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ASSUMPTIONS IN PENSION PLAN VALUATIONS

Arnold F. Shapiro

INTRODUCTION

Most defined benefit pension plans require an actuarial valuation at least once every three years, so valuations are a very common procedure. Nonetheless, many of the criteria of a valuation are only vaguely defined, much less quantified. The IRS, for example, requires the actuaries of all private plans to certify that

To the best of my knowledge, the information supplied in the schedule and on the accompanying statement, if any, is complete and accurate, and in my opinion the assumptions used in the aggregate (a) are reasonably related to the experience of the plan and to reasonable expectations, and (b) represent my best estimate of anticipated experience under the plan.¹

No guidance is given as to how these requirements are to be met prospectively, although there are serious potential consequences for noncompliance in retrospect.

This apparent lack of specificity creates problems. The first problem relates to the traditional actuarial mandate of substituting facts for appearances. This is an untenable mandate with respect to criteria which are either vaguely defined or not defined at all.

The second concern relates to the area of communication and consensus. If

¹Form 5500, Schedule B

criteria are defined too broadly, the danger exists that they include mutually exclusive subcategories from the point of view of obtaining a consensus on the overall criteria. The term "best estimate," for example, has been envisioned by some as involving the minimum absolute deviation while others feel that it should provide for a conservative element. The problem here is that if both of these criterion are regarded as being embodied in the notion of best estimate, there is unlikely to be a consensus.

The final consideration has to do with credibility. The credibility of the pension actuary is in jeopardy when the public senses that underlying parameters of the valuation are ill-defined, ambiguous, or arbitrary.

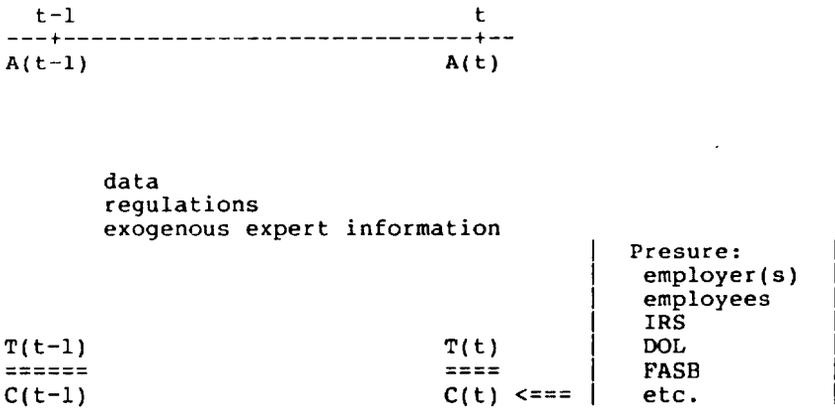
The purpose of this paper is to set the stage for further discussion and research into actuarial assumptions in pension plan valuations by quantifying some of the criteria which underly the choice of these assumptions. The development is not definitive and represents only a preliminary stage of inquiry.

THE VALUATION PROCESS

The valuation process is depicted in Figure 1. In year $t-1$, last year, there are assets, $A(t-1)$ and a population, $T(t-1)$. Using that information, the actuary decided on the contribution, $C(t-1)$. In the following year, year t , there are new assets $A(t)$, and population, $T(t)$. During the course of the year, data has been obtained and various regulations have come out. There are a number of self-interest groups applying pressure, including the employer, the employees, the IRS,²

²The service has begun to formalize its requirement and has "developed and will be implementing guidelines for determining whether actuarial assumptions are reasonable in the aggregate." *BNA Pension Reporter*, Vol. 11, No. 35 (August 27, 1984), p. 1110.

**FIGURE 1
THE VALUATION PROCESS**



the Department of Labor, and accountants. Additionally, there is exogenous expert information that has to be co-ordinated with the valuation. The purpose of the valuation is to find the appropriate current year's contribution, C(t).

ACTUARIAL GAINS AND LOSSES

A concept which is critical to a quantification of actuarial assumptions is that of actuarial gains and losses.

