

**Blue Cross.**

**INTER OFFICE MEMORANDUM**

**TO:** Bob Duncan

**SUBJECT:** Actuarial Data Support  
NCS Claims Study -  
Statistical Comment

**FROM:** Doug Smith ~~DS~~  
Assistant Actuary  
Woodland Hills, B-14 Ext. 3708

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After reviewing the NCS claims study prepared by Actuarial Data Support, my first thought was that perhaps not enough claims had been reviewed to conclude that claims are being paid correctly. As I sought to find some argument to support this hypothesis I ended up showing that a small sample size of claims is actually all that is needed in this case; the converse of what I hoped to prove.

The attached two pages are food for thought on the matter. The material is definitely intended for the "purist" in that it is pretty theoretical, yet the result is quite interesting.

I welcome any comments you may have.

DS/k1

Attachments

PURPOSE: To establish the validity of a small sample size

MODEL: total claims = y

|                                |                              |
|--------------------------------|------------------------------|
| incorrectly<br>paid claims = z | correctly<br>paid claims = x |
|--------------------------------|------------------------------|

$$y = z + x$$

$$x = b \cdot y$$

where b is the percentage of the total claims that were paid correctly  
q is the sample of claims selected that were correctly paid

LET: P = probability of choosing q correctly paid claims from y total claims

GIVEN:  $P = b^q$  (see attachment for proof)

TABLE OF P VALUES:

| $\begin{matrix} q \\ b \end{matrix}$ | 100    | 200       | 250     |                           |
|--------------------------------------|--------|-----------|---------|---------------------------|
| 0.99                                 | .366   | .134      | .081    |                           |
| 0.95                                 | .006   | .00004    | .000003 |                           |
| 0.90                                 | .00003 | .00000001 | c       |                           |
| 0.50                                 | c      | c         | c       |                           |
| 0.25                                 | c      | c         | c       | with $c < .0000000000036$ |

- CONCLUSION:
1. If one assumes that a large percentage of claims are paid correctly, say  $b = .95$ , the P value is less than 1% for sample size  $q > 100$  (see table). A value of P less than 1% means that the probability of choosing q correctly paid claims from y total claims is very small, i.e., it would be unlikely that one would randomly draw  $q > 100$  correctly paid claims. Since the survey conducted by Actuarial Data Support sampled over 100 MCS claims and all were correctly paid, it follows that we can be  $100\% - P = 100\% - 1\% = 99\%$  certain that the claims are being paid correctly.
  2. On the other hand, if one assumes that a smaller percentage of claims were paid correctly, say  $b = .5$  or  $.25$ , then the P value is effectively zero (see table). That is, it would be extremely unlikely to randomly draw q correctly paid claims from a set of claims that had a high number of incorrectly paid claims.

Note how the arguments of 1. and 2. work in opposite directions and thus force the sample size to decrease.

Hence, in this instance, a relatively small sample size is sufficient to establish a surprisingly high degree of certainty in addressing the question of whether the claims are being paid correctly.

PROOF OF THE APPROXIMATION  $P = b^q$

Let  $P =$  the probability of choosing  $q$  correctly paid claims from  $y$  total claims

$$P = \frac{\text{(number of ways to choose } q \text{ correctly paid claims from } x \text{ correctly paid total claims)}}{\text{(number of ways to choose } q \text{ correctly paid claims from } y \text{ total claims)}}$$

$$P = \frac{\binom{x}{q} \frac{x!}{q!(x-q)!}}{\binom{y}{q} \frac{y!}{q!(y-q)!}} = \frac{x! \cdot (y-q)!}{(x-q)! \cdot y!} = \frac{(x-q+1)(x-q+2) \dots (x)}{(y-q+1)(y-q+2) \dots (y)}$$

where  $y = x + z$

- $z =$  number of incorrectly paid claims
- $x =$  number of correctly paid claims
- $y =$  total number of claims

Now,  $x = by$  where  $b$  is the percentage of the total claims that are paid correctly

$$\text{Hence, } P = \frac{(by-q+1)(by-q+2) \dots (by)}{(y-q+1)(y-q+2) \dots (y)}$$

$$\text{So, if } y \gg q \text{ then } P \text{ is approximately equal to } \frac{(by)(by) \dots (by)}{(y)(y) \dots (y)}$$

which gives the final result of  $P = b^q$

i.e.,  $P$  depends only on the percentage of correctly paid claims and the number of claims in the sample.