

Optimizing Risk Retention

Quantitative Retention Management for Life Insurance Companies



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AUTHORS

Kai Kaufhold, Aktuar DAV
*Ad Res Advanced Reinsurance Services
GmbH*
kai.kaufhold@adreservices.com

Werner Lennartz, Ph.D.
inducto GmbH

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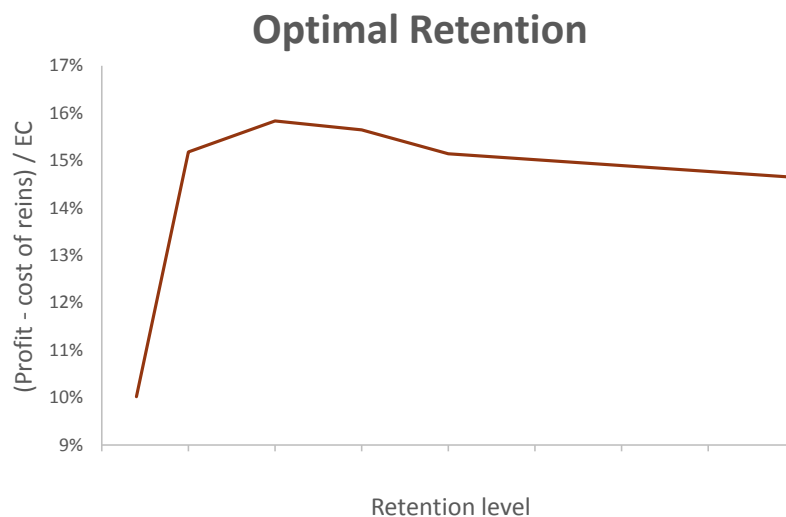
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I. Executive Summary

A life insurer can apply per person excess reinsurance as a risk management tool to reduce the volatility of its claims. This can be utilized to increase the company's return on economic capital. In our case study we apply quantitative retention management analysis that you can customize to your company's specific circumstances to determine the reinsurance retention limit that produces the optimal risk profile and maximum return on economic capital. Our case study illustrates a company's return on economic capital increased from 12% to 16%. Figure 1 depicts the analysis results and measures the return on economic capital over the wide range of retention-level choices:

- Reinsure too little, and the risk of large claims increases the amount of capital you need to hold. The cost of capital puts a drag on your profitability.
- Reinsure too much, and the cost of reinsurance outweighs the benefit of reducing the cost of capital.

Figure 1 Return on Economic Capital for Different Levels of Retention



Source: Case study for a block of term life business; hypothetical return on capital for fully retained policies 12%.

Rules of thumb cannot be applied for guessing the optimal retention limit. Each life insurer has to carry out the analysis for its own portfolio of risks. The optimal retention levels vary by:

- a. Size of the company
- b. Business mix by product and duration
- c. Heterogeneity of the individual risks
- d. Competitiveness of the reinsurance premiums
- e. Reserve and capital requirements and cost of capital

II. Introduction

This research study was sponsored by the SOA Reinsurance Section, the SOA Financial Reporting Section and the SOA Committee on Life Insurance Research with the objective to:

- Investigate the impact of reinsurance retention limits on retained reserves and solvency capital of a life insurer under modern reserving and solvency capital frameworks.
- Demonstrate the use of reinsurance as a tool to manage free surplus and risk.
- Outline a roadmap for companies to optimize their retention limits and manage their life insurance risk profile.

The Project Oversight Group consisted of Kevin Trapp (Chair), Min Mercer, Tom Edwalds, Clark Himmelberger, Lloyd Spencer, Ronora Stryker (SOA Research Actuary) and Jan Schuh (SOA Sr. Research Administrator).

The Challenge

Within a modern valuation and solvency capital framework, the reserves and capital that a life insurance company has to hold are directly tied to the riskiness of its business. In other words, reserves and capital in a principles-based framework must explicitly allow for the uncertainty associated with future claims. A life company's reinsurance coverage influences the level of riskiness of its business. Therefore, reserve margins and capital requirements have to be adjusted to reflect the impact of reinsurance.

Our aim is to investigate how reinsurance changes the risk profile of the retained life business and affects reserves and capital, and to propose a method by which life insurance companies can quantify the impact of reinsurance and thus choose an optimal retention level.

The Solution

By creating a statistical model that describes the insurance risk of a life book, we are able to quantify the uncertainty associated with a life company's insurance risk and can thus determine the necessary margins for reserves and capital relative to best-estimate liabilities. We derive the statistical model directly from the claims experience of a life company to create portfolio-specific valuation assumptions and capital requirements. Different reinsurance retention levels are modeled explicitly to quantify their impact on reserves and required capital.

Limitations

Before describing our research in detail, it is important to clarify the limits of the analysis. In our calculations, we have carried out neither a Principles-Based Reserve Valuation in accordance with the NAIC's Valuation Manual VM-20 nor a calculation of Solvency II capital requirements, both of which would have required creating a comprehensive cash flow model to project revenue, benefits and expenses. Instead, we have focused exclusively on the projection of benefits. The focus of our research has been twofold:

- a) How to set prudent assumptions for mortality and policyholder lapse (which VM-20 refers to as "Prudent Estimate Assumptions") and
- b) How the margins within the prudent assumptions should be adjusted to reflect the influence of reinsurance.

In our derivation of mortality and lapse assumptions, we have implicitly assumed the method of Aggregate Margins, as discussed in Neve (2013). Furthermore, we have deviated from the

mortality calculation method outlined in VM-20, which implicitly assigns greater credibility to industry tables than company experience. We have done so for two reasons:

1. According to Hardy and Panjer (1998), the basic premise for applying credibility theory to blend company-specific experience with industry tables is that the company experience must be a true subset of the industry experience and that the industry experience be relevant to the mortality of the lives being modeled. Given the limited participation of life companies in the industry studies from which industry mortality tables are derived, and due to the very limited differentiation by risk classes provided within industry tables, we believe that these basic prerequisites for applying credibility theory do not generally hold.
2. In relying entirely on company-specific experience and using statistical methods to quantify the uncertainty caused by estimation error and adverse deviation risk, we do not need to rely on industry tables at all. One could argue that the use of industry tables leads to underestimating the uncertainty associated with the projected mortality outcomes, and that for a small block of life insurance risks the uncertainty implied by the confidence intervals of its company-specific mortality experience is more appropriate.

We acknowledge the fact that for new business for which there are little or no experience data the actuary must indeed rely on external sources, such as industry experience or even general population data. The model risk introduced by having to apply actuarial judgement in such situations is beyond the scope of this report.

The roadmap presented here is designed to be general to any modern reserving and capital framework. It can be applied to analyze reinsurance in multiple regulatory jurisdictions and company structures.

III. The Framework

In this report, we outline a method that a U.S. life insurer can use to derive mortality and lapse assumptions for a principles-based valuation or for economic capital calculations. We assess the impact of different reinsurance retention levels on a life company’s in-force business.¹ Our research shows the reduction of uncertainty when a life insurer purchases excess of retention reinsurance. While the impact on benefit reserves is moderate and depends on the business mix, in particular on the duration of the company’s liabilities, the impact of reinsurance on capital is substantial. This enables the company to optimize its reinsurance retention with respect to its return on economic capital.

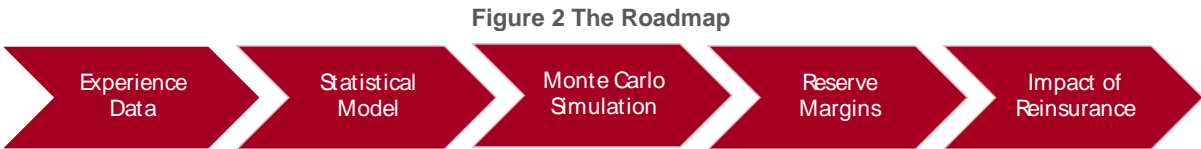


Figure 2 schematically shows the framework put forward in this report. Beginning with the policy-level *Experience Data* of a life company, which contains historical information on the deaths, lapses, surrenders and conversions affecting the company’s life insurance business, we build a *Statistical Model* that describes these multiple decrements. Using the method of

¹ The techniques shown here fall under the broad topic of *Predictive Modeling* and include nonlinear models.

parametric survival models, we are able to quantify the influence of different risk factors on mortality and persistency.

In the next step, we generate a distribution of possible outcomes for claims by running a *Monte Carlo simulation*. The distribution of annual claims and total liabilities is consistent with the company's experience and allows us to calculate the *reserve and capital margins*, with which the company has to load best-estimate liabilities for prudent valuation and for solvency capital requirements. By running the simulation against a range of different reinsurance retention levels we can study the *impact of reinsurance* on reserves and capital margins.

The Data



The starting point for our framework is the life company's historical experience data. To extract the maximum amount of information from the data, we utilize the seriatim policy-level information. For each life leaving the portfolio, we know the exact date and type of exit.

Essentially, the individual insured lives data give us a longitudinal view of the historical experience and allow us to build a statistical model with which we can identify different risk factors and quantify their impact on future claims and premiums. One important point to keep in mind is that we should ideally model insured lives and not policies, which we can do only by combining different policies covering the same life, a process referred to as *deduplication*.² Deduplication is routinely carried out by life companies when they check retention under their reinsurance treaties.

The Model



Life companies require a set of robust best-estimate assumptions for their life insurance risks. Such best-estimate assumptions must take into account all relevant risk factors that influence the mortality and persistency risk to which the company is exposed. This is important, because the distribution of a company's business across the different risk classes largely explains why company-specific experience deviates from the industry average. In addition, trends within a single company's overall experience will in part be due to changes in the composition of its business.

Portfolio-specific assumptions including multiple risk factors can be derived using *parametric survival models*, which describe mortality and lapse risk in continuous time and thus generate a set of smooth graduated tables consistent with the company's experience data.³ In practice, a continuous-time model means modeling mortality and lapses daily rather than the historical actuarial approach of analyzing annual mortality and monthly lapse rates.

² See Richards (2012).

³ The advantages of modeling mortality at the individual level in continuous time are discussed in detail in Richards, Kaufhold and Rosenbusch (2013).

Hazard Rates

We denote the continuous-time mortality rate as μ_x , which is known to actuaries as the force of mortality.⁴ The model we have used for this investigation is the simple time-varying Gompertz law:

$$(1) \quad \mu_{x,y,\tau} = e^{\alpha_\tau + \beta x + \delta(y-2000)},$$

where x denotes age, y denotes calendar time and τ stands for time since policy issue. Age is measured as exactly as the data allow in fractions of a year. Time is measured similarly; for example, April 1, 2003, is rendered as 2003.25. The offset of year 2000 is to keep the parameters well scaled.

We used the LongevityTM software⁵ to fit⁶ the mortality law to the data, that is, to estimate the parameters α , β and δ . The parameter α represents the level of mortality, which can be raised or lowered by the main effect of various risk factors such as gender, underwriting class or policy face amount (a.k.a., sum assured). β represents the rate of change of mortality with age and can be modified by interactions with the main effects; for example, the rate of aging might differ between males and females. The parameter δ describes a constant time trend⁷ over the study period. We also include the duration since policy inception as another continuous-time variable τ . We incorporate different durational patterns by expressing the intercept parameter α_τ as a step function.

MultiDecrement Model

In addition to mortality, the claims experienced in a life insurance portfolio are also influenced by other decrements: lapse, surrender, conversion or disability. These multiple decrements potentially impact the run-off behavior of the portfolio and therefore the total liabilities of the life company. For this reason, we decided to model the lapse hazard as a risk competing with mortality. Using continuous-time survival models allows us to do so without having to make additional assumptions about the respective timing of lapses and deaths, like we would have to for q_x -type discrete-time models.