The gain at time t, resulting from the j-th assumption is

$$G(t,j) = A'(t,j) - E\{A(t,j)\},$$

where A'(t,j) is the actual assets at time t, resulting from the j-th assumption, and E{A(t,j)} is the expected assets at that time. If actual assets exceed expected assets there is a gain; a loss occurs if the converse is true.

SUMMARY OF CRITERIA

The actuarial assumptions of a valuation are either demographic or economic.

The demographic assumptions are related to the pension population and encompass mortality, termination, disability, retirement, distribution by sex, age, employment status, and so on. Economic assumptions are those which impact investment returns and salary progression.

A preliminary list of the major criteria associated with the assumptions in a pension plan include conservativeness, consistency, best estimate, prudence, precision and flexibility. Each of these is discussed in detail below.

CONSERVATIVENESS

The first consideration is conservativeness. This has been a long standing major consideration for actuaries. An assumption is conservative if it tends to produce actuarial gains.

Thus, on an individual basis, the j -th assumption used at times s , $a(s,j)$, is conservative if it generates a gain at time t , that is,

$$a(s,j) \Rightarrow G(t,j) > 0, \quad s < t.$$

This is a prospective consideration.

It is important to note that the regulations and the IRS does not recognize the need for a contingency reserve, stressing, instead, tax deductible contributions. However, contributions that fall within deductibility ranges may not be sufficient to guarantee plan solvency. Many actuaries have circumvented this problem by using "conservative" assumptions, where, as often as not, the degree of conservativeness has not been quantified. A direct approach would be to revise the tax code³ to allow for a contingency reserve.

³IRC Sections 404 and 412.

Conserativeness in the Aggregate

Conservativeness in the aggregate is simply an extension of conservativeness. Assumptions are conservative in the aggregate if they tend to produce actuarial gains, that is,

$$(a(s,j), \text{ all } j) \Rightarrow G(t) > 0,$$

where $G(t)$ is the union of the gain from all sources. The distinguishing feature is that it is possible for some assumptions to produce gains while others produce losses. Here, the concern is the net effect.

The amount of gain which will result from assumptions which are conservative in the aggregate is quite speculative. The direction is known but not the magnitude. There is a need to investigate the extent of this conservativeness and the credibility of any estimates thereof.

RETROSPECTIVE CONSISTENCY

An assumption has retrospective consistency if it is reasonably related to the experience of the plan. The concern here is that current assumptions should not be significantly different from past assumptions. Thus, if $a'(t,j)$ represents the plan experience to date, with respect to assumption j , an assumption has retrospective consistency if

$$| a(t,j) - a'(t,j) | < e(t,j),$$

where $e(t,j)$ is an acceptable tolerance level. Here, $a'(t,j)$ may be some purely objective data point or trend or a subjective hybrid of past experience and current wisdom.

An example of a purely objective trend would be an actuarial value of assets

defined as an n-year moving average of the market value of plan assets.⁴ A subjective hybrid approach might be used in the development of a salary scale or decrement rates.

Retrospective Consistency in the Aggregate

Assumptions have retrospective consistency in the aggregate "[if they] are reasonably related to the experience of the plan." Thus, the assumptions are retrospective consistency in the aggregate if

$$(a(s,j), \text{all } j) \Rightarrow |G(t)| > e(t), s < t$$

then

$$(a(t,j), \text{all } j) \sim (a(s,j) + \Delta a(s,j), \text{all } j)$$

such that

$$(a(t,j), \text{all } j) \Rightarrow E|G(u)| \leq e(u), s < t < u.$$

That is, if experience indicates inordinate gains or losses, assumptions will be revised to reduce expected future deviations.

It should be noted that the retrospective consistency criterion takes a different form in the aggregate than it did on the individual basis. In the latter, each current assumed value of a parameter is compared with the past experience of that parameter. The former, on the other hand, concerns itself not with the relative value of the assumptions but, rather, with the overall impact of assumptions on gains or losses.

