industry fixed effects | No | Yes | No | Yes |
| Number of observations (employee-grant-days) | 362,535,204 | 362,535,204 | 217,348,920 | 217,348,920 |

Table 7: Valuation Results: Volatility, Dividend, and Correlation Effects

The table shows option values for a 42-year old male employee based on the estimated exercise rate function in specification 4 of Table 6. The left side shows an option at grant: the stock price and strike price are $100 and the option expires in ten years. The right side shows the case of an option that is 40% out of the money and has eight years to expiration. The base case assumes 8.5% annual termination rate, two-year vesting period, 50% volatility, zero dividend rate, and beta of one. The riskless rate is 5% and the expected market return is 11%. Prob Vest is the probability that the option vests given the assumed termination rate. Expected Term Given Vest is expected life of the option conditional on vesting, assuming option exercise according to the estimated exercise function and employment termination at the given termination rate. Modified Black-Scholes is the probability that the option vests times the Black-Scholes value of the option assuming expiration at the option's expected term conditional on vesting.

| | Ten-Year At-the-Money Option, $S = K = 100$ | | | | | Eight-Year Underwater Option, $S = 60, K = 100$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Changing Parameter | ESO Value | Prob Vest | Expected Term Given Vest | Modified Black Scholes | Percent Error | ESO Value | Prob Vest | Expected Term Given Vest | Modified Black Scholes | Percent Error | A-T-M/ Under. |
| Volatility Effects | | | | | | | | | | | |
| 0.30 | 28.61 | 0.84 | 4.92 | 29.83 | 4% | 8.49 | 1.00 | 4.73 | 9.78 | 15% | 3.37 |
| 0.40 | 34.78 | 0.84 | 5.44 | 37.49 | 8% | 12.72 | 1.00 | 4.81 | 15.21 | 20% | 2.73 |
| 0.50 | 40.47 | 0.84 | 5.87 | 44.75 | 11% | 17.07 | 1.00 | 4.90 | 20.64 | 21% | 2.37 |
| 0.60 | 45.64 | 0.84 | 6.22 | 51.46 | 13% | 21.41 | 1.00 | 4.99 | 25.89 | 21% | 2.13 |
| 0.70 | 50.20 | 0.84 | 6.51 | 57.48 | 15% | 25.60 | 1.00 | 5.08 | 30.84 | 20% | 1.96 |
| Dividend Rate Effects | | | | | | | | | | | |
| 0.00 | 40.47 | 0.84 | 5.87 | 44.75 | 11% | 17.07 | 1.00 | 4.90 | 20.64 | 21% | 2.37 |
| 0.01 | 37.70 | 0.84 | 5.89 | 41.03 | 9% | 15.86 | 1.00 | 4.92 | 18.94 | 19% | 2.38 |
| 0.03 | 32.71 | 0.84 | 5.93 | 34.30 | 5% | 13.68 | 1.00 | 4.96 | 15.87 | 16% | 2.39 |
| 0.05 | 28.47 | 0.84 | 5.95 | 28.48 | 0% | 11.80 | 1.00 | 4.99 | 13.20 | 12% | 2.41 |
| 0.07 | 24.87 | 0.84 | 5.95 | 23.51 | -5% | 10.18 | 1.00 | 5.02 | 10.90 | 7% | 2.44 |
| Correlation Effects | | | | | | | | | | | |
| 0.00 | 39.33 | 0.84 | 5.61 | 43.84 | 11% | 16.26 | 1.00 | 4.77 | 20.20 | 24% | 2.42 |
| 0.20 | 39.91 | 0.84 | 5.74 | 44.30 | 11% | 16.67 | 1.00 | 4.83 | 20.42 | 23% | 2.39 |
| 0.40 | 40.47 | 0.84 | 5.87 | 44.75 | 11% | 17.07 | 1.00 | 4.90 | 20.64 | 21% | 2.37 |
| 0.60 | 41.01 | 0.84 | 5.99 | 45.19 | 10% | 17.45 | 1.00 | 4.96 | 20.85 | 19% | 2.35 |
| 0.80 | 41.54 | 0.84 | 6.11 | 45.61 | 10% | 17.81 | 1.00 | 5.02 | 21.04 | 18% | 2.33 |