We shall denote the lapse hazard rate $\lambda_{x,y,\tau}$ which depends on age x , calendar time y and duration τ , just like the mortality hazard. We can write the total hazard rate $h_{x,y,\tau}$ simply as the sum of the force of mortality $\mu_{x,y,\tau}$ and the lapse hazard rate $\lambda_{x,y,\tau}$:

$$(2) \quad h_{x,y,\tau} = \mu_{x,y,\tau} + \lambda_{x,y,\tau}.$$

Because we are building a continuous-time model we could theoretically also include the interdependencies between mortality and lapse. However, we have made the simplifying assumption that the instantaneous mortality and lapse hazard rates are independent of each other and can fit separate models for each hazard. For this purpose, we have ignored the concept of antiselective lapse. If we included experience after the end of the level-term period, this simplification would not be appropriate, and the lapse and mortality models would have to incorporate some sensitivity to the other decrement.

⁴ Engineers call the same thing the *failure rate*, and statisticians call it the *instantaneous hazard rate*, but the concept is identical.

⁵ LongevityTM is a commercial software for the analysis of mortality and other demographic risks. For more details, see <http://www.longevity.co.uk/site/ourservices/survivalmodelling/>.

⁶ By fit we refer to estimating parameters of the survival models by maximizing the likelihood function. The parameter estimates shown in this report are therefore all Maximum Likelihood Estimates (MLEs).

⁷ Multidimensional trend models (e.g., Age-Period-Cohort model or Lee-Carter model) have a very large number of parameters and require more data than a typical life company would have. Therefore, they are usually calibrated to the mortality data of an entire country's population.

Risk Factors and their Impact

So far, we have discussed only the basic risk factors age and time. In parametric survival models, we can include any number of risk factors for which we have consistent data in the experience study. We can investigate whether a risk factor has a statistically significant impact on mortality or lapse hazard. All we have to do is to define risk categories and adjust the parameters of the mortality or lapse hazard rates at the level of the individual. For instance, if we want to distinguish between male and female mortality and smokers versus nonsmokers, we would pick the largest group as the baseline—male nonsmokers in our case—and fit parameters for females and smokers. The intercept α_i and slope β_i in the log-linear Gompertz law for each individual i in the file can then be written as

$$(3) \quad \begin{aligned} \alpha_i &= \alpha_0 + I_{Female}\alpha_{Female} + I_{Smoker}\alpha_{Smoker}, \\ \beta_i &= \beta_0 + I_{Female}\beta_{Female} + I_{Smoker}\beta_{Smoker}. \end{aligned}$$

Here α_0 and β_0 refer to the baseline (male, nonsmokers). The indicator function I_{Female} takes on the value 1 for females and 0 for males, and I_{Smoker} takes on the value 1 for smokers and 0 for nonsmokers. In this manner, the model can be extended to include numerous risk factors, such as underwriting class and policy size band. We estimate values for these parameters using the Maximum Likelihood method.

Having built a statistical survival model describing the mortality and lapse behavior of the insured lives in a specific portfolio, we can calculate the actuarial present value of future benefits and future premiums for each individual with policies in force at a particular valuation date y_0 . The sum of actuarial present values over all lives in-force on the valuation date will represent our best-estimate present value (PV) of future liabilities.

For the purpose of this research, we have ignored future premiums in the calculation of policy benefit reserves. We have thus implicitly assumed that all policies are paid up and the present value of liabilities simplifies to the present value of future claims. This may impact the quantitative results, but we believe that this simplification will not affect our conclusions.

Recall that policy benefit reserves are calculated as present value of future claims less present value of future premiums. In situations when mortality is heavier and therefore the present value of future claims is greater, the portfolio will run off more quickly and the present value of future premiums will be smaller, leading to greater benefit reserves. We can therefore assume that variability in present value of liabilities will be at least as great as the variability in the present value of claims.

The Simulation



Now we can reap the benefit of having built a statistical model, because survival models allow us to quantify how uncertain the central estimate is. There are two main reasons for the uncertainty⁸ of the claims outcome:

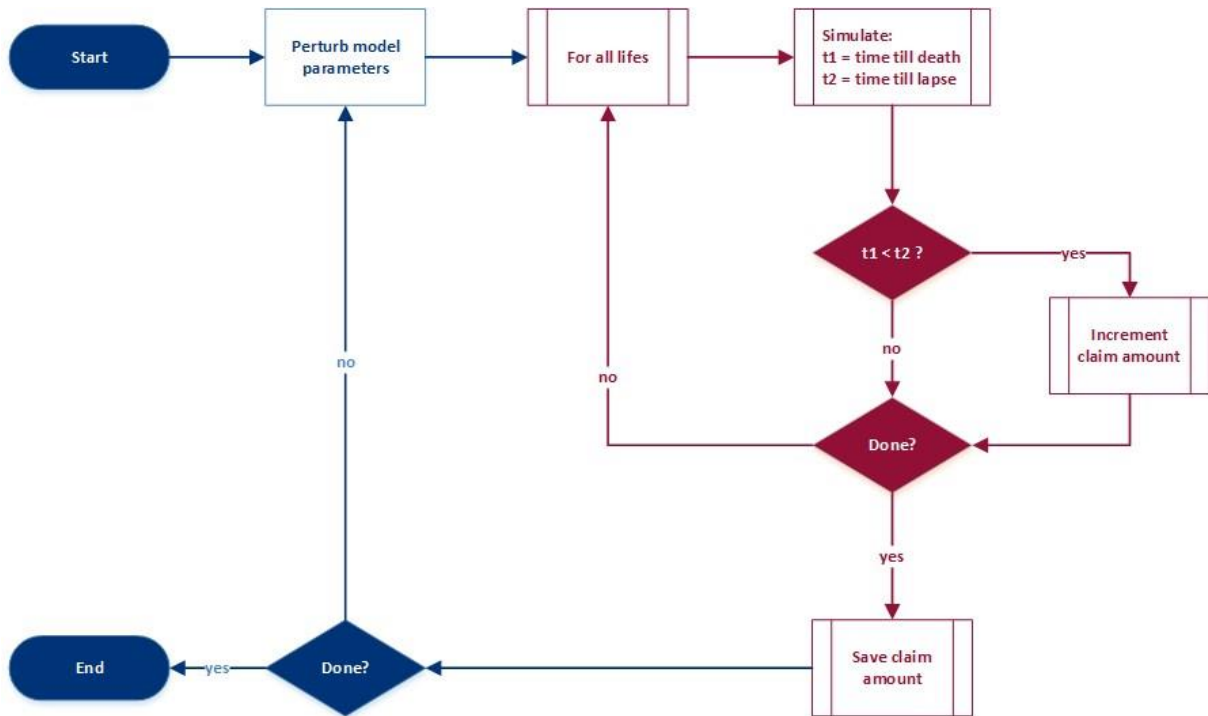
- *Misestimation risk*: The parameters of the model are estimated from a finite set of data, which means that their true value may differ from the estimated parameter values. This estimation error depends on the size and nature of the business. The larger the data set, the closer the parameter estimates will be to the true parameter values, which means that misestimation risk will be smaller for large, stable blocks. Conversely, misestimation risk will have a larger impact for smaller companies with less experience data.
- *Idiosyncratic risk*: Deaths and lapses are random events, and therefore the claims outcome will always be uncertain, even if we did know the true parameter values for the model. The stochastic uncertainty depends on how the different risk factors, like face amount, smoker status etc., are distributed within the block of insurance risks. The more concentrated the risk is upon a small group, the greater the potential variability of results.

We run a Monte Carlo simulation to quantify the impact of both sources of uncertainty. Generally speaking, a Monte Carlo simulation is a stochastic experiment, in which we simulate the death or lapse of each individual and calculate the financial result for many different random scenarios. The aim is to generate the random events in a way consistent with the model so that the Monte Carlo simulation gives us a probability distribution of the financial outcomes for the specific block of business.

The Monte Carlo simulation in our framework includes two nested steps: one for misestimation risk and one for idiosyncratic risk. The simulation is depicted schematically in Figure 3.

⁸ Further contributors to the overall uncertainty include *model risk* as well as *mortality trend risk*. For the purpose of this report, we do not consider any trend assumption beyond the end of the experience study period. Therefore mortality trend risk is excluded from the scope.

Figure 3 Schematic of Monte Carlo Simulation



Step 1: Perturbation

First we perturb our existing models for mortality and lapse in a way consistent with the experience data, to which they were fitted. We follow the calculation method described by Richards (2016). The important thing to observe is that we do not perturb (“shake”) each parameter in isolation but have to consider its correlations with all other parameters.

Simple Example

If our survival model only contained the parameters α and β with estimated values $\hat{\alpha}$ and $\hat{\beta}$, then we would need to draw two Normal-distributed random numbers r_1, r_2 and could generate two perturbed parameters α' and β' in the following way:

$$(4) \quad \begin{aligned} \alpha' &= \hat{\alpha} + \tilde{\sigma}_{\alpha\alpha}r_1 + \tilde{\sigma}_{\alpha\beta}r_2, \\ \beta' &= \hat{\beta} + \tilde{\sigma}_{\beta\beta}r_1 + \tilde{\sigma}_{\beta\alpha}r_2. \end{aligned}$$

$\begin{pmatrix} \tilde{\sigma}_{\alpha\alpha} & \tilde{\sigma}_{\alpha\beta} \\ \tilde{\sigma}_{\beta\alpha} & \tilde{\sigma}_{\beta\beta} \end{pmatrix}$ are correlation coefficients derived from the variance-covariance matrix.⁹

Step 2: Lifetime Simulation

In the second step, we use the perturbed survival model to simulate the random future lifetime of each individual.

We can calculate the individual’s survival curve¹⁰ using Equation 2:

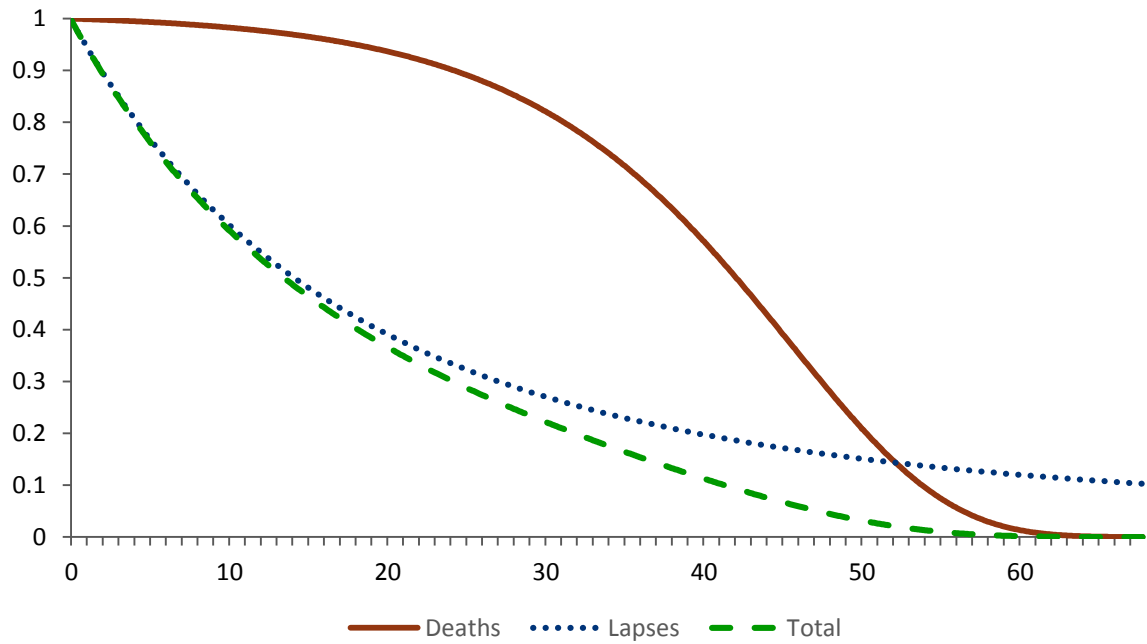
$$(5) \quad {}_t p_x = \exp\left(-\int_0^t h_{x+s} ds\right) = \exp\left(-\int_0^t \mu_{x+s} ds\right) \times \exp\left(-\int_0^t \lambda_{x+s} ds\right).$$

⁹ More details on the perturbation and simulation method in Appendix B.

¹⁰ Here “survival curve” describes the remaining time before the individual exits the portfolio, either by lapsing or due to death.

Figure 4 Survivorship Functions for Mortality and Lapse

Survival within Portfolio



Source: Simple example for a male aged 52 in 2012, residual standard nonsmoker, policy year 10 (past selection period).

