

VALUATION AND ACTUARIAL FUNDAMENTALS,
PRINCIPLES AND STANDARDS

by

James C. Hickman

1. Views of Valuation

Valuation is the act of placing a monetary value on a property or enterprise. Each day markets are engaged in the process of valuation.

To an audience educated in mathematics, the problem of valuing an economic enterprise can be thought of in terms of measure theory. That is, real numbers are assigned to members of a collection of sets of economic well being. The analogy cannot be carried too far for sometimes it is not clear how to interpret the basic elements in the sets of economic well being.

To those educated in decision theory, an analogy can be made with the construction of a utility function. A utility function assigns real numbers to members of a collection of very general sets of states of the world. The construction of a utility function is built on several axioms for coherent individual preferences. The valuation of an economic enterprise for public financial reporting is not an individual matter. Instead many individuals with distinct preferences have interests in the valuation and conflict is inevitable. This conflict should be anticipated as a practical consequence of Arrow's impossibility theorem.

To those with training in basic bookkeeping, the valuation of an economic enterprise is a natural consequence of classifying and recording past transactions. This traditional view is, like the more abstract analogues, insufficient. Past transactions may be irrelevant. An enterprise has value, because of its ability to generate goods and services that create human satisfactions in the future. Shakespeare wrote "What's past is prologue," and so it is in business affairs.

Because of inherent uncertainty about the future and the human preference for immediate over deferred consumption, it is clear that the calculation of actuarial present values, to summarize in one number the numerical values attached to possible future changes in economic well being, provides a general approach to valuation.

The net value of an enterprise is the difference between the numerical value attached to economic changes derived from rights (assets) currently owned and promises (liabilities) made to others. Changes in the net value of an enterprise during a time period are the earnings associated with the period.

2. Uses of Valuations

It is helpful to identify some of the interests that converge on the issues of valuing an insurance enterprise. Management is especially interested in earnings, the first difference of net value indexed by a measure of time. Earnings provide a scale for management success. Management also has an intense interest in the amount of vitality surplus. Vitality surplus is that part of net value, or surplus, of an insurance enterprise that can, with relative safety, be employed to expand operations or enter new lines of business.

Insurance regulators also have an interest, sometimes a competing interest with management, in valuation. The regulation of insurance enterprises has been directed toward maintaining solvency and promoting equity among classes of policyholders. This interest in solvency is not the typical goal of business regulation. The threat of insolvency is believed to be an incentive to efficiency in a capitalistic economic system. The question becomes, can capital markets operate efficiently if one class of businesses, insurance companies, are protected from insolvency by a web of laws and regulations? The political decision has been made in the United States and Canada to sacrifice some of the discipline of the market that enforces efficiency in return for protection of policyholder expectations.

This deviation from free markets is based on two ideas. (1) Insurance is to protect insureds from possible adverse financial consequences over which they have no control and which prudent management cannot alone eliminate. (2) The benefits of insurance are deferred. The price is paid before, perhaps long before, all of the financial protection is enjoyed. In this protracted period the insured usually has only limited ability to protect his interest in the insurance contract. If solvency is in the public interest, regulators have a stake in valuation.

The regulatory enforcement of equity among classes of policyholders is more controversial and traditionally a second priority behind solvency. Nonetheless, any political decision to measure equity among classes of policyholder's requires the valuation of blocks of insurance policies and their associated assets.

In a capitalist economy the allocation of savings into alternative investments is made, at least in part, in open markets. The efficiency of these capital markets depends on accurate economic information available to all market participants. The statistical agencies of government and the accounting profession, along with its various standards and enforcing agencies, are justified in large part by their roles in facilitating efficient capital and commodities markets. The mistakes on economic development made by many third world countries can be partially attributed to the absence of good economic information. If insurance enterprises are going to seek capital from the markets, market participants have an interest in insurance valuation.

The interests of these protagonists are not identical. Flowing from this diversity is some of the controversy surrounding insurance valuation.

