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Mr. Charles S. Fuhrer
Co-Editor, ARCH
Benefit Trust Life Ins. Co.
1771 West Howard Street
Chicago, IL 60626

Dear Sir:

I have enclosed solutions to Examples 14.6 and 15.3 of the New Life Contingencies text that I feel will be of interest to your readers.

I have sent copies of this material to the co-authors of the new text. I have also suggested to the five authors that their handling of the new Canadian reserve standard is not the best. I do not feel that they are well-advised to use the τ^{com} , β^{com} approach. I have enclosed the Act that defines the new system.

Yours sincerely,

A handwritten signature in cursive script, appearing to read "Robert L. Brown".

Robert L. Brown

RLB:mk
Encl.

c.c. Newton L. Bowers, Jr.
Hans U. Gerber
James C. Hickman
Donald A. Jones
Cecil J. Nesbitt

Ex. 14.6

Solution: Question, is $\beta^{\text{com}} > {}_{19}P_{x+1}$?

β^{com} is payable from age 36 on.

$$\therefore \frac{\beta^{\text{com}}(N_{36} - \frac{1}{2}N_{45} - \frac{1}{2}N_{65})}{D_{36}} = 50,000 \frac{(3M_{36} - M_{55} - 2M_{65} + 2D_{65})}{D_{36}}$$

$$\frac{50,000(5505.6909)}{120,984.78}$$

$$= 2275.37$$

Is $\beta^{\text{com}} > {}_{19}P_{36}$?

Find the level insurance in force

$$+ 50,000 \left(\frac{3M_{36} - M_{55} - 2M_{65}}{M_{36} - M_{65}} \right)$$

$$= 130,153.30$$

For a \$1, ${}_{19}P_{36} = .0116543$

$$\therefore \text{for this policy: } F \cdot {}_{19}P_{36} = 130,153.30(.0116543)$$

$$= \$1516.85$$

$$\therefore \beta^{\text{com}} > {}_{19}P_{36}$$

$$\therefore \beta^{\text{com}} \neq \beta^{\text{FPT}}$$

α^{com} must be chosen so that the first year expense is the same as on a 20-pay life using Full-Preliminary Term.

$$\therefore {}_{19}P_{x+1} - A'_{x:\overline{1}|} + 1516.85^{*+} - 150,000(A'_{x:\overline{1}|})$$

$$= 1516.85 - 284.94$$

$$= 1231.91$$

$$\text{Now } \beta^{\text{com}} = p + \frac{{}_{19}P_{x+1} - A'_{x:\overline{1}|}}{\ddot{a}_{x:\overline{n}|}}$$

$$\text{where } P = \frac{50,000(3M_{35} - M_{55} - 2M_{65} + 2D_{65})}{N_{35} - \frac{1}{2}N_{45} - \frac{1}{2}N_{65}} = \frac{50,000(5575.5396)}{133,241.55}$$

$$= 2092.27$$

$$\therefore \beta^{\text{com}} = 2092.27 + \frac{1231.91D_{35}}{133,241.55} = 2205.59 \text{ to Dur 10}$$

$$\&1102.79 \text{ Dur 11,30}$$

The issues that arise with nonforfeiture benefits for policies with non-level gross premiums and/or non-level benefit amounts are similar to those discussed under modified reserves.

The 1980 law refers to an average amount of insurance (AAI) and indicates that this amount is based on benefit amounts at the beginning of the first ten policy years. That is,

$$AAI = \frac{\sum_{j=0}^9 b_{j+1}}{10}$$

We then have:

$$E_1 = 1.25P + .01(AAI) \quad \text{if } P < .04(AAI)$$

$$= .06(AAI) \quad \text{if } P \geq .04(AAI)$$

where P is the level premium equivalent for the policy. (Note: equal if $P = .04AAI$.)

Ex. 15.3: Find the minimum cash values at durations 1, 2, 10, and 20 for the special 30-year endowment policy of Ex. 14.6. Benefit is 150,000 for the first 20 years and 100,000 thereafter with a maturity benefit of 100,000. The gross premium is a level $\frac{1}{2}G$, 2,500 for 10 years and thereafter is 1250 (G then $\frac{1}{2}G$).

Solution: The AAI is based on the first 10 policy years and is, therefore, \$150,000.

The net level premium for this policy at issue age 35 is:

$$p = \frac{50,000(3M_{35} - M_{55} - 2M_{65} + 2D_{65})}{N_{35} - N_{65}}$$

$$= 1622.9358$$

since $P = 1622.94 \leq .04(AAI) = 6000$ then:

$$\begin{aligned} E_1 &= 1.25P + .01(AAI) \\ &= 3528.6698 \end{aligned}$$

For nonforfeiture value purposes, the initial net unlevel premium is:

$$P = \frac{50,000(3M_{35} - M_{55} - 2M_{65} + 2D_{65})}{N_{35} - \frac{1}{2}N_{45} - \frac{1}{2}N_{65}}$$

$$\begin{aligned} \text{and } P^A &= \frac{50,000(3M_{35} - M_{55} - 2M_{65} + 2D_{65}) + E_1 \cdot D_{35}}{N_{35} - \frac{1}{2}N_{45} - \frac{1}{2}N_{65}} \\ &= \frac{50,000(5575.5396) + (3528.6698)(12256.76)}{133.241.55} \\ &= 2416.8665 \quad (\beta^{\text{com}} = 2275.37) \end{aligned}$$

so P^A starts at 2416.8665 and goes to 1208.4333 after 10 years.