We draw two uniformly distributed random numbers u_1 and u_2 and use them to calculate random remaining *time until death* t_1 and *time until lapse* t_2 :

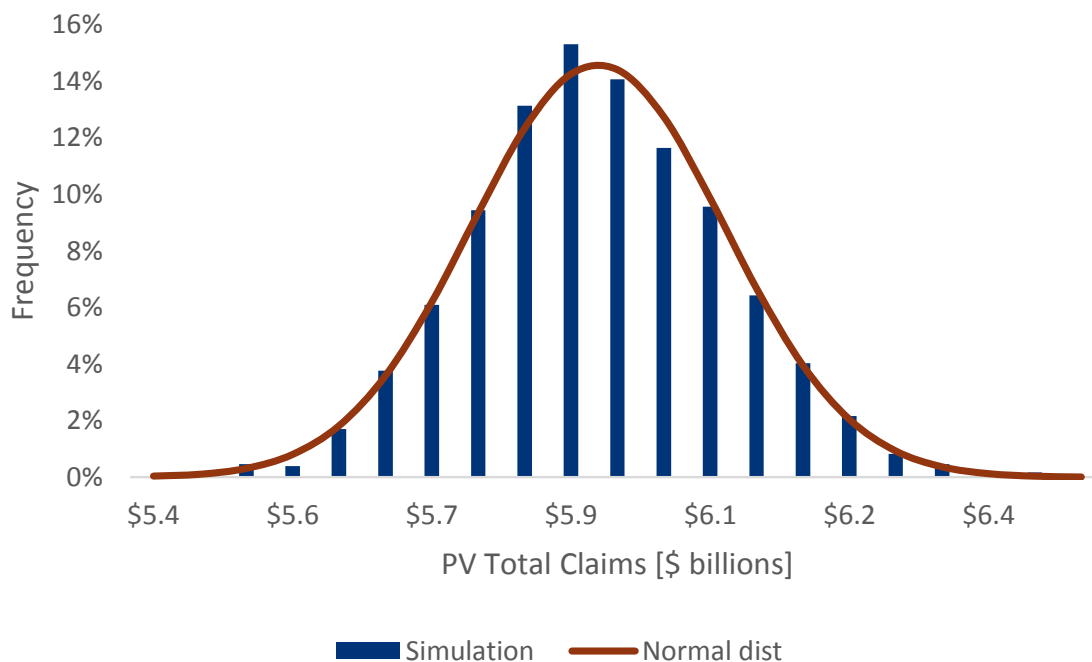
$$u_1 = 1 - \exp\left(-\int_0^{t_1} \mu_{x+s} ds\right) = F(t_1) \mapsto t_1(\text{death}) = F^{-1}(u_1),$$

$$u_2 = 1 - \exp\left(-\int_0^{t_2} \lambda_{x+s} ds\right) = F(t_2) \mapsto t_2(\text{lapse}) = F^{-1}(u_2).$$

Now, all we need to do is check whether lapse or death occurred first, that is, whether $t_1 > t_2$ or $t_1 < t_2$, and calculate the appropriate benefit reserve for that scenario. We repeat this valuation for each individual and then for each perturbation of the survival model.

The result of the simulation is a distribution of liabilities, as illustrated in Figure 5.

Figure 5 Monte Carlo Simulation of Claims



Source: Sample Monte Carlo simulation of term life case study. The simulated results have greater skewness and fatter tails than the corresponding Normal distribution.

The Principles-Based Reserves and Capital Requirements



In its *Model Standard Valuation Law—VM-05*, the National Association of Insurance Commissioners (NAIC) states the *Requirements of a Principles Based Valuation (Section 12.A)*:

- (1) Quantify benefits ... and their risks at a level of conservatism that reflects conditions that include unfavorable events that have a reasonable **probability** of occurring during the lifetime of the contracts. ...
- (2) Incorporate assumptions, risk analysis methods ... that are **consistent** with ... those utilized by **the company's overall risk assessment process**. ...
- (3) Incorporates assumptions that are derived in one of the following manners:
 - a. The assumption is prescribed in the Valuation Manual.
 - b. For assumptions that are not prescribed, the assumptions shall:
 - i. Be established using **the company's available experience**, to the extent that it is relevant and statistically credible; or
 - ii. To the extent that company data is not available, relevant or statistically credible, be established utilizing other relevant, statistically credible experience.
- (4) Provide **margins for uncertainty, including adverse deviation and estimation error**, such that the greater the uncertainty the larger the margin and the resulting reserve.

The NAIC Valuation Manual 20 (VM-20) prescribes three different methods of calculating principles-based reserves: net premium reserve, deterministic reserve and stochastic reserve. Each of these methods requires *prudent estimate assumptions* for mortality, lapse and

expenses. In general, principles-based valuation requires that these prudent estimate assumptions be determined by:

- Applying *probabilistic* methods for quantifying insurance risks
- Ensuring their consistent *use* in the management of insurance business and
- Deriving assumptions from a *statistical analysis of company experience*, where possible.

The framework put forward in this report is consistent with the general aims of the NAIC stated above. However, we do not follow the calculation method laid out explicitly in VM-20. We start out with a statistical analysis of the life company's experience data and then carry out a Monte Carlo simulation. Our aim within this project is to apply the results of the statistical analysis to managing the insurance risk of the insurer by means of per person excess reinsurance. Beyond this application, the results can also be used within other contexts as well, such as product development and marketing.

Within this context, it might be interesting to note that under the new *Solvency II* regulatory framework in the European Union, a life insurer can use a so-called *Internal Model* for calculating its *Solvency Capital Requirements* only if it can demonstrate that it puts the model to use in its risk management and business practices. Such a *Use Test* is fulfilled by the framework in this report, as we use the framework to actively decide upon the company's retention, which is an important risk management application. The framework can also be applied to in pricing and product development.

Reserve MARGIN

We derive prudent estimate assumptions by using the distribution of liabilities from our Monte Carlo simulation to calculate confidence intervals around best-estimate liabilities and risk measures such as the Conditional Tail Expectation $CTE(X)$ for a given percentile X .

Assuming a CTE_{70} reserving requirement, which is the conditional expected value of all those claims that exceed the 70th percentile, we can express the valuation margin for adverse deviation and estimation error in terms of the best-estimate liabilities (BEL) as follows:

$$(6) \quad \text{Reserve Margin} := \left(\frac{CTE(70) \text{ of liabilities}}{BEL} - 1 \right).$$

Economic capital

Life insurers are subject to regulatory requirements to hold a certain amount of solvency capital to protect the company against ruin should it incur disastrous losses in any given year. Different regulatory regimes have different methods of defining the solvency capital requirements. Most of the recently introduced capital frameworks operate on a probabilistic basis, measuring required capital in terms of either the Value-at-Risk (VaR) in a one-in-200-year event, that is, 99.5th percentile of the total annual loss distribution or the expected shortfall given a certain percentile loss.

The new European regulatory regime *Solvency II* is based on the VAR approach, as were the *Individual Capital Assessments* (ICA) in the United Kingdom. The expected shortfall is also referred to as Conditional Tail Expectation (CTE) and is used within the Canadian *Minimum Continuing Capital and Surplus Requirements* (MCCSR) and the *Swiss Solvency Test* (SST).

For the purpose of this report, we will not consider all the different capital frameworks but will apply a simplified method, which is based on the CTE99 of the distribution of annual claims. We will refer to this figure as “Economic Capital.” We assume that a life company will hold at least enough Economic Capital (EC) to cover an annual loss at the level of CTE99. We express the EC in terms of the best-estimate liabilities:

$$(7) \quad EC \text{ Margin} := \frac{CTE99(Annual \text{ Loss}) - \text{mean}(Annual \text{ Loss})}{BEL}$$

Impact of Reinsurance Retention on Reserves and Capital



Having established an integrated method for deriving reserve and capital margins from a life company’s own experience data, we can apply this method gross and net of reinsurance. This shows us how a particular reinsurance coverage changes the risk profile of the retained business and what the impact of reinsurance is on a life insurers retained net liabilities and Economic Capital.

IV. Case Study: U.S. Term Life Insurance

The Data



For our case study, we used seriatim policy experience data for term life insurance collected from the U.S. life insurance industry by a leading consulting firm for the years 2000 to 2011. The data provided to us were sampled randomly from one life insurer’s data to ensure that it was impossible to identify the company or the individual lives.¹¹

For each policy, we had information on age, gender, date of issue, type of product, smoking status, underwriting class and policy face amount. For the purpose of this analysis, we decided to model only the experience up to the end of the level-term period. Post-level-term experience is an important factor that should be considered when applying this framework in real life; however, it raises issues that would have exceeded the scope of this project. All exposure times and decrements (deaths and lapses) stated in the tables below are with respect to the level-term period only. Furthermore, we limited the analysis to the range 30 to 86 years of age, outside of which only very limited data on deaths and lapses were available.

We further simplified the analysis by considering only the run-off of the in-force business at the end of the study period. No new business was considered. In our case study, we have also combined lapses and surrenders to be one type of decrement that is modeled simultaneously

¹¹ There was no information available by which the data could be deduplicated. Therefore, our analysis is on a policy basis only.

with death. Furthermore, we chose to treat term conversions as a censoring event, which means that these lives exited without decrement.

The following tables summarize the case study data by gender, underwriting class, smoking status, product and amount band.

Table 1 Summary by Risk Group, Ages 30 to 86 Years Only

Risk class	Risk group	Number of policies	Total face amount [US\$ millions]	Exposure ¹² [time lived]	Deaths	Lapses
Gender	Female	650,906	\$174,512	4,438,518	4,021	180,255
	Male	944,353	\$407,996	6,276,808	9,864	333,591
Rated	Standard	1,530,354	\$566,494	10,344,206	12,734	485,272
	Substandard	64,905	\$16,014	371,120	1,151	28,574
Product	T10	396,958	\$128,601	2,070,848	4,057	185,315
	T15	303,164	\$97,204	2,224,775	3,995	106,544
	T20	671,708	\$272,655	4,890,347	4,329	151,238
	T30	189,900	\$75,007	1,348,023	672	44,457
	Other Term	33,529	\$9,041	181,334	832	26,292
Underwriting Class	Preferred	590,062	\$257,668	4,345,446	3,056	143,397
	Residual standard	894,851	\$300,663	5,733,886	8,629	316,519
	Smoker	110,346	\$24,178	635,993	2,200	53,930
Total		1,595,259	\$582,508	10,715,326	13,885	513,846

Table 2 Summary by Face Amount Band

Amount band	From US\$	To US\$	Number of policies	Total face amount [US\$ millions]	Exposure [time lived]	Deaths	Lapses
1	0	99,999	135,865	\$6,976	824,701	3,026	52,496
2	100,000	124,999	335,052	\$33,626	2,245,455	3,629	110,015
3	125,000	249,999	271,160	\$47,598	1,903,095	2,478	91,123
4	250,000	299,999	249,850	\$62,605	1,702,191	1,524	76,336
5	300,000	499,999	163,429	\$55,823	1,140,202	1,019	50,170
6	500,000	749,999	244,294	\$125,540	1,621,494	1,242	72,694
7	750,000	1,999,999	166,525	\$167,137	1,096,445	803	50,243
8	2,000,000	150m	29,084	\$83,203	181,742	164	10,769
Total			1,595,259	\$582,508	10,715,326	13,885	513,846

The highest amount band contained fewer than 2% of all policies. However, these policies accounted for more than 14% of the total face amount. The two highest bands together included fewer than 12.5% of policies but 43% of the total face amount, indicating a high level of concentration risk, which contributes to the overall volatility of the mortality results of the portfolio and influences the impact of reinsurance on these results.