3. Issues

The basic issues in insurance valuation are old. We will identify three such issues and illustrate them.

(a). In determining the actuarial present value of assets and liabilities, shall the models be simple or complex? The issue is which of the many possible future cash flows will be incorporated explicitly into the models used to calculate actuarial present values.

(b). Will the assumptions made to calculate the actuarial present values used in valuations remain invariant over time? On the asset side this issue is the familiar one of market versus cost or amortized values. On the liability side the issue is whether valuation assumptions should be locked in when a block of policies is sold or should they reflect the current view of the future? In realism or continuity of greater importance?

(c). If one adopts solvency as a goal, it follows that an insurance enterprise must not reduce the value of its assets by paying dividends to owners (mutual policyowners or stockholders) or increase risk by investing in more uncertain projects unless it is fairly certain that the obligations to current policyholders can be fulfilled. This principle of conservatism creates several issues.

- (i). Should policy reserves greater than the actuarial present value of future losses be held, or should actuarial present values be augmented by contingency surplus to provide an acceptable assurance of solvency?
- (ii). Should the level of conservatism be measured on a probability scale? Is there an alternative to probability?
- (iii). Should the financial margin for conservatism be explicit or implicit? For example, possible gains can be omitted or pos-

sible losses can be consistently overstated to provide an implicit margin of conservatism.

Examples abound in actuarial practice of the handling of these issues. Unearned gross premium reserves introduce an implicit margin of conservatism into reserves by ignoring that most expense cash flows occur early in policy years. Net level premium reserves ignore expense and loading cash flows as well as cash flows from withdrawals. These ignored cash flows presumably create an implicit margin of conservatism. The argument against using discounted expected cash flows in loss reserves is that by not recognizing expected investment income an implicit element of conservatism is introduced into liability valuation.

We will illustrate some of these ideas using a general life insurance example. We consider a loss function, given that the insured life has survived until time t . The random variable U is the time until death, measured from the time already lived since the policy was issued. That is $U = T - t$, where T is the random variable time until death.

$$L = B_{t+U} e^{-\int_0^U \delta(t+s) ds} - \pi \int_0^U a(t+s) e^{-\int_0^s \delta(t+y) dy} ds,$$

$$0 < t \leq U,$$

where:

B_{t+U} is the benefit paid upon death at time $t+U$ measured from issue,
 $\delta(t+s)$ the force of interest at time $t+s$ measured from issue,
 $\pi a(t+s)$ premium payment rate at time $t+s$ measured from issue.

For a collection of n such policies, the value of future losses is given by the sum of loss functions of this type. In this formulation the sum is a func-

tion of n random variables, each interpreted as a time until death. The salient point is that this summation has a distribution. In elementary actuarial theory the expected values of this distribution is reported as a reserve liability. As an alternative, the actuary can build an explicit element of conservatism into reserves by reporting the 75th or 99th percentile of the distribution as the reserve liability.

Clearly the example can be made more elaborate, and realistic, by recognizing additional possible future cash flows and more sources of randomness. The following list illustrates these elaborations.

Original Component	Elaborated Component	Effect
B_{t+u}	$B_{t+u}^{(1)}$	To allow for benefits paid in the event of decrement from causes other than death.
$\pi a(t+s)$	$\pi_G a(t+s) - e(t+s)$	To allow for a gross premium (π_G) and expenses. The expense rate is denoted by $e(t+s)$.
$\delta(t+s)$	$\hat{\delta}(t+s)$	To permit random interest rates.

Once again the salient point is that even after the elaborations, the sum of n loss functions has a distribution which can be used with a reserve (decision) principle in valuing insurance liabilities.

4. Recent Developments

There have been three themes in developments in the theory and practice of valuing life insurance liabilities in the past fifty years. The first of these themes is the improved matching of expenses and associated revenues. The various preliminary term valuation methods have the effect of altering the reserve liability and surplus of an insurance firm on valuation dates from that which would have been reported under the level premium valuation method. By the simple act of allocating more of the first year premium to expenses, than under the

net level premium method, a better match with the actual incidence of expenses is achieved. The application of General Accepted Accounting Principles (GAAP) to life insurance with its deferred acquisition expense asset account has the same goal.