PROSPECTIVE CONSISTENCY

An assumption has prospective consistency if it is "reasonably related ... to

⁴In practice, this would be subject to the regulatory requirement that the actuarial value resides within a 20 percent corridor of current market value.

reasonable expectations." This criterion embodies consideration of both the short-term elements and long-run elements of pension plans, and actuaries have traditionally struggled to explain the implications of the latter.

The term "reasonable expectation," of course, could mean various things. However, if it is taken to mean current wisdom with respect to expectations, in the sense that it is based on authoritative information, then an assumption has prospective consistency if its assumed value at some future time t is

$$L[a(t,j)] < a(t,j) < U[a(t,j)],$$

where $[L,U]$ is the prospective range of expectations for this assumption. The bounds may or may not be symmetric.

Prospective consistency can be an extremely dynamic consideration. This is particularly true for assumptions that have an inflation component, like interest rates and salary rates, or assumptions that react to the economy or changing legislation, like termination and retirement rates. Yet, even estimates of a new or changing environment are expected to be prospectively consistent in some senses. This, of course, presents a dilemma, since the value of a parameter may not converge over time.

Prospective Consistency in the Aggregate

Assumptions have prospective consistency in the aggregate at time s , if the gains they produce at time t , $s < t$, are bounded by a tolerance level based on both the most pessimistic set of reasonable expectations and the most optimistic set. Thus, prospective consistency is obtained in the aggregate if

$$(a(s,j), \text{all } j) \Rightarrow L[G(t)] \leq G(t) \leq U[G(t)].$$

where $[L,U]$ is the prospective range of reasonable expectations for gains.

The predetermination of future gains and losses is a problem, of course. Because of this, deterministic scenarios are often used, generally based on

optimistic, pessimistic, and some ordinary projections.

BEST ESTIMATE

An assumption is a best estimate if it represents the "best estimate of anticipated experience." Here, the term "best" refers to the actuary's best estimate, as opposed to "the" best estimate, in some collective wisdom sense.⁵ Thus, the criterion is satisfied at time s , if, for some assumption at time t , $s < t$,

$$| a(t,j) - m[a(t,j)] | < e(t,j),$$

where $m[a(t,j)]$ is the actuary's measure of best with respect to assumption j , at time t . Possible measures of best, depending on the distribution involved, might be the mean, mode, and so on.

Best Estimate in the Aggregate

Assumptions in the aggregate result in a best estimate if they are based on the best estimate of anticipated experience. Thus, this criterion is satisfied at time t , if

$$(a(t,j), \text{ all } j) \Rightarrow C'(t) = E[C(t)],$$

where $C'(t)$ is the actual contribution and $E[C(t)]$ is based on $m[a(t,j)]$. Here, the contribution is based on the best estimate if it is equivalent to the contribution derived using the best estimate.

Some have suggested⁶ that the foregoing could be a working definition and provides a vehicle for implementing best estimate notions. The approach suggested is to first do an explicit calculation to estimate the cost, and then to do

⁵It is not clear in what sense an actuary's best estimate can be wrong if the yardstick is the actuary's best estimate.

⁶See, for example, Winklevoss ([7], p. 267).

an implicit calculation. The first would entail an array of assumptions while the later would involve only selected assumptions.

An implicit notion in the foregoing discussion is that the best-estimate concept does not embody the concept of conservativeness, that is, there is no contingency charge built into the best estimate. This is consistent with the notions of Winklevoss⁷ and Anderson.⁸ It is inconsistent with Berin ([7], p. 270) however, when he asserts that best estimate does not mean above or below each assumption half the time.

PRUDENT

An assumption is prudent if it is appropriate, in the sense that it would be used by a prudent actuary in similar circumstances. This is the essence of the herd instinct, which find security in uniformity, and feels uncomfortable with the prospect of being an outlier. Thus, this criterion is satisfied at time t if

$$L[a(t,j)] < a(t,j) < U[a(t,j)],$$

where $[L,U]$ is the range of values currently used for this assumption.