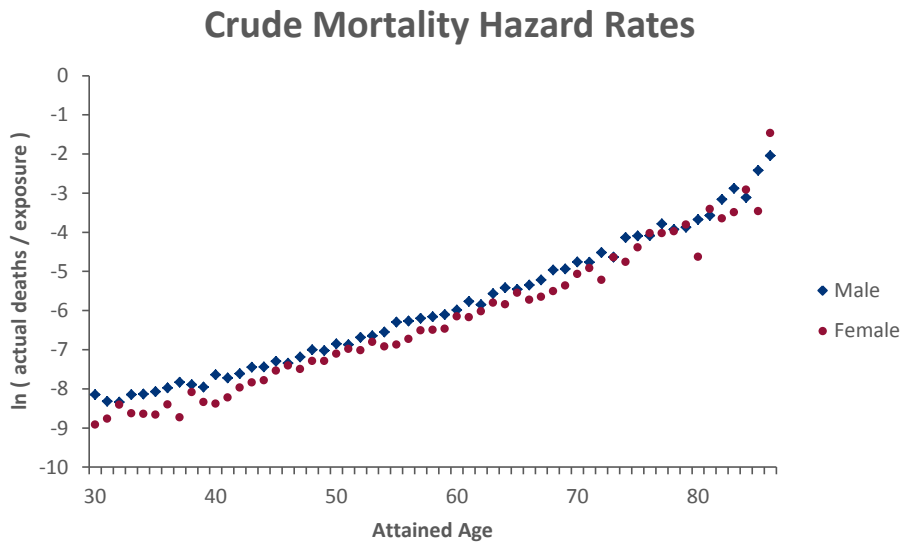
The Model



We chose a simple log-linear Gompertz function for both the mortality and the lapse hazard rates, because both decrements largely follow a linear pattern on a logarithmic scale within the core age range.

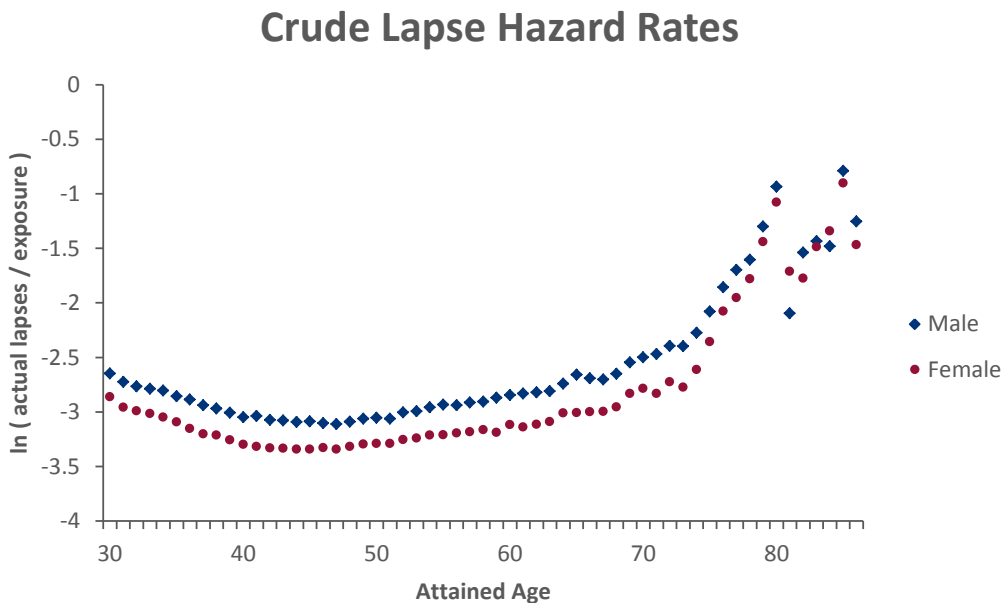
¹² A policy entering the study on January 1, 2000, and still exposed to risk on December 31, 2011, would have contributed 12 life-years of exposure.

Figure 6 Crude Hazard Rates against Age, Natural Logarithm of Actual Deaths over Exposure Time



The crude mortality hazard rates in Figure 6 show a log-linear behavior between ages 40 and 75. For older ages, the crude mortality rates appear to increase faster, which means that a more complex mortality law might have been more appropriate.

Figure 7 Crude Hazard Rates against Age, Natural Logarithm of Actual Lapses over Exposure Time



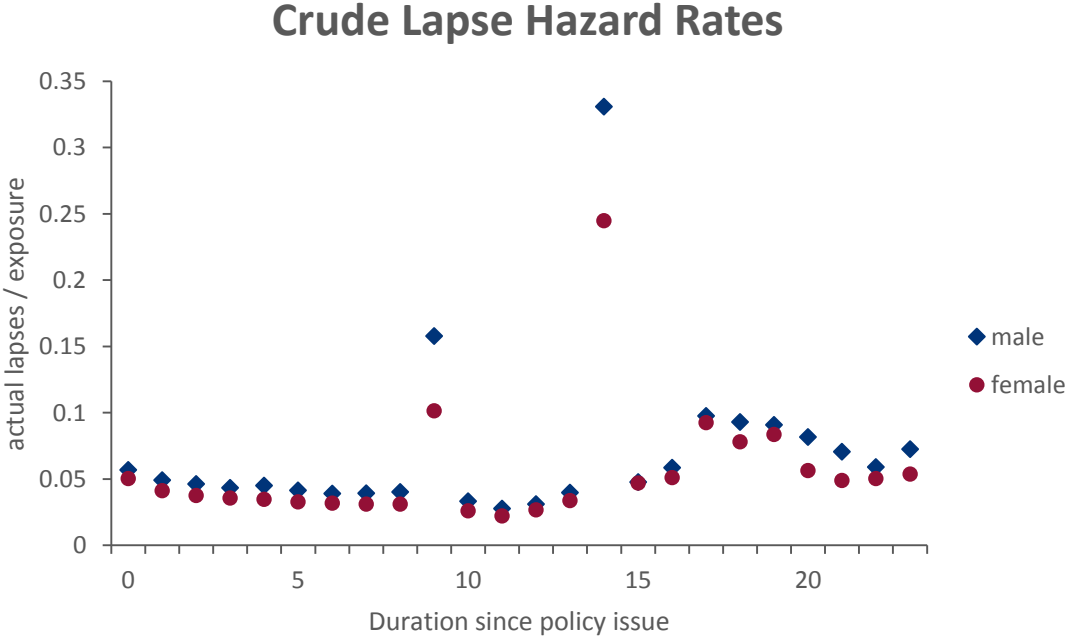
Since this case study is for illustration purposes only, we limit the analysis to the log-linear function.¹³ The lapse hazard rates by age in Figure 7 are roughly log-linear between ages 45 and 75. Outside this age range the age pattern of lapses takes on a trough-like shape, which is often referred to as “bathtub hazard,” meaning that lapses are higher than average for younger ages as well as for advanced ages. A more rigorous analysis, which would also be based on lives instead of policies, would use a more complex function for lapses. However,

¹³ Generalized Linear Models commonly assume either a logarithmic or a logistic link, which implies that mortality rates decelerate at advanced ages. Clearly the logistic function would not be appropriate for the experience data shown here.

we did not have the necessary information to deduplicate the policies for a lives-based analysis.

For the purpose of our case study, however, we ignore the curvature, mainly because the age pattern of lapses is likely less important than the shape by duration since policy issue. Figure 8 shows two lapse spikes in policy years 10 and 15, which is consistent with the prevalence of T10 and T15 products in the portfolio. We note briefly that there is no corresponding spike for T20. This is simply due to the fact that there are hardly any lives left at the end of policy year 19, that is, 3,200 lives compared to 1.2 million lives in duration 1 or 165,000 lives at the end of policy year 14, because most of the observed policies were written less than 20 years ago. Note that the magnitude of the lapse spikes shown in Figure 8, which is aggregate across all guarantee periods, depends on the composition of the portfolio by term. Within our model, which is for the level-term period only, we project mortality and lapses only until each individual policy’s maturity date, when it is censored. Thus, the magnitude of the lapse spike itself has only a relatively small overall impact. When considering post-level-term mortality and its correlation with lapses at the end of the level term, it would be necessary to model the lapse spikes separately for each term product.

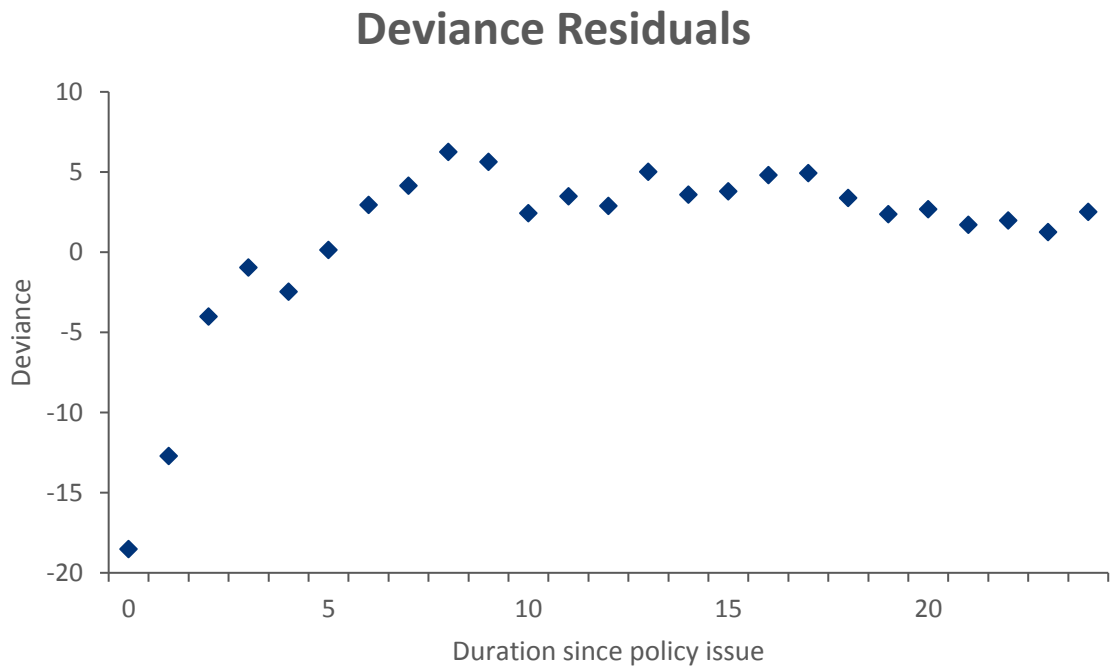
Figure 8 Crude Hazard Rates against Duration



Duration is an important risk factor for mortality as well. Insured lives mortality during early policy years is typically substantially lower than average due to the selection effect of the underwriting process. In Figure 9 we show the deviance residuals¹⁴ by duration for a simple model that does not include any allowance for duration. The residuals show how actual experience differs from the modeled (“expected”) deaths. A selection period of at least six years is apparent from the pattern of mortality by duration.

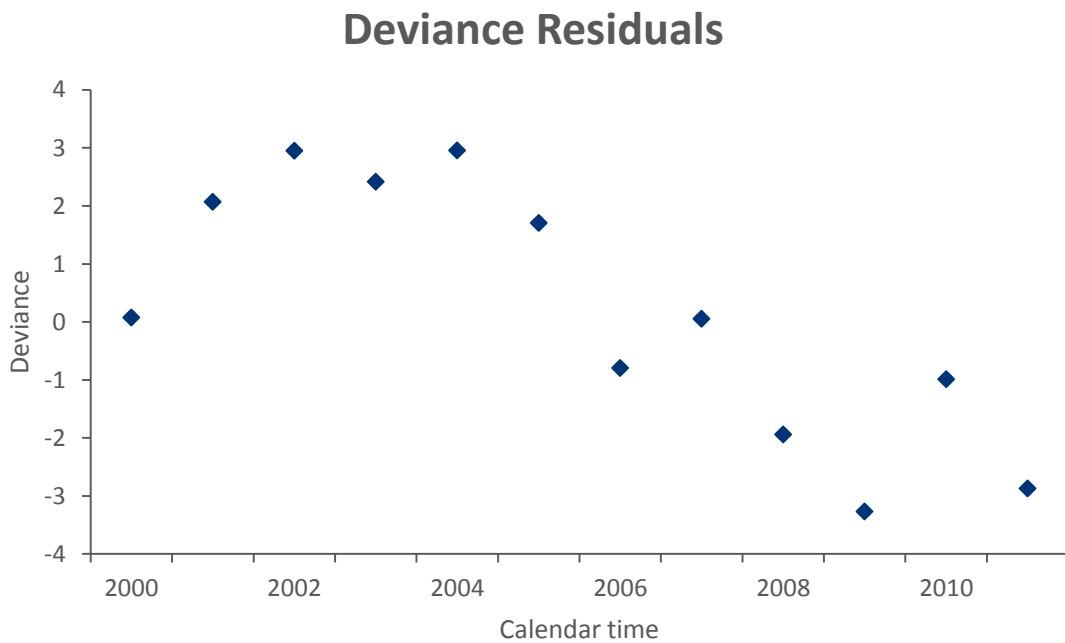
¹⁴ See Appendix A on Model Choice for the definition of deviance residuals.