The second theme has been the necessity for rather frequent revisions of valuation probability distributions. Many actuaries still active have worked with four statutory life insurance tables, American Experience, 1941 CSO, 1958 CSO, and 1980 CSO. The most interesting aspect in the construction of the CSO tables was the introduction of margins for conservatism. The discussion of these margins at times was confusing. Should margins be in reserves or surplus and should the margins be explicit or implicit? In addition, the discussion was confusing for technical reasons. One goal is to increase valuation premiums so that variations in the actual cost of insurance can be absorbed by valuation premium income. A second goal might be to introduce an element of conservatism, consistent overstatement, in reserve liabilities. A set of q_x 's that are large but relatively flat will produce large valuation premiums while a steeply increasing set of q_x 's tend to produce high reserves.

The third and dominant theme in the recent history of insurance valuation is the response to increased interest rate volatility. There has been three elements in this response.

- (a). Insurance products have been redesigned to ameliorate the adverse financial consequences of interest rate volatility. Examples of this response include variable policy loan interest rates and market value adjustments for withdrawal benefits.
- (b). More flexibility has been introduced into statutory valuation interest rate assumptions. A sequence of patchwork adjustments culminated in the 1980 Standard Valuation Law which provided for

dynamic interest rate assumptions for valuation. The maximum valuation rate for a particular class of policies became an interpolated value between a rate derived from current market interest rates and a fixed long-term rate. The interpolation weights depend on the presumed duration of the classes of policies. Short-term insurances and single premium immediate annuities are permitted to use valuation rates closer to those in the market at the time of issue. Whole life contracts use a rate closer to the fixed long-term rate.

- (c). Insurance enterprises have engaged in increasingly complicated plans to match asset and liability cash flows to manage the adverse effects of interest rate change.

Associated with these three themes in the history of valuation has been research activity. By far the most active area of research has been associated with the third theme. The main ideas in this cash flow matching exercise have been known for some years. However, estimating the response of asset and liability cash flows to interest rate changes is an empirical task that is an important component of this research activity. In the original work on immunization (duration matching), cash flows were assumed to be independent of interest rates. With guaranteed withdrawal benefits in many insurance contracts, bond call provisions and accelerated repayments permitted on mortgages, it is obvious that the assumption of independence is violated. These interest rate response functions depend on the particular assets held by an insurance company, the products sold and the financial sophistication of the company's customers. Because these response functions can differ significantly among companies, it is important that each company examine the sensitivity of its cash flows to interest rate changes.

The same fundamental forces that have driven revisions in valuation methods and forced research activity have influenced the financial markets. The new products sold by insurance companies have tended to reflect more directly investment performance. In an era of volatile performance there has been a tendency to shift investment risk to the customer. Variable life insurance and annuity policies are examples. Unbundled life insurance policies in which cash value funds are credited with interest at rates closely related to a particular index, expected costs of insurance are deducted from these funds, and flexibility is permitted in the amount of insurance and the periodic premium have come to dominate the market. Single premium deferred annuity policies in this era have been marketed mainly as tax deferred interest sensitive investment products rather than as a tool in retirement planning. Guaranteed investment contracts were developed to provide interest sensitive investment instruments for large blocks of pension assets. These products created an obligation for a high degree of coordination in the actuarial design of insurance products and investment management.

Where there is financial risk and markets are open, it is almost certain that investment and insurance contracts will evolve by which the risk can be transferred, for a price, from those who are risk averse to those who have greater capacity for risk or are less averse to it. This has happened in the investment world. A plethora of new investment instruments and markets in which they are traded have been created. Options and future contracts have been added to the kit of tools for managing investment performance risk. The fact remains that risk is transferred for a price. The resulting management issue is to compare the cost of risk transfer to that of risk management by investment diversification in conjunction with a careful study of interest sensitive cash flows.

