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Dear Editor cont'd

uses the term "risk surplus" rather than "required surplus" and defines it as being just sufficient to keep the probability of ultimate ruin sufficiently low, with 1% being a common standard. We can apply this definition to Tan's example to see that the riskiness of the two products may be surprisingly similar.

Using Tan's definition of R and defining F(R) as the probability distribution function:

E[R] = 110 for Product A
E[R] = 115 for Product B
and
F(−5) = .01 for Product A
F(−10) = .01 for Product B

If the probability density functions of financial results for the two products have the same shape, differing only by expected value and standard deviation (for example, if both are normal), then the risk of adverse deviations from Product B is identical to Product A scaled up by 125/115 = 1.087 (since the ratio of standard deviations equals the ratio of differences between expected values and one-percentile values.) This hardly justifies a risk premium (that is a demanded ROE less the risk-free rate) for Product B of approximately twice that of Product A.

With a little reflection, it should be clear that if the risk premium inherent in the expected ROE is there to compensate for the risk of adverse deviation from expected results, and if required surplus is defined as above, then the theoretically correct standard for ROE can differ materially between products only if the p.d.f.'s of financial results differ materially in kind. We must actually believe that the risk profiles differ in shape, not just in magnitude. In such a situation, several values of the respective p.d.f.'s would be compared in order to judgmentally arrive at appropriate overall ratios of risk to attribute to the products. One possible example of this might be products with substantial AIDS risks being compared to products thought to have normally distributed p.d.f.'s. In many cases, once all factors are considered, the estimation of required surplus will already be speculative, and further speculation on differences in shape of p.d.f.'s will not really add to the process.

David L. Creswell

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After reading Joseph Tan's article on varying the ROE target by profit center depending on risk, I found myself probably agreeing with the conclusion, but using a different structure for the analysis. My first point of departure is to differentiate between two types of risk. The first risk relates to the volatility of returns on the product (return risk). The second risk is that an "intolerable" event occurs (intolerance risk). The proposed solution to the intolerance risk is to invest sufficiently in riskless assets to shift the expected return distribution of the product plus riskless asset to the right and eliminate (or at least acceptably so) the intolerance risk.

Let us analyze this situation using the Capital Asset Pricing Model (CAPM). Under CAPM, Products A and B can be considered as two risky securities, and RS (the required surplus to eliminate intolerance risk) is invested in risk-free assets. In Tan's example, the expected return on A (R_A) was 10%, on B (R_B) was 15% and on RS 5%. Under CAPM, this would translate to a riskless return (R_F) of 5% and $\beta_B = 2.0\ \beta_A$. For example, assume the expected return on the market (R_M) also equalled 10%, so $\beta_A = 1$ and $\beta_B = 2.0$. The security market line under CAPM in this example is

$$E(R_i) = R_F + [E(R_m) - R_F]\beta_i$$

for Product A

$$E(R_A) = 5\% + [10\% - 5\%]1 = 10\%$$

and for Product B

$$E(R_B) = 5\% + [10\% - 5\%]2.0 = 15\%.$$

Under CAPM, each investor will assess the return risk in the same manner and arrive at the same required expected return from Products A and B as every other investor. However, not all people tolerate risk equally well for a variety of reasons, including statutory requirements. For that reason, in the example, a pure portfolio of only Product A or Product B is unacceptable. The products need to be mixed with some amount of riskless asset to eliminate the intolerable risk and allow the resulting expected return distribution to become acceptable. The expected return of the mix of product and riskless asset is a weighted average of their individual expected returns.

Let w be the proportion of the total investment that is in riskless securities in order to eliminate the intolerance risk. If all portfolios of product and riskless assets must have the same ROE, then for all products i,

$$wR_F + (1 - w)R_i = \text{ROE}$$

or

$$w = \frac{R_i - \text{ROE}}{R_i - R_F}$$

Under CAPM, this also leads to the result that if ROE is the same for each portfolio, β for each portfolio is the same. The effect of the addition of riskless assets is to dilute the β of the product to the level necessary to attain the corporate ROE.

This holds true only if the intolerable risk and return risk are the same. Recall that under CAPM, return risk is related to undiversifiable portfolio return volatility. As described in Tan's article, the intolerable risk is not related directly to that volatility, but also includes the need to attain minimum levels of acceptable return. I will leave it up to those more familiar with reserve setting to ascertain whether these risks are, in fact, one and the same or are different as I suspect. If they are different, then varying ROE target by profit center is appropriate.

James M. Jackson

CIA offers reports on AIDS

The Canadian Institute of Actuaries (CIA) recently produced four reports on AIDS. These are:

- Memorandum covering AIDS from the Chairman of the Committee on Life Insurance Company Financial Reporting
- 1988 Guidance Notes for Valuation Actuaries, Report of the Subcommittee on Valuation, Task Force on AIDS
- First Report of the Subcommittee on Modeling, Task Force on AIDS
- Second Report of the Subcommittee on Modeling, Task Force on AIDS, An Analysis of USA Data

Copies of the Memorandum can be obtained free from the CIA. Copies of the other three reports can be obtained for $10.00 each ($US or $CDN). The address of the Canadian Institute of Actuaries is Suite 405, 360 Albert Street, Ottawa, Ontario, K1R 7X7, Canada.