Figure 9 Deviance Residuals for Mortality against Policy Duration



The data used for our case study relate to a 12-year observation period. Reviewing the residuals of a simple model that does not incorporate time as a risk factor, we can observe a mortality improvement trend. For this reason, our models incorporate age, calendar year and policy duration as real-valued risk factors.

Figure 10 Deviance Residuals for Mortality against Time



We model the changes in mortality and lapse rates with step functions, because this makes it straightforward to model both the selection shape and the lapse spikes by duration. Both mortality and lapse hazard functions have the same structure: they include parameters β^{mort} , β^{lapse} and δ^{mort} , which describe the increase of the hazard rates by age and a uniform

decrease over calendar time. In addition, we include adjustment factors for each selection period α_{Select}^{mort} and α_{τ}^{lapse} :

$$\mu_{x,y,\tau} = \exp\left(\alpha^{mort} + \alpha_{Select}^{mort} + \beta^{mort}x + \delta^{mort}(y - 2000)\right),$$

$$\lambda_{x,\tau} = \exp\left(\alpha^{lapse} + \alpha_{\tau}^{lapse} + \beta^{lapse}x\right).$$

The adjustment factors α_{Select}^{mort} and α_{τ}^{lapse} describe the durational effects. As seen in Figure 8 and Figure 9, mortality and lapse rates have distinct patterns by duration. We have modeled this by determining in advance which selection periods to fit and then estimating the values for α_{Select}^{mort} and α_{τ}^{lapse} .

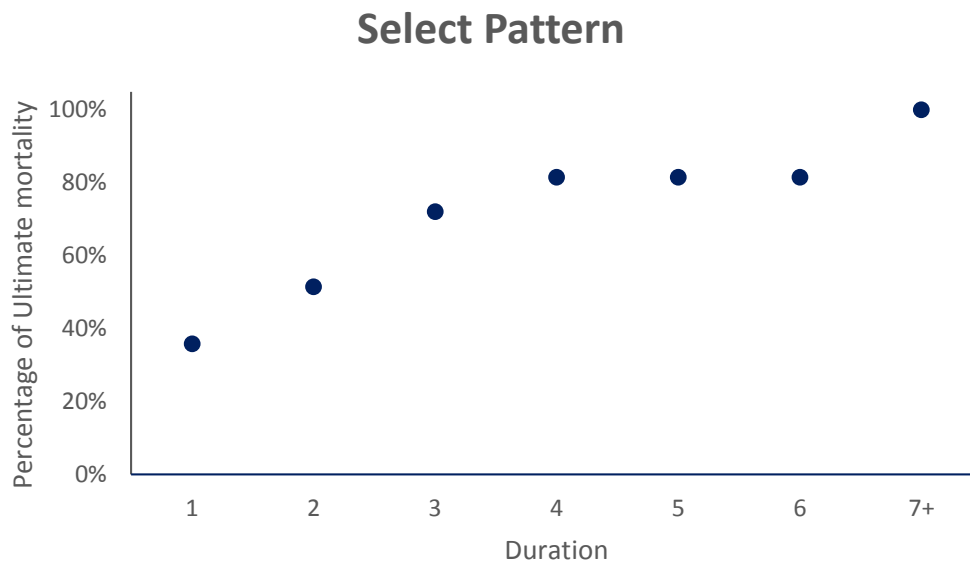
Table 3 Basic Parameter Estimates

Model	Type	Parameter	Estimate	Std. error	Significance ¹⁵
Mortality	Intercept	α^{mort}	-11.41	0.06	***
	Age	β^{mort}	0.094	0.001	***
	Time	δ^{mort}	-0.023	0.003	***
Lapse	Intercept	α^{lapse}	-2.94	0.02	***
	Age	β^{lapse}	-0.0113	0.0002	***

Table 4 Parameter Estimates for α_{Select}^{mort}

Parameter	Policy year	Estimate	Std. error	Significance
α_0	1	-1.03	0.05	***
α_1	2	-0.66	0.04	***
α_2	3	-0.33	0.03	***
α_3	4-6	-0.20	0.02	***
$\alpha_{ultimate}$	7+	-	-	Baseline

Figure 11 Adjustment Factors for Mortality by Duration

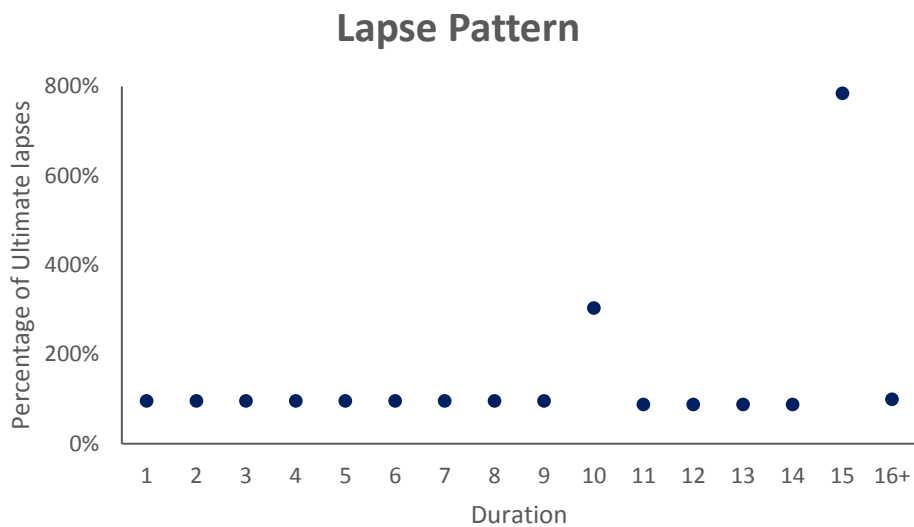


¹⁵ Statistical significance in relation to p -value: *** $p < 0.001$, ** $p < 0.01$, * $p < 0.05$.

Table 5 Parameter Estimates for α_{τ}^{lapse}

Parameter	Policy year	Estimate	Std. error	Significance
α_0	1–9	-0.038	0.02	.
α_1	10	1.11	0.02	***
α_2	11–14	-0.12	0.02	***
α_3	15	2.06	0.02	***
$\alpha_{ultimate}$	16+	-	-	Baseline

Figure 12 Adjustment Factors for Lapse by Duration



Additional Risk Factors

To capture the mortality differentials by risk group, we have incorporated categorical variables as described by Equation 3 for gender, product, underwriting class, policy size and whether a policy was issued standard or with a substandard rating. The results are shown in Table 6 and Table 7.

Table 6 Parameter Estimates in Mortality Model for Different Risk Categories

Category	Risk group	Estimate	Std. error	Significance
Gender	Male	0	-	Baseline
	Female	-0.33	0.02	***
Underwriting	Preferred	-0.25	0.02	***
	Residual standard	0	-	Baseline
	Smoker	1.09	0.02	***
Rated	Standard	0	-	Baseline
	Substandard	0.50	0.03	***
Policy size	Amount band 1	0.17	0.03	***
	Amount bands 2, 3	0.07	0.02	***
	Amount bands 4–8	0	-	Baseline
Product	T10, T15, T20	0	-	Baseline
	T30	0.16	0.04	***
	Other term	0.32	0.04	***

Table 7 Parameter Estimates in Lapse Model for Different Risk Categories

Category	Risk group	Estimate	Std. error	Significance
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Gender	Male	0	-	Baseline
	Female	-0.164	0.003	***
Underwriting	Preferred	-0.302	0.003	***
	Residual standard	0	-	Baseline
	Smoker	0.298	0.005	***
Rated	Standard	0	-	Baseline
	Substandard	0.334	0.006	***
Policy size	Amount band 1	0.052	0.005	***
	2	0	-	Baseline
	3	0.042	0.005	***
	4	0.062	0.005	***
	5	0.056	0.006	***
	6	0.109	0.005	***
	7	0.157	0.006	***
	8	0.372	0.01	***
Product	T10	0.993	0.004	***
	T15	0.273	0.004	***
	T20	0	-	Baseline
	T30	0.070	0.005	***
	Other term	1.316	0.007	***

The figures in Table 6 imply that someone in the preferred underwriting class would have $\exp(-0.25009) \approx 22.1\%$ lower mortality than the baseline residual standard underwriting class; a smoker has up to 197% higher mortality than a standard nonsmoker. Insureds with the largest policies have the lowest mortality, with the smallest policy sizes being associated with nearly 18.6% higher mortality rates. All term products had similar levels of mortality, apart from the T30 and Other categories, which had higher than average mortality.

As far as lapses go, Table 7 shows that females appear to be less likely to lapse than males, a preferred policy is less likely to lapse than a residual standard underwritten policy, and a smoker is 35% more likely to lapse their policy. Interestingly, the larger a policy is, the greater the propensity of its policyholder is to lapse.

Simulation Results and Reserve Margins



Figure 13 shows the distribution of future claims that we obtain by running the Monte Carlo simulation for the entire term portfolio in our case study. The perturbation and simulation process was carried out 5,000 times. The resulting distribution for the PV of future claims is shown in Figure 13 and has mean \$6.59 billion and standard deviation \$220 million. We calculate percentiles and conditional tail expectation values for the purpose of determining principles-based reserves and capital. Note that these percentiles and CTE values are greater than the corresponding values for a Normal distribution. This means that the distribution of

liabilities for this case study has “heavy tails,” and therefore using the Normal distribution as an approximation would risk systematically underestimating reserves and capital.

Figure 13 Simulated Distribution of Total Claims (PV at Interest = 0%)

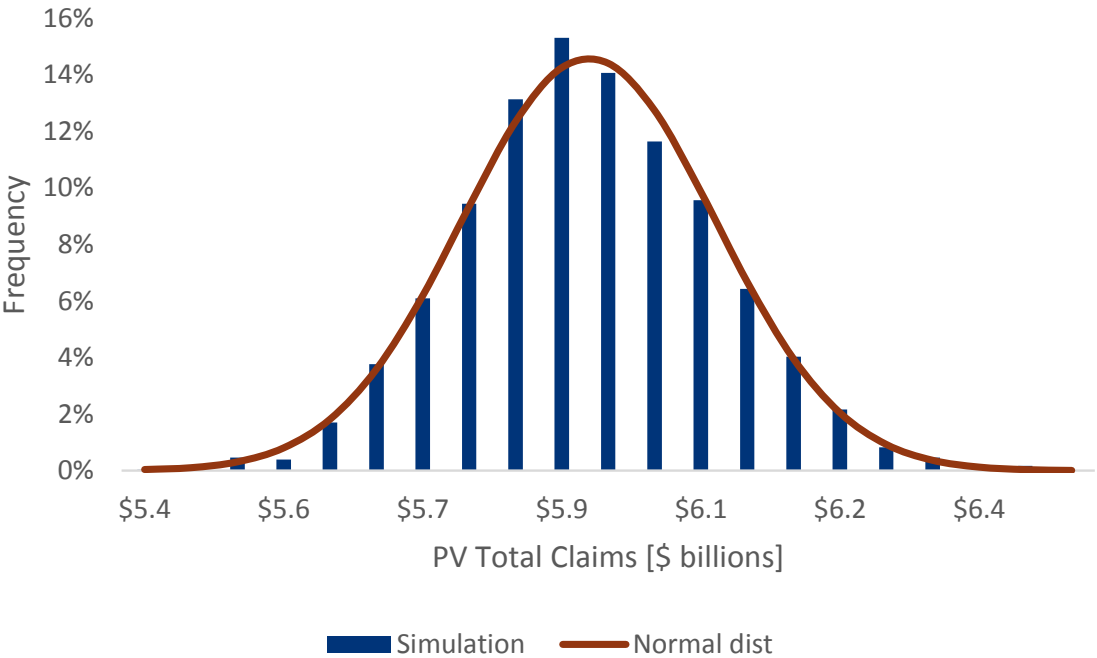


Table 8 Reserve and Capital Margins

Measure	Total portfolio	50% of portfolio	25% of portfolio	10% of portfolio
Mean PV of claims (BEL) [\$ millions]	\$5,992	\$2,812	\$1,481	\$556
Standard deviation	\$143	\$93	\$67	\$43
CTE70	\$6,162	\$2,921	\$1,559	\$607
Reserve margin	2.8%	3.86%	5.3%	9.1%
Mean annual claims [\$ millions]	\$453.7	\$208.7	\$111.6	\$43.9
Standard deviation	\$22.1	\$14.4	\$10.6	\$6.6
99.5th percentile	\$516.4	\$248.0	\$144.5	\$64.4
CTE99	\$518.8	\$249.5	\$145.4	\$66.5
Economic capital	\$65.0	\$40.7	\$33.7	\$22.6
Capital margin as percentage of BEL	1.1%	1.5%	2.3%	4.1%

We evaluated the term life portfolio as a whole and ran the entire analysis on three subsets of the portfolio: randomly chosen 10%, 25% and 50% samples. The mortality and lapse models were fitted to each subset of the data, which means that both estimation error and idiosyncratic risk are taken into account for the three additional scenarios.