5. Organized Research

There has been a massive response within actuarial organizations to these insurance and investment market developments. In 1976 the Committee on Valuation and Related Areas (COVARA) was created by the Society of Actuaries to organize and direct the creation of new valuation ideas in response to the new market realities. In its early years COVARA was headed by C. L. Trowbridge. In recent years it has been chaired by D. D. Cody.

The early work of COVARA is summarized in a preliminary report published in RSA, Vol. 5, No. 1 (1979). The report stressed the need for coherence between the values attached to assets and liabilities. It also proposed a classification system for risks and the associated contingency funds. C-1 denotes the risk of losses due to drops in asset values attributed to the inability of the asset to generate cash in the future. C-2 denotes the risk of losses caused by insurance payments in excess of those anticipated when premiums were set. These losses could be due to random fluctuations, risk misclassification or bad judgment in fixing premiums. C-3 denotes the risk of losses due to changes in interest rates.

At the time of the 1979 preliminary report, the C-3 risk was the dominant concern. The following development illustrate some of the main ideas in the report.

Let

$a(t)$ denotes the expected cash flow rate from assets at time t , measured from the valuation date.

$l(t)$ denotes the expected cash flow rate from liabilities (claims plus expenses minus premiums) at time t , measured from the valuation date.

$A(\delta)$ value of assets at the valuation date at force of interest δ .

$L(\delta)$ value of liabilities at the valuation date at force of interest δ .

$S(\delta) \equiv A(\delta) - L(\delta)$, surplus at valuation date.

In this simplified illustration, it is assumed that $a(t)$ and $i(t)$ are independent of δ . As discussed in Section 3, this assumption is frequently violated in practice.

We have

$$S(\delta) = \int_0^{\infty} e^{-\delta t} [a(t) - i(t)] dt$$

$$S'(\delta) = - \int_0^{\infty} t e^{-\delta t} [a(t) - i(t)] dt$$

$$S''(\delta) = \int_0^{\infty} t^2 e^{-\delta t} [a(t) - i(t)] dt.$$

Figure 1 traces three typical graphs of $S(\delta)$ where δ_1 is the minimum feasible valuation force of interest and δ_2 is the maximum feasible valuation rate.

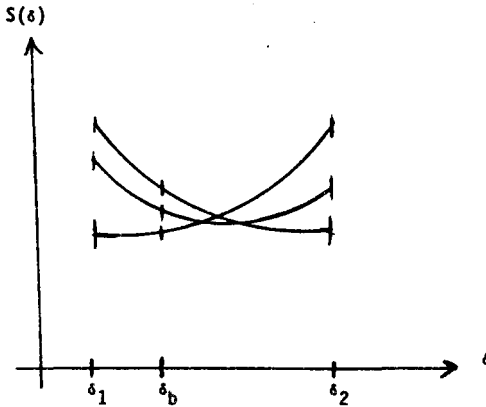


Figure 1

The valuation rate might be fixed by regulation at δ_b to produce "bookvalues." In this case the 1979 report suggests the possibility of using $A(\delta_b)$ and $L(\delta_b)$ in the balance sheet but requiring a C-3 contingency fund of

$$S(\delta_b) - \min_{\delta_1 \leq \delta \leq \delta_2} S(\delta).$$

In the years following the 1979 report, COVARA spawned four task forces. One task force was devoted to each of C-1, C-2, and C-3 and the fourth task force was assigned to work on the combination of risks issue. Reports of the C-3 task forces have appeared in RSA.

The work of these task forces has involved massive amounts of computing. In this work two themes have been clear. First, the models used have been disaggregated in the sense that many components for possible cash flow have been built into the models. Second, is the almost universal adoption of scenarios to describe economic variability that is reflected in the ultimate financial results. Figure 2 illustrates the scenario idea.