Table 8: Valuation Results: Termination Rate, Vesting Period, and Equity Premium Effects

The table shows option values for a 42-year old male employee based on the estimated exercise rate function in specification 4 of Table 6. The left side shows an option at grant: the stock price and strike price are $100 and the option expires in ten years. The right side shows the case of an option that is 40% out of the money and has eight years to expiration. The base case assumes 8.5% annual termination rate, two-year vesting period, 50% volatility, zero dividend rate, and beta of one. The riskless rate is 5% and the expected market return is 11%. Prob Vest is the probability that the option vests given the assumed termination rate. Expected Term Given Vest is expected life of the option conditional on vesting, assuming option exercise according to the estimated exercise function and employment termination at the given termination rate. Modified Black-Scholes is the probability that the option vests times the Black-Scholes value of the option assuming expiration at the option's expected term conditional on vesting.

| | Ten-Year At-the-Money Option, $S = K = 100$ | | | | | Eight-Year Underwater Option, $S = 60, K = 100$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Changing Parameter | ESO Value | Prob Vest | Expected Term Given Vest | Modified Black Scholes | Percent Error | ESO Value | Prob Vest | Expected Term Given Vest | Modified Black Scholes | Percent Error | A-T-M/ Under. |
| Termination Rate Effects | | | | | | | | | | | |
| 0.05 | 45.71 | 0.91 | 6.41 | 50.79 | 11% | 19.25 | 1.00 | 5.64 | 23.08 | 20% | 2.37 |
| 0.07 | 43.01 | 0.87 | 6.13 | 47.69 | 11% | 18.13 | 1.00 | 5.25 | 21.83 | 20% | 2.37 |
| 0.09 | 40.47 | 0.84 | 5.87 | 44.75 | 11% | 17.07 | 1.00 | 4.90 | 20.64 | 21% | 2.37 |
| 0.11 | 38.07 | 0.80 | 5.62 | 41.98 | 10% | 16.09 | 1.00 | 4.57 | 19.49 | 21% | 2.37 |
| 0.13 | 35.81 | 0.77 | 5.40 | 39.36 | 10% | 15.16 | 1.00 | 4.26 | 18.40 | 21% | 2.36 |
| Vesting Period Effects | | | | | | | | | | | |
| 1.00 | 40.56 | 0.92 | 5.17 | 46.11 | 14% | 17.07 | 1.00 | 4.90 | 20.64 | 21% | 2.38 |
| 2.00 | 40.47 | 0.84 | 5.87 | 44.75 | 11% | 17.07 | 1.00 | 4.90 | 20.64 | 21% | 2.37 |
| 3.00 | 39.58 | 0.77 | 6.53 | 42.99 | 9% | 16.69 | 0.92 | 5.26 | 20.01 | 20% | 2.37 |
| 4.00 | 38.13 | 0.70 | 7.16 | 41.01 | 8% | 17.04 | 0.84 | 5.74 | 19.58 | 15% | 2.24 |
| 5.00 | 36.58 | 0.64 | 7.77 | 38.88 | 6% | 17.19 | 0.77 | 6.23 | 19.05 | 11% | 2.13 |
| Equity Premium Effects | | | | | | | | | | | |
| 0.00 | 40.47 | 0.84 | 6.16 | 45.77 | 13% | 17.07 | 1.00 | 5.08 | 21.25 | 25% | 2.37 |
| 0.02 | 40.47 | 0.84 | 6.06 | 45.45 | 12% | 17.07 | 1.00 | 5.02 | 21.06 | 23% | 2.37 |
| 0.04 | 40.47 | 0.84 | 5.97 | 45.11 | 11% | 17.07 | 1.00 | 4.96 | 20.86 | 22% | 2.37 |
| 0.06 | 40.47 | 0.84 | 5.87 | 44.75 | 11% | 17.07 | 1.00 | 4.90 | 20.64 | 21% | 2.37 |
| 0.08 | 40.47 | 0.84 | 5.76 | 44.38 | 10% | 17.07 | 1.00 | 4.83 | 20.41 | 20% | 2.37 |