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More on Premiums and Interest Rate Change

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In the October 1984 and January 1985 issues of The Actuary, there was an interesting discussion of whether the statement

$$\frac{dP_x}{di} < 0$$

is a theorem. The result of the discussion was to establish that the suggested theorem is false.

These notes were developed in connection with that discussion. They were not submitted to The Actuary for they involve a bit more mathematics than is typical of that publication. The example which follows contradicts the basic idea of the proposed theorem. It involves the Pareto distribution, which is a fruitful source of counter-examples in life contingencies.

The key elements of the example are as follows:

1. Mortality model
   a. Force of mortality at age x+t is $(1 + x + t)^{-1}$, $0 < t$.
   b. Conditional survival function, given survival to age x is $(1 + x)/(1 + x + t)$.
2. Actuarial present values
   a. $\bar{a}_x = \int_0^\infty e^{-\delta t}[(1 + x)/(1 + x + t)]dt$
   b. $\bar{A}_x = 1 - \delta\bar{a}_x$
   c. In Exercise 2, Chapter 4, Actuarial Mathematics, it is shown that

$$\frac{d}{dx}\bar{A}_x < 0,$$

which means that

$$\frac{d}{dx}\bar{a}_x = -\frac{1}{\delta}\left(\frac{d\bar{A}_x}{dx}\right) > 0.$$

d. Since ${}_t\bar{V}(\bar{A}_x) = 1 - \bar{a}_{x+t} / \bar{a}_x$, from 2.c. we have ${}_t\bar{V}(\bar{A}_x) < 0,\ 0 < t$.

3. Whole life premiums

a. Whole life premiums are weighted averages,

$$\bar{P}(\bar{A}_x) = \int_0^\infty w(t,\delta)\, \mu_{x+t}\, dt$$

where

$$w(t,\delta) = e^{-\delta t}\,{}_t p_x \,/\, \bar{a}_x.$$

b. Properties of the weight function

(i) $\frac{\partial}{\partial t} w(t,\delta) < 0$

(ii) We have $\frac{\partial w(t,\delta)}{\partial \delta} = e^{-\delta t}\,{}_t p_x \left[-t\bar{a}_x + (\bar{I}\bar{a})_x\right] / \bar{a}_x^{\,2}$,

which is positive for small values of t and then becomes and remains negative.

4. Conclusion

a. By 3.b(ii) an increase the force of interest increases the weight attached to the force of mortality for small values of t and decreases the weight attached to the force of mortality for large values of t in the annual whole life premium rate.

b. In our example, the force of mortality is a monotone decreasing function. Therefore, because of the interpretation of the whole life premium rate as a weighted average, an increase in the force of interest will produce an increased premium rate.