The main finding summarized in Table 8 is that the size of the portfolio has a substantial impact on the level of uncertainty, which in turn increases both reserve and capital margins.

Impact of Reinsurance Retention on Reserve Margins



Proportional reinsurance

There are two basic types of proportional reinsurance: first dollar quota share reinsurance, where the same proportion is reinsured for each policy, and surplus reinsurance, where the ceding company retains the risk up to a fixed retention per life, and the surplus above the retention level is reinsured. Surplus reinsurance is also known as excess reinsurance. The reinsurer receives premiums for the proportion of each risk exceeding the retention and covers that same proportion in the event of a claim.

First dollar quota share does not affect the reserve margin, because the distribution of retained claims is simply scaled to the quota share proportion retained. For excess reinsurance, we have the prior expectation that surplus reinsurance has an impact on reserve margins and capital, because limiting the maximum claim amount to the retention reduces the variability of claims in each period.

We ran the simulation for a series of different levels of retention. Figure 14 summarizes the results for reserves, which we have defined to be CTE70 of the PV of claims. It shows that reinsurance has less impact on reserve margins than we expected. The required margin for a CTE70 reserve level hardly varies from the scenario without reinsurance, unless the company's retention is below \$1,000,000, and even then, the relative impact is small; for example, to achieve a 15% reduction in reserve margin, this particular life company would have to reinsure all risks above \$100,000, which amounts to around 69% of its business.

Figure 14 Relative Distance of CTE70 Reserve to Best-Estimate Liabilities (BEL) for Various Retention Levels

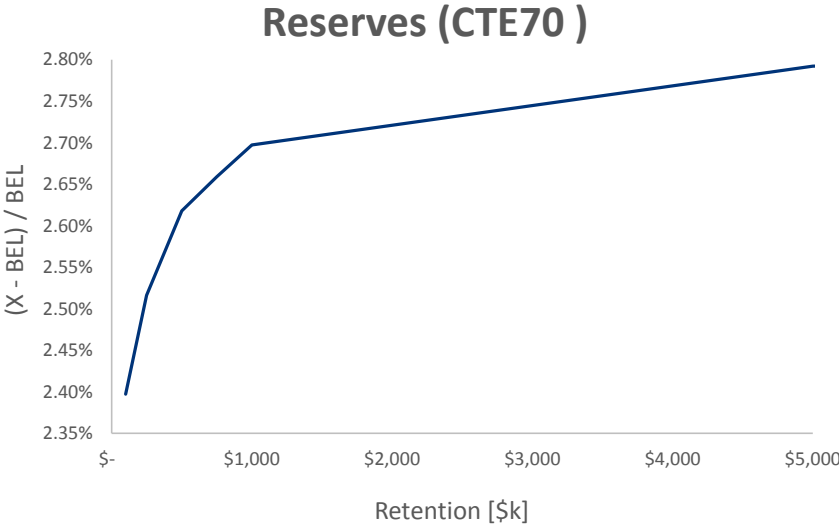


Figure 15 Ratio of Reserves and Capital to Face Amount

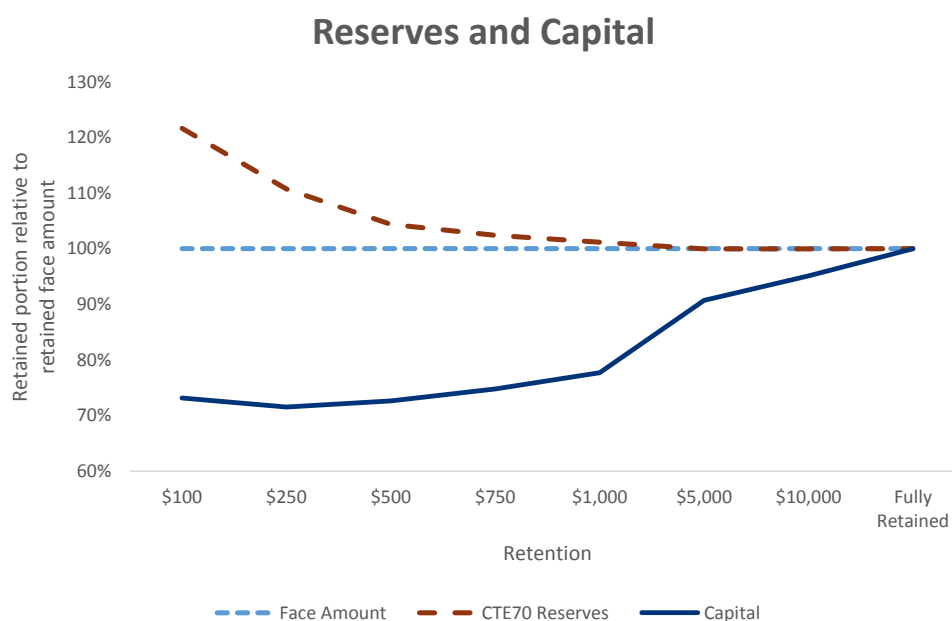


Figure 15 shows that the retained portion of reserves based on our simulated distribution of future claims is greater than the retained portion of policy face amount. This is somewhat surprising, because we know that the reserve margin for the retained reserves is slightly lower than the gross reserve margin before reinsurance. The explanation for this phenomenon is that for excess reinsurance, the reinsurer is covering only a portion of larger policies, which have systematically lower mortality and higher lapse rates, as we saw in the previous sections. The mortality rates for lower policy sizes are higher than average and the lapse rates lower than average, which means that the reserves per unit of face amount for lower policy sizes are higher than average.

In contrast, reinsurance has a much stronger impact on capital, which Figure 15 demonstrates clearly. The relative reduction in capital is much greater than the adverse impact of retaining policies with lower face amounts, which exhibit higher average mortality and lower average lapse rates. By purchasing excess reinsurance a life company can reduce its economic capital by a greater proportion than the ceded portion of face amount. This means that for the retained business the capital is relatively lower, and thus the cost of capital is reduced. The results of our analysis are summarized in Table 9 and Table 10.

Table 9 Reserve Margins for Excess Reinsurance at Different Retention Levels

Retention [US\$ thousands]	Total face amount [US\$ billions]	Retained portion of amounts	Best-estimate liabilities (BEL) [US\$ millions]	CTE70 reserves [US\$ millions]	Reserve margin [% of BEL]	Retained portion of reserve
Fully retained	\$359	100.0%	\$5,992	\$6,162	2.83%	100.0%
\$10,000	\$358	99.9%	\$5,985	\$6,153	2.81%	99.9%
\$5,000	\$357	99.4%	\$5,958	\$6,124	2.79%	99.4%
\$1,000	\$324	90.3%	\$5,483	\$5,631	2.70%	91.4%
\$750	\$299	83.2%	\$5,117	\$5,254	2.66%	85.3%
\$500	\$265	73.9%	\$4,630	\$4,751	2.62%	77.1%
\$250	\$184	51.4%	\$3,423	\$3,509	2.52%	57.0%
\$100	\$91	25.5%	\$1,866	\$1,910	2.40%	31.0%

Table 10 Capital Margins for Excess Reinsurance at Different Retention Levels

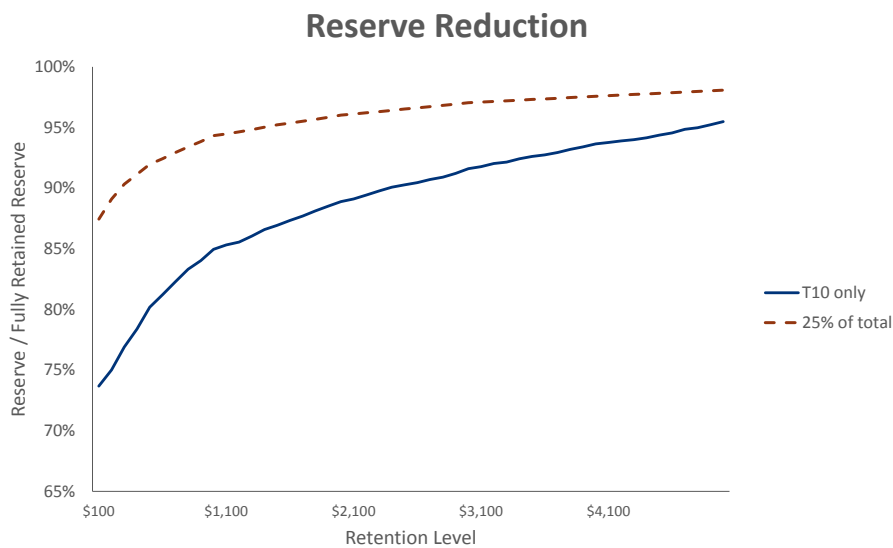
Retention [US\$ thousands]	Best-estimate reserves [US\$ millions]	Best-estimate annual claims [US\$ millions]	Economic capital [US\$ millions]	Capital margin [% of BEL]	Retained portion of capital
Fully retained	\$5,992	\$453.7	\$65.0	1.09%	100.0%
\$10,000	\$5,985	\$452.8	\$61.8	1.03%	95.0%
\$5,000	\$5,958	\$449.6	\$58.7	0.98%	90.2%
\$1,000	\$5,483	\$410.6	\$45.7	0.83%	70.2%
\$750	\$5,117	\$382.6	\$40.5	0.79%	62.3%
\$500	\$4,630	\$343.4	\$34.9	0.75%	53.7%
\$250	\$3,423	\$251.7	\$23.9	0.70%	36.8%
\$100	\$1,866	\$134.5	\$12.1	0.65%	18.6%

Why Does Reinsurance Affect Reserves Much Less than Capital?

We know that reinsurance reduces volatility in the annual claims amounts. However, when calculating reserves, we benefit from the fact that the business runs off over an extended period of time. The volatile annual claims amounts are smoothed out over time when we calculate the present value of liabilities.

We illustrate this with a simple example: The shortest duration business, the 10-year term product, is analyzed on its own. T10 makes for roughly a quarter of the entire portfolio. We therefore compare the impact of reinsurance on the T10 business with a random sample of 25% of the total business. We observe that the T10 policies remaining in-force at the end of the observation period (year-end 2011) have an average remaining duration of 3.9 years in comparison to an average remaining time till maturity¹⁶ of 11.5 years for the total portfolio. Therefore, the impact of reinsurance is substantially greater for the T10 business than for a block of the same size with a different business mix.

Figure 16 Simulation Results: CTE70 Reserves against Retention Level for T10 and 25% of Entire Portfolio



¹⁶ Time until the end of the level-term period.