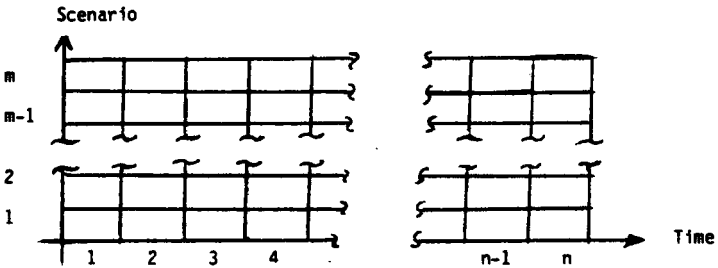


Figure 2

Each cell is filled with assumptions about the economic and demographic experience for period i , $i=1,2,\dots,n$, and scenario j , $j=1,2,\dots,m$. The model which

generates financial results at the end of each period is then run for each scenario.

6. Criticisms of Research

Most of the recent valuation research has emphasized process rather than results. Because the interest rate response functions may differ significantly for different companies, reports on the C-3 risk have stressed that results about the required level of assets to guard against insolvency are not universal. The decision to emphasize process rather than results is in accord with the new reality that valuation, even for regulatory purposes, can no longer be an application of fixed rules, formulas and assumptions. However, the decision has had some deleterious effects. Actuaries without the resources to replicate the massive simulations reported in actuarial literature are perplexed.

A partial solution to the problem of summarizing the vast output of simulations is to proceed as we would in summarizing experimental or observational data. That is we can use regression analysis to fit a function that estimates the required initial surplus using a small number of explanatory or regression variables. The use of these statistical methods also would lead naturally to the identification of the salient factors that determine required surplus. The elaborate models used and the large number of assumptions that make up each scenario obscure the fact that typically a few variables that capture the most salient features of the insurance product and the investment policy can explain most of the variability in required surplus. For example, simulation research in the case of single premium deferred annuities shows that the C-3 contingency fund is approximately a linear function of the difference between the durations of assets and liability cash flows. In the notation of Section 5 this difference is $A^*(\delta)/A(\delta) - L^*(\delta)/L(\delta)$. The task of summarizing a man-made simulation universe can be done using the same statistical tools as are used in summarizing observations of the natural world.

A second criticism of recent valuation research is that in the formulation of scenarios the discipline of data has usually been avoided. There are several advantages of doing some data analyses. For example, a fitted time series model and a generator of normal independent variates, with variance equal to the error variance of the model, can be used in stochastic simulation without the task of writing scenarios to subjectively evaluate possible future variability. The avoidance of data analyses leads to the implicit assumption that all scenarios, even those without historical precedent, are equally likely. The slogan that the future will not duplicate the past should not be used as an excuse for avoiding an analysis of data.

The economic volatility of recent years has provided valuable data on the interrelationships among economic variables and this information should not be wasted. This observation leads to a third criticism of some valuation research, which is the unwarranted use of independence assumptions in models. Economic variables and company results, when viewed as a time series, are seldom independent. Likewise economic variables and company results for various lines of business are usually intertemporally correlated. If in fact these variables were mutually independent, the management of an insurance enterprise would be much easier because diversification of investments and insurance products would solve most risk problems. Independence is a convenient assumption but for the problem of setting C-1, C-2 and C-3 contingency funds and providing for combinations of these risks it is frequently unwarranted. A simple model incorporating autocorrelations and intertemporal correlations may capture more of the reality of the situation than a disaggregated model with unsupported independence assumptions.

Finally there has been an understandable but regrettable confusion as to whether models used to estimate C-1, C-2 and C-3 contingency funds are marginal

or conditional models. The difference can be subtle, but it is important. A marginal model considers deviations from expected results from a particular source, when deviations from all other sources are averaged over all possible outcomes. With conditional models deviations from other sources are assumed fixed, usually at zero. If one uses a multinormal model, or if one is willing to use it as an approximation, one can select either the marginal or conditional mode of thought and pass from one to the other rather mechanically. One would conjecture that most models for generating contingency funds for a single risk are conditional in nature. One of the reasons for classifying deviations from expected results by cause is to simplify model building by permitting the actuary to concentrate on one type of threat to solvency at a time. In combining separate contingency funds to provide an overall estimate of needed contingency funds very different calculations are needed depending on whether the component contingency funds are derived from conditional or marginal models.