$$\begin{aligned} {}_1CV^{\text{PROSP}} &= 50,000 \frac{(3M_{36} - M_{55} - 2M_{65} + 2D_{65})}{D_{36}} - 2416.8665 \left(\frac{N_{36} - \frac{1}{2}N_{45} - \frac{1}{2}N_{65}}{D_{36}} \right) \\ &= 23,855.433 - 25,338.966 \\ &= -1483.5331 \end{aligned}$$

$$\begin{aligned} {}_2CV^{\text{PROSP}} &= 50,000 \frac{(3M_{37} - M_{55} - 2M_{65} + 2D_{65})}{D_{37}} - 2416.8665 \left(\frac{N_{37} - \frac{1}{2}N_{45} - \frac{1}{2}N_{65}}{D_{37}} \right) \\ &= 25,019.272 - 24,349.539 \\ &= 669.7334 \end{aligned}$$

$$\begin{aligned} {}_{10}CV^{\text{PROSP}} &= 50,000 \left(\frac{3M_{45} - M_{55} - 2M_{65} + 2D_{65}}{D_{45}} \right) - 2416.8665 \left(\frac{\frac{1}{2}N_{45} - \frac{1}{2}N_{65}}{D_{45}} \right) \\ &= 36,507.541 - 13,987.734 \\ &= 22,519.807 \end{aligned}$$

etc.

APPENDIX A

Excerpt from the Canadian and British Insurance Companies Act
as amended by Bill S-3, having to do with life reserves.

"82. (1) Subject to section 82.2, the liabilities shown in the annual statement of the company shall include a reserve for all unmatured obligations under the terms of its policies dependent on life, accident or sickness or on any other contingencies or on a term certain.

(2) The reserve referred to in subsection (1) shall be calculated on the basis of a rate or rates of interest and a rate or rates of mortality, accident, sickness or other contingencies

(a) that, in the opinion of the valuation actuary, are appropriate to the circumstances of the company and the policies in force; and

(b) that are acceptable to the Superintendent.

(3) The reserve referred to in subsection (1) for a life insurance policy shall be calculated in accordance with the methods specified in subsections (4) and (7) or in accordance with any other method that produces a reserve not less, for any policy at any duration, than the reserve calculated by the method specified in those subsections.

(4) In the case of a life insurance policy that provides for a uniform annual premium, the reserve on any date shall be calculated by deducting from the present value on that date of the unmatured obligations under the terms of the policy the present value on that date of the valuation premium assumed to be payable on each anniversary of the policy following that date during the term for which premiums are required to be paid under the terms of the policy.

(5) The amount of the valuation premium to be used for the purposes of subsection (4) shall be equal to the net level annual premium for the policy, increased by an amount that, if assumed to be payable at the beginning of the second and each subsequent policy year for which a premium is payable under the terms of the policy, has a present value at the date of issue of the policy equal to the lesser of

(a) one hundred and fifty per cent of the net level annual premium or such other amount as may be prescribed by regulation, and

(b) such amount as in the opinion of the valuation actuary represents the costs incurred by the company in connection with the issuance of the policy,

but the valuation premium shall not in any case be greater than the portion of the annual premium specified under the terms of the policy that remains after deducting from the premium an amount that, if available from each future annual premium, would, in the opinion of the valuation actuary, be sufficient to provide future administrative expenses and to enable the company to meet dividend expectations on the current scale of dividends modified in accordance with changes, if any, described in paragraph 82.1(2)(e).

(6) For the purposes of this section, the net level annual premiums and present values shall be calculated on the basis of the rate or rates of interest and rate or rates of mortality, accident, sickness or other contingencies used for the calculation of the reserve.

(7) In the case of a life insurance policy that does not provide for a uniform annual premium and in the case of an annuity contract, the reserve shall be calculated in accordance with the method described in subsection (4) with such modifications as the Superintendent considers appropriate in the circumstances.

(8) Where the valuation actuary uses a method of calculation that produces a reserve different from the reserve produced by the methods described in subsections (4) and (7), he shall also calculate

(a) a reserve in accordance with the methods described in those subsections; or

(b) a reserve in accordance with the methods described in those subsections, modified by

(i) assuming in the calculation that withdrawal rates are zero, and

(ii) for any policy, substituting the amount of the cash surrender value, if any, at a particular date for the calculated reserve at that date where the reserve is less than the cash surrender value,

and the reserve so calculated shall be shown in the annual statement in such manner as the Minister may direct.

(9) If the method of calculating a reserve is other than the net level premium method, the company shall also calculate the reserve in accordance with the net level premium method and such reserve shall be shown in the annual statement in such manner as the Minister may direct.

(10) The reserves required to be calculated in accordance with subsections (8) and (9) may be determined by any method of approximation satisfactory to the Superintendent.