Optimizing return on economic capital

We can now apply the finding that reinsurance has a substantial impact on Economic Capital (EC), which we have defined as the difference between the CTE99 of annual claims and the expected annual claims. We illustrate the optimization of EC with an example as follows:

- As target return on EC before reinsurance we assume, say, 12%. This means that without any reinsurance the life company expects the business to yield a first-year profit of 12% of \$65.0 million, that is, \$7.8 million.
- We further assume that expected profits are proportional to expected annual claims.
- Cost of reinsurance is modeled as a percentage of ceded profits. We aim to show that lowering the company’s retention increases the cost of reinsurance, which in part offsets the benefit from reducing the required amount of EC.
- We can thus find an optimum for given cost of reinsurance and cost of capital.

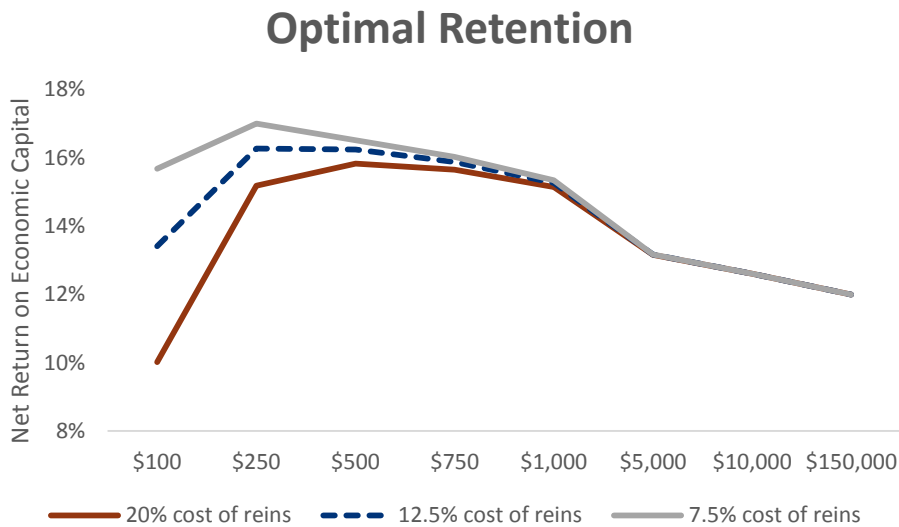
In Table 11 we have assumed the cost of reinsurance to be 20% of expected annual profits for the ceded portion. The return on EC can thus be lifted by purchasing reinsurance, but when the retention drops below \$500,000, the increased cost of reinsurance takes over and reduces the return on EC again. Thus, in this particular example for this specific portfolio the optimal retention for the life insurer would be \$500,000 per life.

Table 11 Return on Economic Capital

Retention	Best-estimate annual claims [US\$ millions]	EC [US\$ millions]	Retained profit	Profit less cost of reinsurance [20% of profit]	Return on EC
Fully retained			\$7.8	\$7.81	12.0%
\$10,000,000	\$453.7	\$65.0	\$7.8	\$7.79	12.6%
\$5,000,000	\$449.6	\$58.7	\$7.7	\$7.72	13.2%
\$1,000,000	\$410.6	\$45.7	\$7.1	\$6.92	15.1%
\$750,000	\$382.6	\$40.5	\$6.6	\$6.34	15.6%
\$500,000	\$343.4	\$34.9	\$5.9	\$5.53	15.8%
\$250,000	\$251.7	\$23.9	\$4.3	\$3.63	15.2%
\$100,000	\$134.5	\$12.1	\$2.3	\$1.22	10.0%

The results for three different assumed costs of reinsurance are shown in Figure 17. We see that by purchasing reinsurance specifically gauged to the portfolio it is possible to improve the return on EC from 12% to nearly 17%. Reinsuring too large a portion of the business then reduces the return on EC.

Figure 17 Optimizing Return on Economic Capital



Assumptions: Target gross return on Economic Capital = 12%, cost of reinsurance as a percentage of annual profits.

Findings

Reserve and capital margins directly reflect the uncertainty associated mortality and lapse risk. The results of our case study show the following:

- i. Prudent valuation assumptions based on the company's own experience differ systematically by size of the life company. Smaller companies require higher reserve margins.
- ii. Reinsurance retention has a relatively low impact on reserves, even for different size companies. The impact of reduced volatility in liabilities can be more than offset by the impact of lighter mortality and higher lapses for larger policies.
- iii. Business mix and duration of the business have a material influence on how reinsurance affects reserves and capital.
- iv. Capital margins are calculated as the CTE99 of losses within any one-year period. Since this removes any time-smoothing effects implicit within the calculation of benefit reserves, reinsurance has a greater impact on the required capital level of a life insurer. This finding can be utilized by a life company to optimize its retention level with respect to return on Economic Capital.

Further Considerations

NONPROPORTIONAL REINSURANCE

Within this context we note that nonproportional reinsurance such as Catastrophe Excess of Loss (Cat XL) reinsurance or Stop Loss reinsurance is not covered by this report. A Cat XL covers events in which multiple insured lives die from a single event. Due to the fact that our statistical model describing the portfolio experience is based on independent events, we cannot quantify the impact of a Cat XL using this framework.

This technical restriction does not strictly apply to Stop Loss reinsurance. However, a Stop Loss, as is common in Property and Casualty reinsurance, covers only the losses arising during a single year or at most a relatively short time period no greater than three to five years. In contrast, the total claims of a block of life insurance policies unfold over the entire term of the business until the last policy has matured or the last insured has lapsed or died. It is

conceivable that reinsurers may one day offer model-based coverage of the performance of a block of life policies that is out-of-the-money like a typical Stop Loss. In fact, some forms of financial reinsurance effectively offer that kind of protection already. However, a discussion of such specialized solutions is beyond the scope of this report. We simply note that the most common method of derisking life insurance business is proportional reinsurance, that is, first dollar quota share and excess of retention reinsurance (aka surplus reinsurance).

EQUIVALENT NET SINGLE PREMIUM CALCULATION

Rather than expressing the reserve margin as a percentage of best-estimate liabilities, it is also possible to express both best estimate liabilities and the prudent reserves corresponding to a specific percentile or CTE value as percentages of a standard table. This could either be the industry valuation tables or a company-specific base table. This method is outlined in Richards (2012) and is based on solving for the percentage of a standard table, which gives the same reserve amount as the portfolio-specific assumptions from our statistical analysis.

ANALYZE LIVES NOT POLICIES

Due to the nature of the experience data provided for our case study, we were not able to ensure that deaths of individuals with several policies were treated as one event. It is important to keep in mind that the process of policy deduplication must be carried out rigorously when applying the proposed framework.

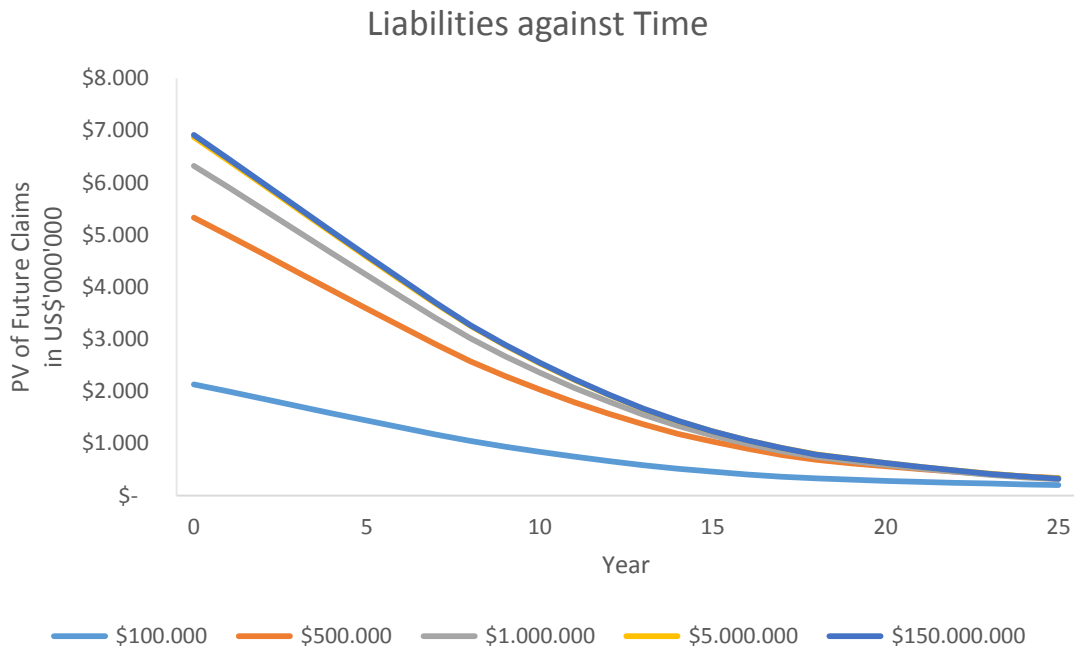
CONSIDER MORE COMPLEX MORTALITY AND LAPSE RATE FUNCTIONS

We also considered using a quadratic log function as the basis for our analysis. However, the improvement in fit was deemed to be spurious in light of the fact that we do not have truly lives-based experience. The choice of mortality and lapse law will have an impact on the absolute level of total liabilities. However, the focus in this study is to show the relative impact of different reinsurance retention levels on the reserve margins.

IMPORTANCE OF REINSURANCE FOR RUN-OFF

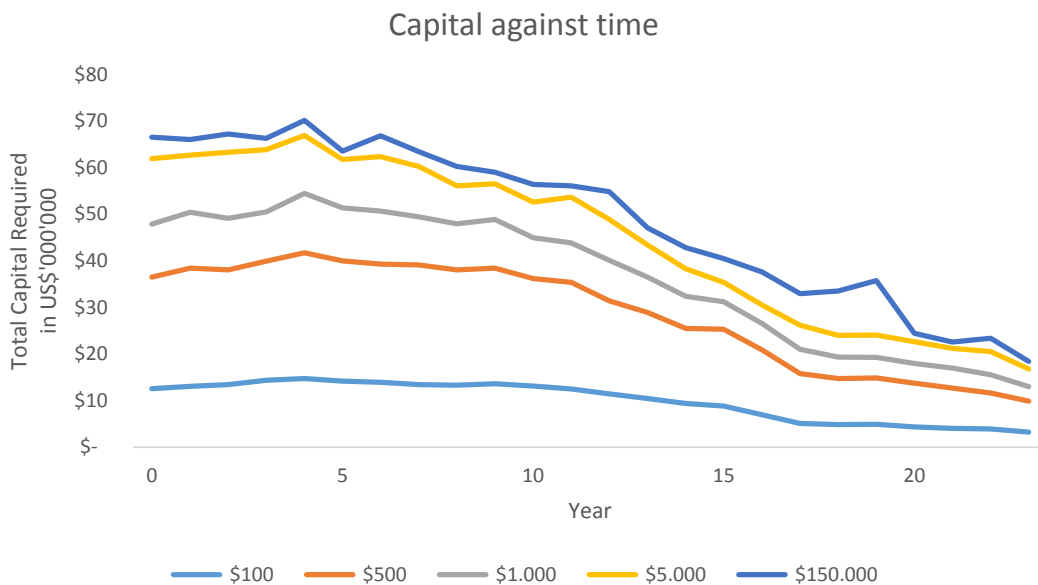
In the above calculations, we considered only the capital margins for the first year. With our simulation, we are also able to show the entire run-off of the business over time.

Figure 18 Run-off of Term Life Business for Five Different Retention Levels¹⁷



We see that reinsurance reduces the overall annual losses. Whether a company has a retention of US\$5 million or no reinsurance at all makes little difference for reserves, as shown in Figure 18. However, required capital runs off in a different manner, leading to a dramatic increase in capital requirements in relation to liabilities. This is shown in Figure 19, where required capital does not reduce in line with the total liabilities as the business volumes drop, because the volatility of annual results increases substantially, as the business runs off and fewer lives remain. Therefore, a reinsurance strategy must not only consider the current level of capital, but should incorporate a projection of capital requirements over the term of the business.