7. Advice to Academic Actuaries

This conference has a splendid mixture of actuaries employed by universities and those employed by insurance companies and consulting firms. The following remarks are appropriate to all actuaries, independent of their employer, but I hope that they will be particularly useful to actuaries involved directly in education.

- a. We need to teach the methods for eliciting subjective probability distributions and attaining consistency through the process. It seems that any alternative to using a probability scale to measure the degree of conservatism in a system of reserves and contingency funds is too vague to be useful or is equivalent to probability. Given the extensive use of the scenario approach in connection with a very complex computer model of the insur-

ance system under study, the use of a probability standard in fixing the size of required surplus can be achieved only if a probability distribution is defined on the set of scenarios. Without disciplined thought, it is natural to slip into the practice of assigning equal probability to each scenario. Part of the responsibility of the actuary who uses the scenario approach is to formulate a distribution that summarizes all available information.

- b. In Section 6 the need to summarize simulation results was discussed. This means that regression needs to be taught as a means of summarizing simulation results as well as real world observations.
- c. The principle of parsimony must be adopted because of the diversity of companies that will use the new valuation ideas. Although the models must be kept simple, the parameters of the models must be estimated for each company. In this volatile era there are no universal rules of thumb. In the combination of risks problem parsimony must once again be the guiding principle. Means, variances and covariances, first and second moments, can usually be estimated from data. If we can get actuarial practice to incorporate first and second moments intelligently in estimating the size of contingency funds, actuarial researchers and educators will have done well.
- d. The three preceding items of advice can be summarized by stating that we face an educational crisis. As actuaries assume greater responsibility for valuation there must be a commensurate increase in actuarial knowledge. This knowledge must be

communicated to all actuaries with the broader responsibility.

Actuarial educators must help with this task.

8. Fundamentals, Principles and Standards

The 1980 Standard Valuation Law probably marked the highwater market in statutory complexity. The legislative process and its statutory output is simply not flexible enough to respond to the stimulus of a volatile economy, rapidly changing insurance products and a plethora of new investment instruments. The alternative is to turn to a body of educated and responsible actuaries, committed to the public interest, to value insurance companies. Despite considerable conversation on the subject, the responsibility for valuation in the United States has not yet passed to a body of actuarial professionals. Experience in the United Kingdom and Canada can help guide this transfer.

An NAIC Committee has adopted a set of guidelines for the process. The goal is to place on the valuation actuary the responsibility to provide assurance that the assets are adequate to mature existing policies and that the valuation is consistent among lines of business. The traditional goals of regulation, solvency and equity among classes of policyholders, are evident in these goals.

Conditions beyond the control of actuaries are conspiring to force the replacement of rigid statutes with valuation standards, built on principles that are grounded on a firm intellectual foundation. Are these standards, principles and fundamentals in place? The answer is no. The distinctions among compliance requirements, standards, principles and foundations have not yet won acceptance in the actuarial profession. Even in the profession's education and examination system, the distinction between the mastery of fundamentals needed to enter the profession and the mastery of current compliance requirements needed to reason in practice has not been made.

The Interim Actuarial Standards Board was created in 1985 by the American Academy of Actuaries. It was charged with systematizing the process of developing and promulgating standards of actuarial practice. The Society of Actuaries and the Casualty Actuarial Society have both held discussions on ventures in the articulation of principles. If the structure of standards and compliance requirements is to be coherent, the task of identifying principle and their associated foundations must be undertaken. In theology there is scripture, interpretations and commentaries. In taxation there is code, regulations and interpretations in a structured form. If the valuation actuary idea is to succeed, it will take devotion to the public interest by those who serve in that role and they must have the intellectual tools to carry out their assignment. This conference will contribute to the sharpening of the tools.