Prudence is differentiated from prospective consistency in that a prudent assumption is based on what people are actually using, as opposed to what people purport to use.

The question of prudence only arises when a problem occurs, and then the

⁷*ibid.*, p. 265, suggests that "...a strict interpretation of the best-estimate requirement under ERISA calls for a given actuary, dealing with a given plan, to arrive at a cost or liability which he believes has an equal change of being over or under the true cost or liability."

⁸Anderson ([3], p. 6.2-10) remarked, "If we interpret 'best estimate' to mean that the expected value of the absolute deviation of the result from the estimate is to be minimized (and what other interpretation could there be?)..."

criterion is applied retrospectively. Examples are situations where funding problems occur and IRS audits.

Prudent in the Aggregate

Assumptions are prudent in the aggregate if the contribution they generate is appropriate, in the sense that it would be developed by a prudent actuary in similar circumstances. Thus, this criterion is satisfied at time t , if

$$L[C(t)] < C(t) < U[C(t)],$$

where $[L,U]$ is the range of prudent contributions. It should be noted that the $[L,U]$ range generally would not be known before the fact.

It is worth mentioning that the IRS is collecting data on assumptions used on the Form 5500, Schedule B, the federal reporting form for actuarial valuations. Undoubtedly this data will be used to develop notions on reasonable assumptions. So, the Service is not concerned with an actuary's knowledge but, rather, how well the actuary's assumptions fit.

PRECISION

An assumption has precision if the difference between actual experience and the assumption is minimized. Thus, if $a'(t,j)$ represents the plan's experience to date, with respect to assumption j , this assumption had precision at time s , $s < t$, if

$$|a(s,j) - a'(t,j)| < e(t,j).$$

where $e(t,j)$ is the acceptable tolerance level at time t .

In effect, the assumption made at time s is judged at some latter time t , so that precision is a retrospective criterion. The questions to be resolved are how and when measurements should take place and the size of an acceptable tolerance level.

Precision in the Aggregate

Assumptions have precision in the aggregate if the difference between actual experience and the anticipated experience is minimal. Thus, if $A'(t)$ represents the plan experience to date and $T[A(t)]$ represents the targeted experience, the plan had precision in the aggregate at time s , $s < t$, if

$$(a(s,j), \text{ all } j) \Rightarrow | A'(t) - T[A(t)] | < e(t),$$

where $e(t)$ is the acceptable tolerance level at time t . Again, the assumption made at time s are judged at some later time t , so that precision is a retrospective criterion.

FLEXIBILITY

An assumption has flexibility if the actuary will certify a plan where the assumption has been arbitrarily drawn from any value within a range for that assumption. Flexibility is commonly associated with an understanding of and tolerance for the "real world." The concern here is how flexible the assumptions of an actuary should be.

It is within the criterion of flexibility that accommodations for the funding objectives of the sponsor is resolved. In the small plan area, for example, where tax sheltering is a primary objective and lump-sum distributions at the normal retirement age are common, it is not unusual to see relatively high annuity purchase rates being used. On the other hand, in large plan situations, where payments to annuitants come directly from the trust fund, annuity purchase rates generally are based on considerably lower values.

Flexibility in the Aggregate

Assumptions are flexible in the aggregate when there is a range of acceptable contribution based on potential ranges for assumptions.

COMMENTS

The two basic considerations in a pension plan valuation are the choice of actuarial cost methods and actuarial assumptions. This paper has dealt with the latter.⁹ The concern in this regard was what actuaries are doing and why.

There are explicit criteria why particular assumptions are chosen, and these have often been discussed in the pension literature. The bibliography indicates a number of such articles. However, there also are implicit criteria, which, while seldom discussed in the literature, can dominate the valuation. The goal of this paper has been to quantify some of these criteria, and thereby provide a basis for further discussion and research.

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⁹For a discussion of actuarial cost methods see Shapiro [29].

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