Figure 19 Run-off of Term Life Business for Five Different Retention Levels



¹⁷ A retention of US\$150 million means that the business is fully retained without reinsurance.

MORTALITY TREND RISK

Our model includes mortality and lapse trends as far as they occur within the period of the mortality investigation 2000–2011. Future trends beyond year-end 2011 and the risks associated with these are outside the scope of this report. The main reason is that incorporating future mortality improvements into life insurance reserves is likely to meet with considerable opposition from regulators. However, to the extent that mortality improvements are incorporated into a life insurer's pricing basis, these assumptions should also be considered within the context of setting economic capital assumptions.

In contrast, a life company's annuity business is subject to a large degree of longevity trend risk. Therefore, the company's reserves and capital for annuity business should allow for longevity trends. An extension of the framework discussed in this report to life annuities will therefore incorporate models for longevity trend risk, such as outlined in Richards, Currie and Ritchie (2014), for instance.

INTEGRATION WITH VALUATION MANUAL 20

VM-20 is fairly prescriptive with regard to setting company-specific assumptions. We have not considered in detail how the proposed framework could be incorporated within the methods outlined in VM-20. This will be the subject of future research.

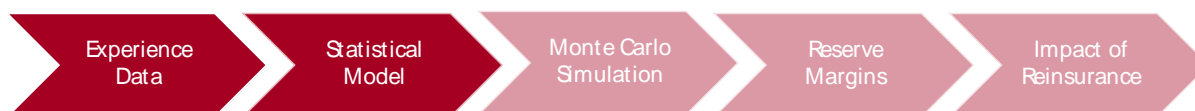
V. Acknowledgments

This project was carried out using the Longevitas™ experience analysis software. The authors are grateful to the Society of Actuaries and the Project Oversight Group for their support and to Prof. Johnny Li and Dr. Stephen Richards for their valuable feedback on the draft report. Any errors or omissions remain the sole responsibility of the authors.

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VII. Appendix A: Model Choice



We decide which risk factors to include in our model by applying a number of different criteria:

1. The model should have the best goodness of fit for a given number of parameters. We compare models with different numbers of parameters using information criteria, such as the Akaike Information Criterion (AIC) or the Bayesian Information Criterion (BIC), which balance the goodness of fit with the number of parameters. The general rule is that the fewer parameters a model needs to describe the data,¹⁸ the better.
2. Each parameter should be broadly statistically significant, which means that the p -value for the parameter estimate should be 5% or less.
3. The parameters should have an intuitive interpretation. For example, we expect smokers to have a higher level of mortality ($\alpha_{smoker} > 0$) relative to the *nonsmoker* baseline.
4. There should be no bias apparent in the residuals.
5. The result should be suitable for financial applications based on a bootstrap test.

Assuming that the count of deaths in each age group follows a Poisson distribution, the deviance residual r is calculated from the actual number of deaths A and the expected deaths E :

$$r = \text{sign}(A - E) \sqrt{2[A \log\left(\frac{A}{E}\right) - (A - E)]}$$

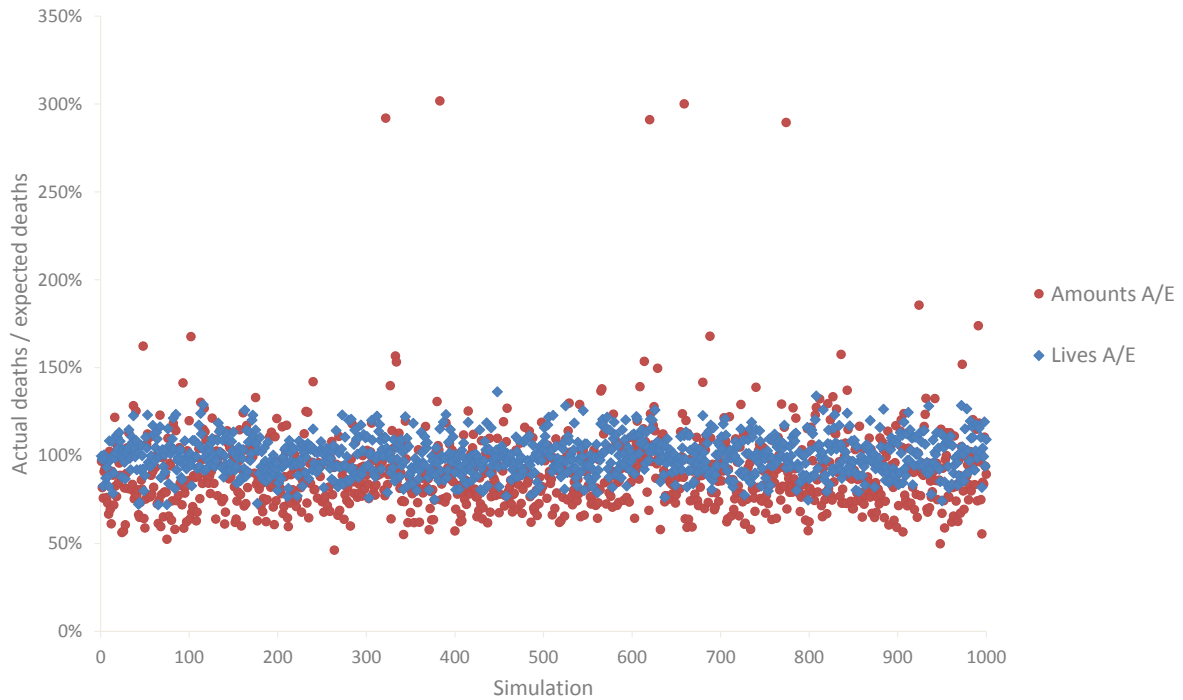
We can see that the lives with the smallest policies have significantly higher mortality than all others, and that there is a systematic trend of decreasing mortality by increasing policy size up to policies of \$250,000 face amount. For larger policies, the size does not appear to make a difference. This example demonstrates that a simple model without size as a risk factor would not be sufficient and that we should include a risk factor that describes the differences in mortality by policy size.

Financial Applicability: Bootstrap Experiment

Criterion 5 reflects the fact that the death or surrender of different policies will not necessarily have the same financial impact, because they differ by risk amount. We can test the financial suitability of our model results by carrying out a simple stochastic test. We draw a random sample of 10,000 lives from our experience data and compare the observed deaths to the deaths predicted by our model. This comparison is done repeatedly for, say, 1,000 iterations.

¹⁸ A model with as few parameters as possible is referred to as being *parsimonious*. As a rule of thumb, a parameter that lowers the AIC score by at least four units can generally be considered a useful extension of the model.

Figure 20 Bootstrapping Simulation Comparing Lives-Based and Amounts-Based Mortality
Bootstrap Experiment



On a lives basis we should always have an Actual-to-Expected ratio centered upon 100%, because we have fitted the model with Maximum Likelihood Estimates. When we weight the observed deaths by amount, a model that does not take face amount into account will tend to overestimate mortality rates. In this case study, amounts-based mortality is on average 10% lower than lives-based mortality. We also see that the amounts-based A/E ratios have a much greater variability than the lives-based A/E ratios. This is a reminder that we have to consider the volatility caused by large face amount policies causing the outliers in Figure 20. It is not unusual to have mortality more than 50% greater than expected, but even three times the expected claims amount can also happen.

VIII. Appendix B: Misestimation Risk



Our aim is to measure the estimation error for the liabilities of a life book. The rationale stated here follows the method described in Richards (2016).

We fit a multidecrement survival model choosing an appropriate parametric form for the force of mortality and lapse hazard rates:

$$h_{x,y,\tau} = \mu_{x,y,\tau} + \lambda_{x,y,\tau}.$$

We have chosen the Gompertz mortality law for both hazards,¹⁹ in which selection is modeled by defining the intercept parameter stepwise:

$$\begin{aligned}\mu_{x,y,\tau} &= e^{\alpha_{\tau}^{death} + \beta^{death}x + \delta^{death}(y-2000)}, \\ \lambda_{x,y,\tau} &= e^{\alpha_{\tau}^{lapse} + \beta^{lapse}x + \delta^{lapse}(y-2000)}.\end{aligned}$$

All in all, we have a hazard function $h_{x,y,\tau} = h(\alpha_{\tau}, \beta, \delta) = h(\underline{\theta})$, which is a function of the parameters that we estimate searching for that set of parameters that maximizes the likelihood function. Call the parameter vector $\underline{\theta}$. From maximum likelihood theory, we know that the error term of each of the maximum likelihood estimates (MLEs) follows a Multivariate Normal (MVN) distribution. This means that we can perturb the parameter set $\underline{\theta}$ in a way consistent with the data against which they were fitted.

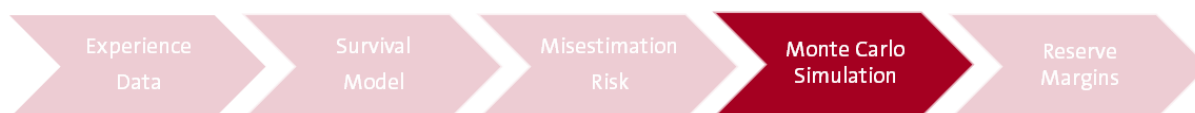
We generate an alternative set of parameters $\underline{\theta}'$ by adding a perturbation term to the vector of parameter estimates $\hat{\underline{\theta}}$. The perturbation term is constructed from the so-called Cholesky decomposition of the variance-covariance matrix, which we obtain as part of the fitting process:

$$\underline{\theta}' = (\alpha', \beta', \delta') = \hat{\underline{\theta}} + \mathbf{C}\underline{z}.$$

Matrix \mathbf{C} represents the Cholesky decomposition of the variance-covariance matrix $\mathbf{V} = \mathbf{C}\mathbf{C}^T$. \underline{z} is a vector of $N(0,1)$ -distributed random numbers. We use the inverse of the Fischer Information matrix $\mathfrak{I} = -\mathcal{H}(\hat{\underline{\theta}})$, that is, the negative Hessian of second partial derivatives of the likelihood function, as an approximation for the variance-covariance matrix. The Cholesky decomposition can be viewed as the multidimensional analogue of a square root of the variance, that is, the standard deviation.

¹⁹ For a more accurate valuation of the business, more complex mortality laws would likely be more appropriate. We have chosen a simple law for the sake of illustrating the method.

IX. Appendix C: Simulation of Future Lifetimes



A Monte Carlo simulation is a stochastic experiment in which we randomly generate events from the realm of all possible events and then calculate the financial result. If we manage to generate the random events in a way consistent with their probability distribution, the Monte Carlo simulation will then give us a probability distribution for the financial results.

In our case, the events in which we are interested are all the possible ways the life insurance book could run off—that is, how the insured lives will either lapse or die. Broken down to the level of the individual, the uncertain event we have to model is how much time each individual remains in the portfolio and whether he or she will exit the portfolio due to death or due to lapse.²⁰ The good news is, we know exactly what the distribution of future lifetimes looks like, because we have already fitted a survival model.

We can simulate the future lifetime of each individual in the portfolio. For each individual, we know the survival function, that is, the probability to remain within the portfolio without lapsing or dying until at least time t :

$${}_t p_x = \exp\left(-\int_0^t h_{x+s,y+s,t+s} ds\right) = {}_t p_x^{\text{death}} \times {}_t p_x^{\text{lapse}}.$$

The cumulative distribution function $F_T(t)$ of future exit dates is therefore

$$F_T(t) = 1 - {}_t p_x = 1 - {}_t p_x^{\text{death}} {}_t p_x^{\text{lapse}}.$$

Since the distribution function is strictly monotonically increasing, it is possible to define its inverse, $F^{-1}: (0,1) \rightarrow \mathbb{R}^+$, which assigns a future lifetime to any given probability between 0 and 1. By randomly drawing uniformly distributed numbers from $(0,1)$ we can generate a set of random numbers that are consistent with the distribution function for future lifetimes, or, rather, times till exit from the portfolio.

Special attention must be given to the fact that the force of mortality and lapse hazard rate functions are defined stepwise for the different selection periods. Therefore, the inverse has to be calculated stepwise too.

²⁰ For simplicity, we combine all forms of surrender or lapse that do not generate a claim into “lapse.”